On The Local Splitting Theorem

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AIMS

(1) TO PRESENT TWO EXTENSIONS OF MADER'S THEOREM ON SPLITTING OFF PRESERVING LOCAL EDGE-CONNECTIVITIES.

(2) TO EXPLAIN THAT SPLITTING OFF RESULTS ARE THE TOOLS FOR SOLVING EDGE-CONNECTIVITY AUGMENTATION PROBLEMS

(3) TO SHOW APPLICATIONS :

- NETWORK DESIGN
- STATICS

APPLICATION: NETWORK DESIGN

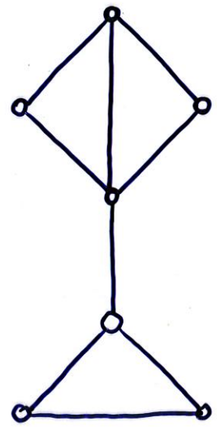
TELEPHONE NETWORK:

TELEPHONE CENTER = VERTEX

CONNECTION = EDGE

RELIABILITY = EDGE-CONNECTIVITY

AUGMENTATION OF RELIABILITY = EDGE-CONNECTIVITY AUGMENTATION



EDGE-CONNECTIVITY

GLOBAL

LOCAL

GLOBAL E-C AUGMENTATION

GIVEN
- GRAPH G
- REQUIREMENT $2 \leq k \in \mathbb{Z}_+$

FIND A SET H OF NEW EDGES OF MIN. CARD.

S.T. $G+H$ IS k-E-C.

LOCAL E-C AUGMENTATION

GIVEN
- GRAPH G
- REQUIREMENT FUNCTION $r(u,v) \geq 2$

FIND A SET H OF NEW EDGES OF MIN. CARD.

S.T $\lambda_{G+H}(u,v) \geq r(u,v) \quad \forall u,v \in V.$

THEOREM: GLOBAL SPLITTING

- $G=(V+s,E)$ GRAPH
- $d_G(s)$ EVEN
- G k-E-C IN V ($k \geq 2$)

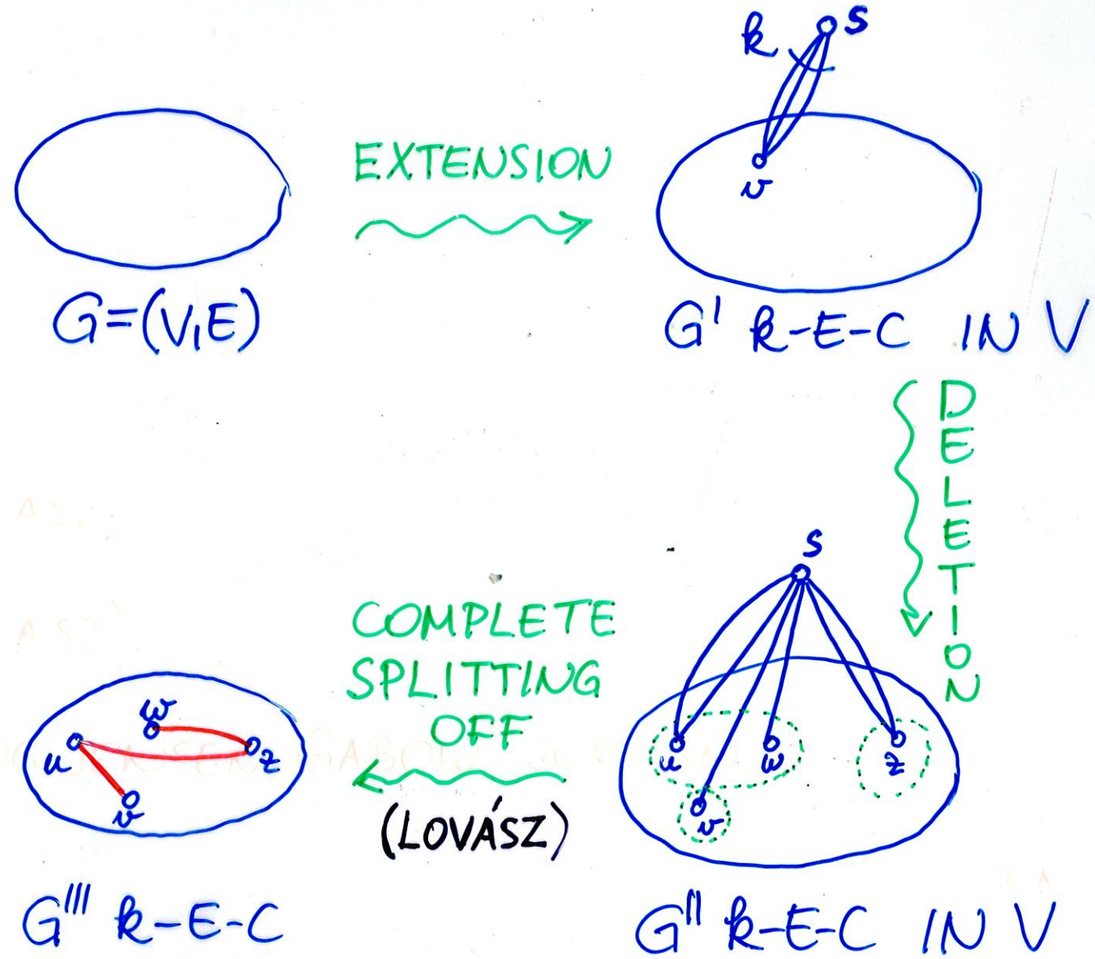
(LOVÁSZ) $\exists$ k-ADMISSIBLE PAIR AT s.

(LOVÁSZ) $\forall su \in E, \exists sv \in E : \{su, sv\}$ k-ADMISSIBLE.

(BANG-JENSEN, GABOW, JORDÁN, Sz.) $\forall su \in E$

$$\left|\{sv \in E : \{su, sv\} \text{ } k\text{-ADMISSIBLE}\}\right| \geq \begin{cases} \frac{d(s)}{2} & k \text{ EVEN} \\ \frac{d(s)}{2} - 1 & k \text{ ODD} \end{cases}$$

ALGORITHM (FRANK)



GLOBAL E-C AUGMENTATION

GLOBAL E-C AUGMENTATION IN BIPARTITE GRAPHS

GIVEN
- BIPARTITE GRAPH G,
- REQUIREMENT $2 \leq k \in \mathbb{Z}_+$

FIND A SET H OF NEW EDGES OF MIN. CARD.

S.T. $G+H$ IS
- BIPARTITE,
- k-E-C.

LOCAL E-C AUGMENTATION IN BIPARTITE GRAPHS

GIVEN
- BIPARTITE GRAPH G,
- REQUIREMENT FUNCTION $r(u,v) \geq 2$

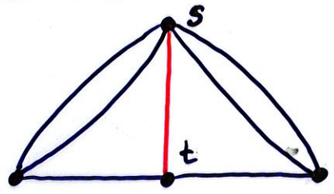
FIND A SET H OF NEW EDGES OF MIN. CARD.

S.T.
- $G+H$ IS BIPARTITE
- $\lambda_{G+H}(u,v) \geq r(u,v) \quad \forall u,v \in V$

THEOREM: LOCAL SPLITTING

- $G=(V+s, E)$ GRAPH
- $d_G(s) \neq 3$
- G 2-E-C

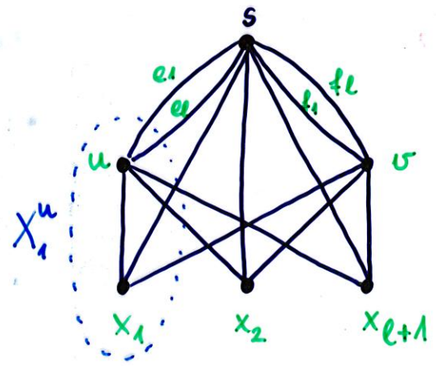
(MADER) $\exists$ λ-ADMISSIBLE PAIR AT s.



(FRANK) AT MOST ONE EDGE INCIDENT TO s BELONGS TO NO λ-ADMISSIBLE PAIR AT s.

(S_2.) $\exists$ $su \in E$: $|\{sv \in E : \{su, sv\}$ λ-ADMISSIBLE$\}| \geq \lfloor \frac{d(s)}{3} \rfloor$

IF $d(s) \geq 4$



$d(s) = 3\ell + 1$

$\lambda(u,v) = 2\ell + 1$

$d(X_1^u) = 2\ell + 2$

(S_2.) CHARACTERIZATION WHEN AN EDGE INCIDENT TO s BELONGS TO NO λ-ADMISSIBLE PAIR AT s.

OPEN PROBLEM

GLOBAL E-C AUGMENTATION
(WATANABE NAKAMURA)

GLOBAL E-C AUGMENTATION
BIPARTITE GRAPHS
(B-G-J-Sz)

LOCAL E-C AUGMENTATION
(FRANK)

LOCAL E-C AUGMENTATION
BIPARTITE GRAPHS
OPEN