ON THE LOCAL SPLITTING THEOREM

Z. SZIGETI

UNIVERSITY PARIS 6
PARIS
APPLICATIONS

AIMS

1. To present two extensions of Mader's theorem on splitting off preserving local edge-connectivities.

2. To explain that splitting off results are the tools for solving edge-connectivity augmentation problems.

3. To show applications:
   - Network Design
   - Statics
APPLICATION: NETWORK DESIGN

TELEPHONE NETWORK:

TELEPHONE CENTER = VERTEX
CONNECTION = EDGE
RELIABILITY = EDGE-CONNECTIVITY
AUGMENTATION OF RELIABILITY = EDGE-CONNECTIVITY AUGMENTATION

EDGE-CONNECTIVITY

GLOBAL

LOCAL
GLOBAL E-C AUGMENTATION

GIVEN \cdot GRAPH G

\cdot REQUIREMENT \quad 2 \leq k \in \mathbb{Z}_+

FIND A SET \(H\) OF NEW EDGES OF MIN. CARD.

s.t. \quad G+H \text{ IS } k-E-C.

LOCAL E-C AUGMENTATION

GIVEN \cdot GRAPH G

\cdot REQUIREMENT FUNCTION \(\gamma(u, v) \geq 2\)

FIND A SET \(H\) OF NEW EDGES OF MIN. CARD.

s.t. \quad \lambda_{G+H}(u, v) \geq \gamma(u, v) \quad \forall u, v \in V.
THEOREM: GLOBAL SPLITTING

- $G = (V + s, E)$ GRAPH
- $d_G(s)$ EVEN
- $G \text{ R-E-C IN } V \quad (k \geq 2)$

(LOVÁSZ) $\exists k$-ADMISSIBLE PAIR AT $s$.

(LOVÁSZ) $\forall s \in E, \exists s \in E: \{s, s'\}$ $k$-ADMISSIBLE.

(BANG-JENSEN, GABOW, JORDAN,Sz.) $\forall s \in E$

$$|\{s \in E: \{s, s'\} \text{ R-ADMISSIBLE}\}| \geq \begin{cases} \frac{d(s)}{2} \quad &\text{if } k \text{ EVEN} \\ \frac{d(s)}{2} - 1 \quad &\text{if } k \text{ ODD} \end{cases}$$
ALGORITHM (FRANK)

$G = (V, E)$

EXTENSION

$G'_{R-E-C} \in V$

COMPLETE SPLITTING OFF (LOVÁSZ)

$G''_{R-E-C}$

GLOBAL E-C AUGMENTATION
GLOBAL E-C AUGMENTATION IN BIPARTITE GRAPHS

Given • Bipartite graph G,

- Requirement \( 2 \leq k \in \mathbb{Z}_+ \)

Find a set \( H \) of new edges of min. card.

s.t. \( G + H \) is • Bipartite,

- \( k \)-E-C.

LOCAL E-C AUGMENTATION IN BIPARTITE GRAPHS

Given • Bipartite graph G,

- Requirement function \( \tau(u,v) \geq 2 \)

Find a set \( H \) of new edges of min. card.

s.t. • \( G + H \) is Bipartite

- \( \Lambda_{G+H}(u,v) \geq \tau(u,v) \quad \forall u,v \in V \)
THEOREM: LOCAL SPLITTING

\cdot G = (V + s, E) GRAPH
\cdot d_G(s) \neq 3
\cdot G 2-E-C

(MADER) \exists \lambda-ADMISSIBLE PAIR AT s.

(FRANK) AT MOST ONE EDGE INCIDENT TO s
BELONGS TO NO \lambda-ADMISSIBLE PAIR AT s.

(S2.) \exists s \in E : |\{s \in E : \{s, u, v\} \lambda-ADMISSIBLE\}| \geq \left\lfloor \frac{d(s)}{3} \right\rfloor

IF d(s) \geq 4

(S2.) CHARACTERIZATION WHEN AN EDGE INCIDENT TO s
BELONGS TO NO \lambda-ADMISSIBLE PAIR AT s.

\begin{align*}
d(s) &= 3l + 1 \\
\lambda(u,v) &= 2l + 1 \\
d(x_i) &= 2l + 2
\end{align*}
LOCAL E-C AUGMENTATION (FRANK)

GLOBAL E-C AUGMENTATION (WATAAIBE NAKAMURA)

LOCAL E-C AUGMENTATION BIPARTITE GRAPHS

GLOBAL E-C AUGMENTATION BIPARTITE GRAPHS (B-G-J-S^2)

OPEN PROBLEM