Orientations of graphs

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INP Grenoble, France

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Outline 1

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Since then, the city was renamed "**The Arcs**".

Outline 2

Orientation

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root-connected, strongly connected orientation

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2-arc-connected orientation

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Orientation problems with connectivity constraints

- root-connected / k-root-connected
- strongly connected / k-arc-connected
- well-balanced
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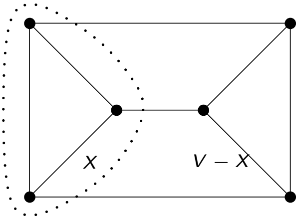
- same problems with prescribed degrees

Notations

- Given an undirected graph G,
  - $d_G(X)$ = number of edges of G entering X,
  - $i_G(X)$ = number of edges of G in X.
- Given a directed graph D,
  - $d_D^-(X)$ = number of arcs of D entering X,
  - $d_D^+(X)$ = number of arcs of D leaving X.



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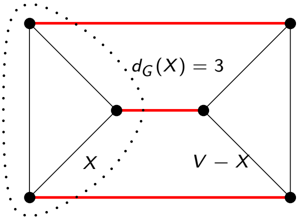

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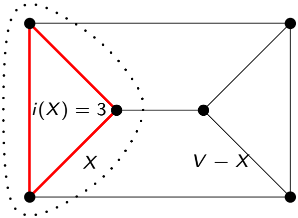

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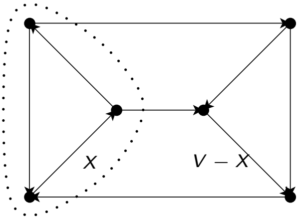

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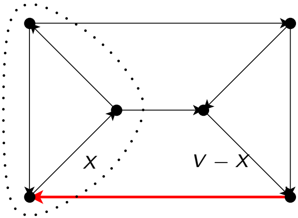

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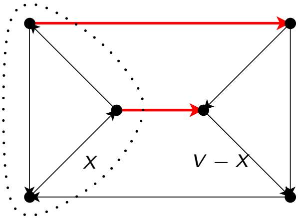

Connectivity properties 1

Definitions

- local-edge-connectivity $\lambda_G(u,v)$ = maximum number of edge-disjoint paths from u to v in G,
- local-arc-connectivity $\lambda_D(u,v)$ = maximum number of arc-disjoint paths from u to v in D.

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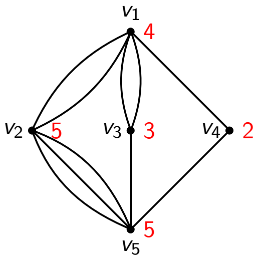

$$\lambda_G(v_i, v_j) = \min\{val(i), val(j)\}$$

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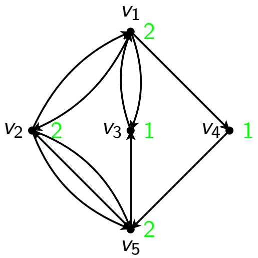

$$\lambda_{\vec{G}}(v_i, v_j) \geq \min\{val(i), val(j)\}$$

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Definitions

- k-edge-connected : $\lambda_G(u, v) \geq k \ \forall (u, v) \in V \times V$,
- root-connected : $\lambda_D(s, v) \geq 1 \ \forall v \in V - s$, s fixed,
- k-root-connected : $\lambda_D(s, v) \geq k \ \forall v \in V - s$, s fixed,
- strongly connected : $\lambda_D(u, v) \geq 1 \ \forall (u, v) \in V \times V$,
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Connectivity properties 2

Theorem (Menger)

- local-edge-connectivity $\lambda_G(u,v)$ = maximum number of edge-disjoint paths from u to v in G $= \min\{d_G(X) : v \in X, u \notin X\}$,
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Theorem (Menger)

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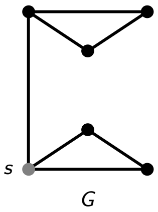
Exercise

Given an undirected graph G and a vertex s of G,

- there exists a root-connected orientation of G at s $\Longleftrightarrow$
- there exists an orientation of G containing an s-arborescence $\Longleftrightarrow$
- there exists a spanning tree of G $\Longleftrightarrow$
- G is connected.



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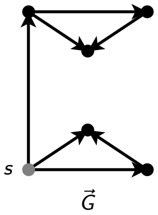

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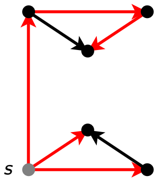

$\vec{G}$

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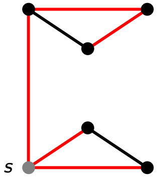

G

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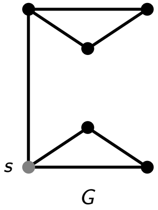

k-root-connected orientation

Theorem (Frank'78)

Given an undirected graph G, a vertex s of G and an integer $k \geq 1$,

- *there exists $\vec{G}$ of G that is k-root-connected at s* $\Longleftrightarrow$ *(Menger)*
- *there exists $\vec{G}$ of G with $d^-_{\vec{G}}(X) \geq k \ \forall X \subset V - s$* $\Longleftrightarrow$ *(Edmonds)*
- *there exists $\vec{G}$ of G containing k arc-disjoint s-arborescences* $\Longleftrightarrow$
- *there exist k edge-disjoint spanning trees of G* $\Longleftrightarrow$ *(Nash-Williams)*
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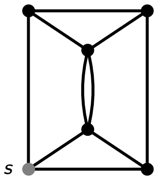

$G, k = 2$

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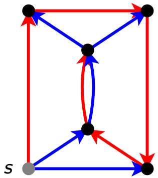

2-root-conn. $\vec{G}$

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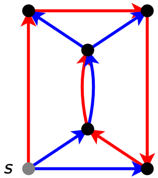

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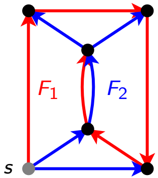

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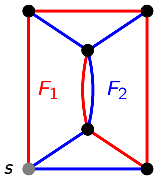
- *there exists $\vec{G}$ of G containing k arc-disjoint s-arborescences* $\iff$
- *there exist k edge-disjoint spanning trees of G* $\iff$ *(Nash-Williams)*
- *for every partition $\mathcal{P}$ of V,*



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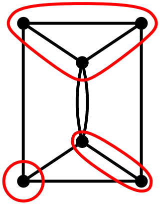

$\mathcal{P}$

k-root-connected orientation

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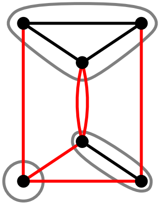

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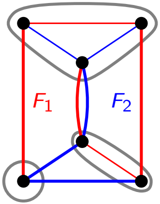

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Given an undirected graph G,

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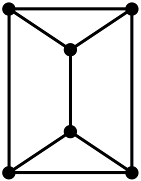

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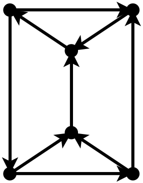

strongly conn. $\vec{G}$

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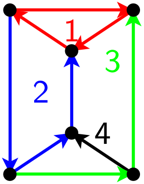

dir. ear-dec.

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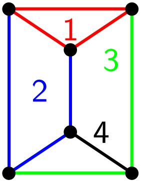

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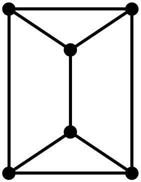

2-e-c G

Theorem (Nash-Williams'60)(weak orientation)

Given an undirected graph G,

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k-arc-connected orientation

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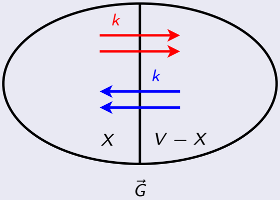

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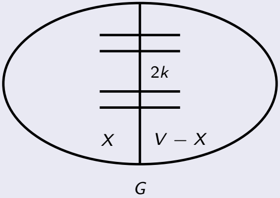

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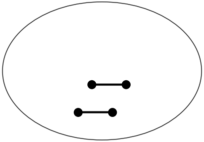

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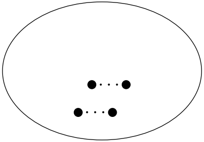

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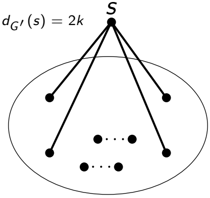

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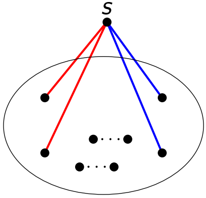

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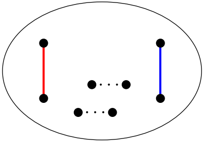

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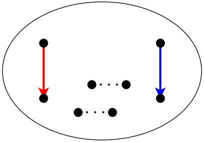

$\vec{G}''$ k-a-c.

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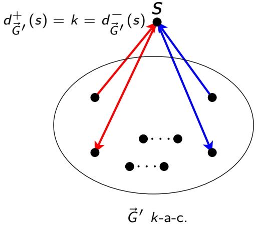

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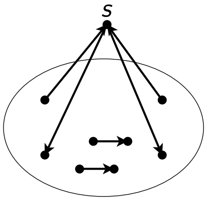
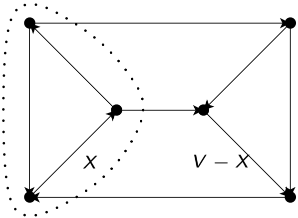


$\vec{G}$ k-a-c.

In-degree vector

- If m is the in-degree vector of D $(m(v) = d_D^-(v)\ \forall v \in V)$, then
$$m(X) - i_D(X) = d_D^-(X).$$
- The in-degree vector characterizes the in-degree function.
- The in-degree function characterizes the connectivity properties.



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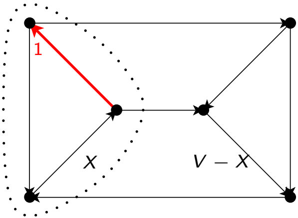

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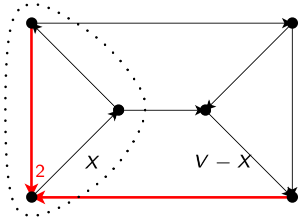

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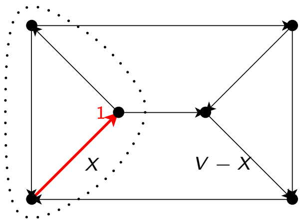

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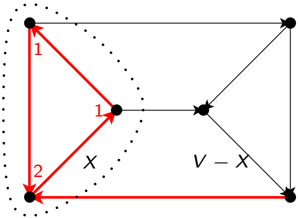

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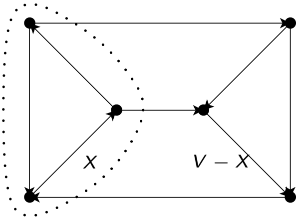

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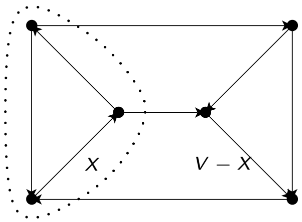

Theorem (Hakimi'65)

Given an undirected graph $G = (V, E)$ and a vector $m\colon V \rightarrow \mathbb{Z}_+$,

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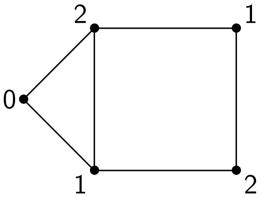

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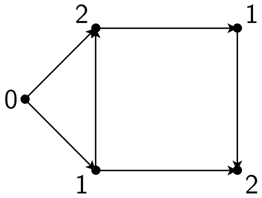

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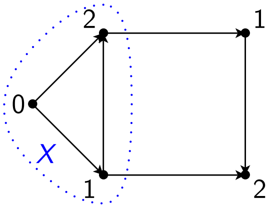

Applications of in-degree constrained orientations

Applications

- Eulerian orientation of an undirected graph (Euler),
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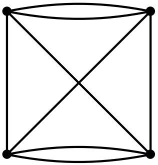

G

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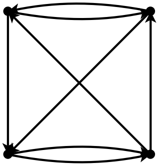

G

$m(v) = \frac{d_G(v)}{2} \ \forall v \in V$

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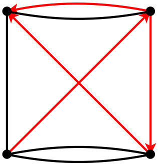

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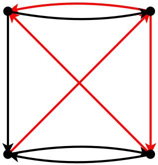

$\vec{G} = (V, \vec{E} \cup A)$

$$m(v) = \frac{d_E(v) + d_A^+(v) + d_A^-(v)}{2} - d_A^-(v) \ \forall v \in V$$

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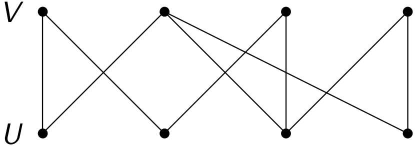

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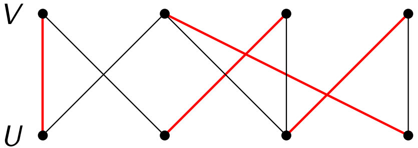

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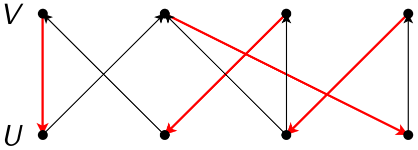

$$G = (U \cup V; E)$$
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Applications of in-degree constrained orientations

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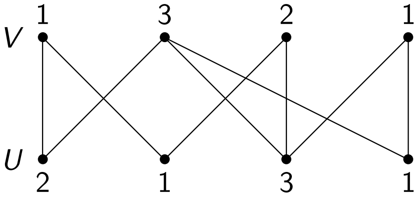

$G = (U \cup V; E), f : V \to \mathbb{Z}_+$

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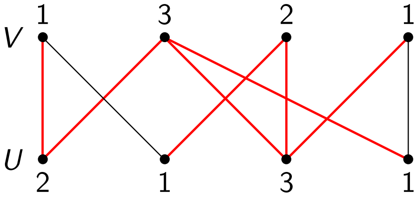

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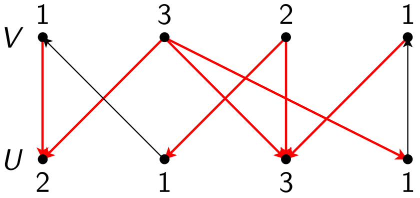

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$m(u) = f(u) \quad \forall u \in U$

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Theorem (Frank'80)

Given an undirected graph $G = (V, E)$ *and a vector* m*:* $V \to \mathbb{Z}_+$*, there exists an orientation* $\vec{G}$ *of* G *with in-degree vector* m *that is* $\Longleftrightarrow$

- *root-connected :* $d^-_{\vec{G}}(X) \geq 1 \ \forall X \subset V - s,\ \ s$ *fixed,*
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Eulerian orientation

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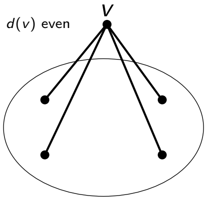

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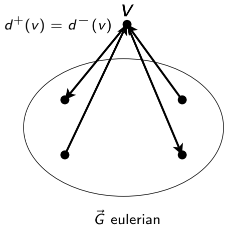

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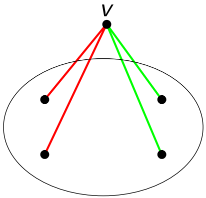

G eulerian, $\mathcal{P}_v$ partition

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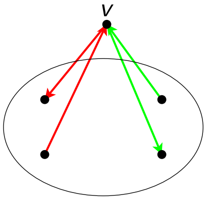

$\vec{G}$ eulerian and compatible with $\mathcal{P}_v$

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An orientation $\vec{G}$ of an Eulerian graph G is Eulerian $\iff$

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An orientation $\vec{G}$ of a graph G is best-balanced if

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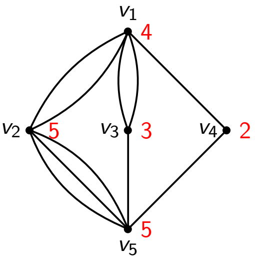

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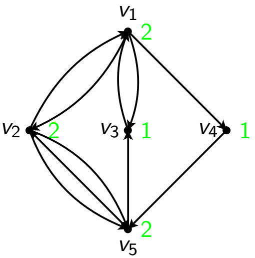

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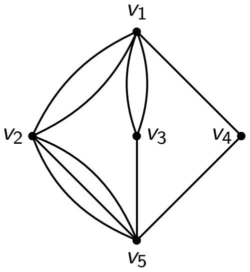

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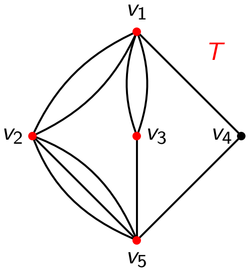

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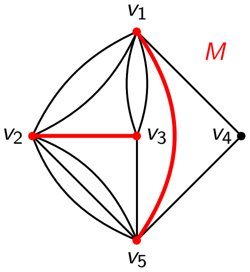

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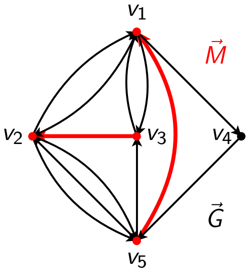

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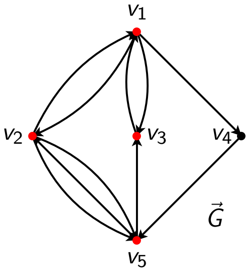

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Theorem (Nash-Williams'60)(pairing theorem)

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Generalizations

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For every subgraph H of a graph G, there exists an orientation $\vec{G}$ of G : $\vec{G}$ and $\vec{G}(H)$ are best-balanced orientations of G and H.

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Easy by pairing theorem.

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4. *Minimum Cost k-Arc-Connected Orientation Problem can be solved in polynomial time.*

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4. *Minimum Cost Well-Balanced Orientation Problem is NP-complete.*

k-connected graphs

Definition

A directed graph $D = (V, A)$ with $|V| > k$ is k-connected $\Longleftrightarrow$ $D - X$ is strongly connected for all $X \subset V$ with $|X| = k - 1$

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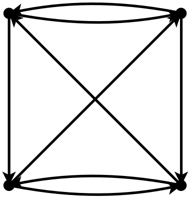

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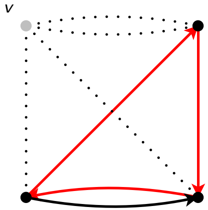

$D - v$ strongly connected $\forall v$

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Given an undirected graph $G = (V, E)$ with $|V| > k$,

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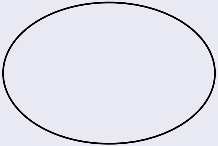

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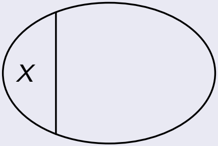

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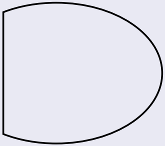

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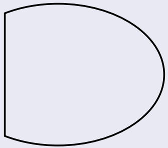

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Theorem (Berg-Jordán'06)

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A generalization

Theorem (Berg-Jordán'06)

Given an Eulerian graph $G = (V, E)$ *with* $|V| > 2$,

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Given an Eulerian graph $G = (V, E)$ with $|V| > k + 1$,

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About the proof :

- very short, applying pairing theorem,
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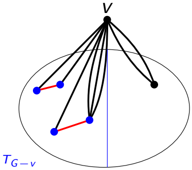
Proof

sufficiency :

- Let M_v be a NW-pairing for $G - v$ for each $v \in V$.
- M_v provides pairs of edges incident to v,
- Let $\vec{G}$ be an Eulerian orientation of G compatible with each $\mathcal{P}_v$.
- $\vec{G}$ provides an orientation $\vec{M}_v$ of M_v.
- $\vec{G} - v + \vec{M}_v$ is Eulerian,



M_v NW-pairing of $G - v$

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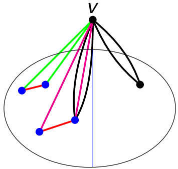

Proof

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- Let M_v be a NW-pairing for $G - v$ for each $v \in V$.
- M_v provides pairs of edges incident to v, the remaining edges has a natural partition into pairs, let $\mathcal{P}_v$ be the partition obtained.
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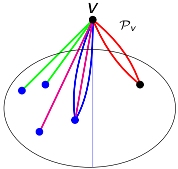

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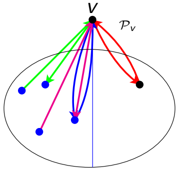

$\vec{G}$ eulerian compatible with $\mathcal{P}_v$

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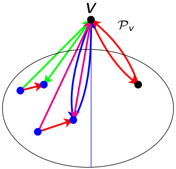

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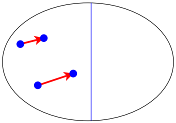

$\vec{G} - v + M_v$ eulerian

Proof

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- Let M_v be a NW-pairing for $G - v$ for each $v \in V$.
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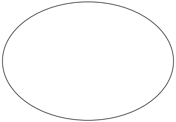
- Let $\vec{G}$ be an Eulerian orientation of G compatible with each $\mathcal{P}_v$.
- $\vec{G}$ provides an orientation $\vec{M}_v$ of M_v.
- $\vec{G} - v + \vec{M}_v$ is Eulerian, ($G - v$ is $2k$-edge-connected and M_v is a NW-pairing for $G - v$) so $\vec{G} - v$ is k-arc-connected for each $v \in V$.



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Every 18-connected graph has a 2-connected orientation, that is $f(2) \leq 18$.

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Frank's conjecture would imply that $f(2) \leq 4$.

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Theorem (Cheriyan-Durand de Gevigney-Szigeti'11)

Every $(12k+2)$*-connected graph has an orientation* $\vec{G}$ *such that* $\vec{G} - v$ *is* k*-arc-connected* $\forall\ v \in V$.

Thank you for your attention !