Hypergraph Edge-Connectivity Augmentation

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Laboratoire G-SCOP
INP Grenoble, France

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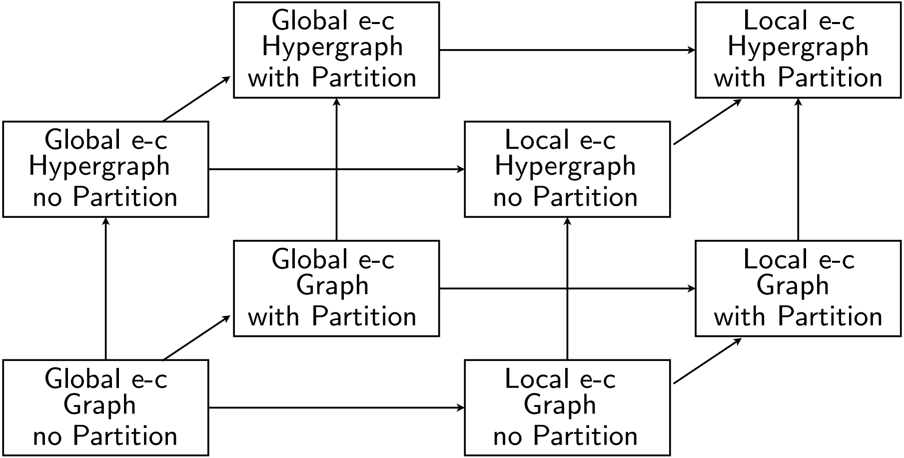
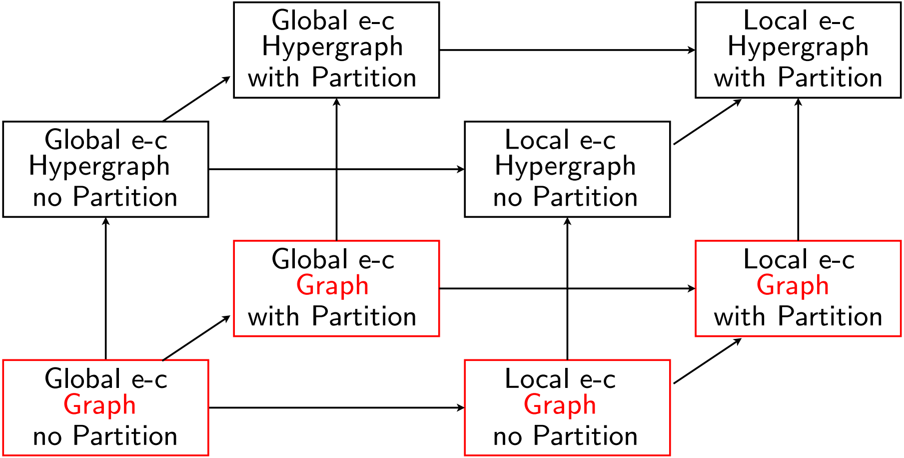
Problems to be considered



Graph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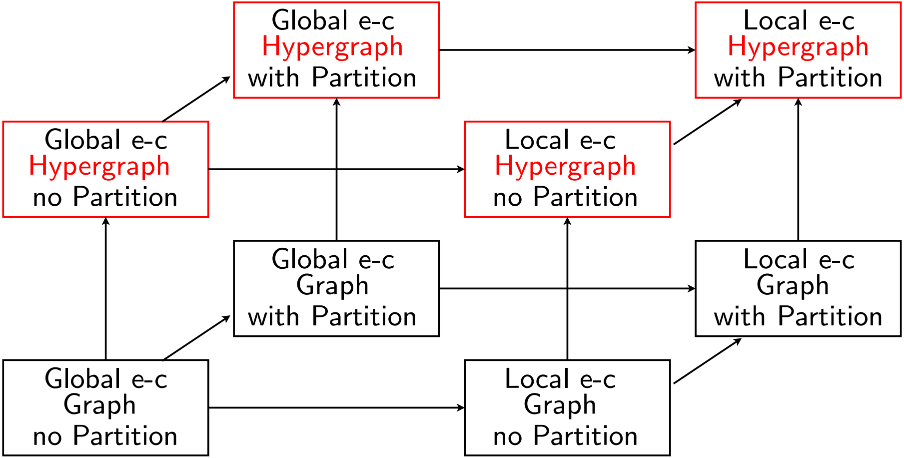
Hypergraph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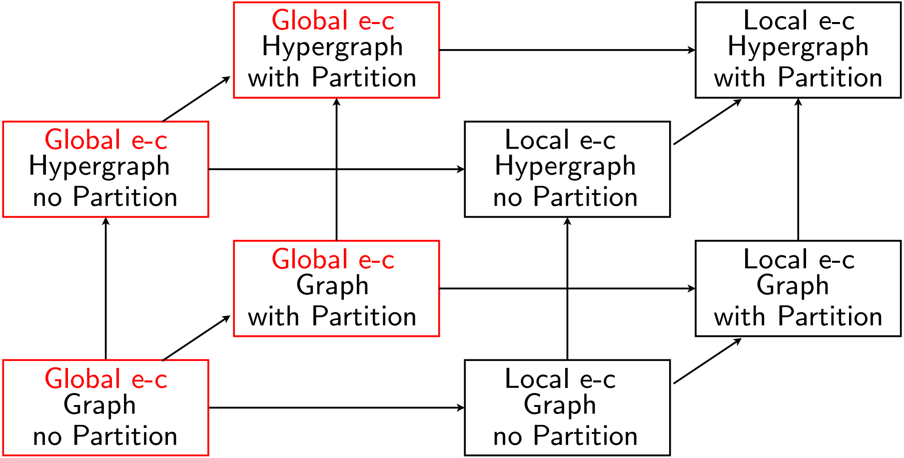
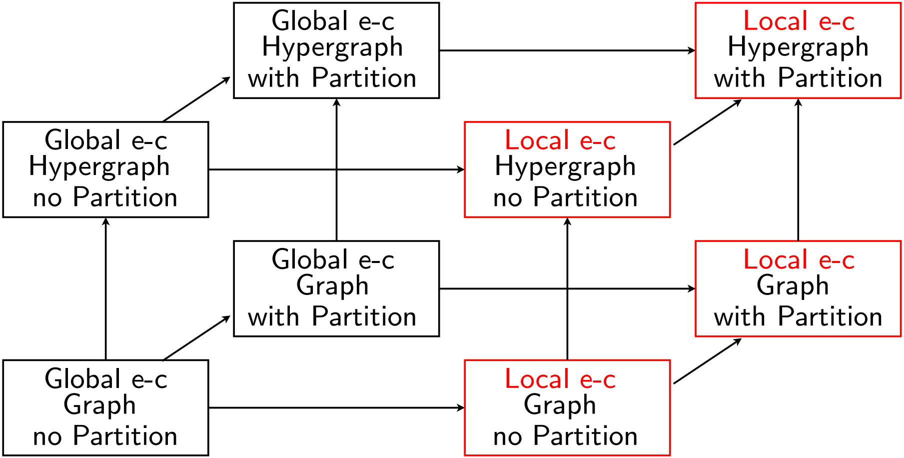
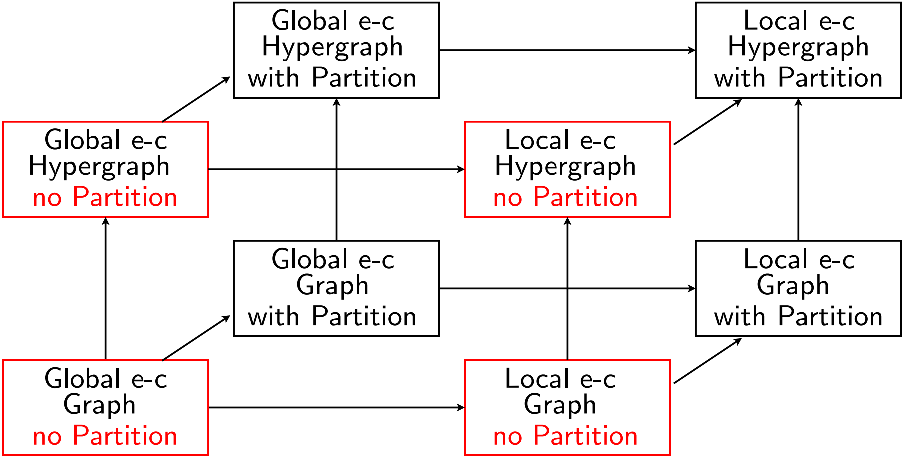
Global edge-connectivity



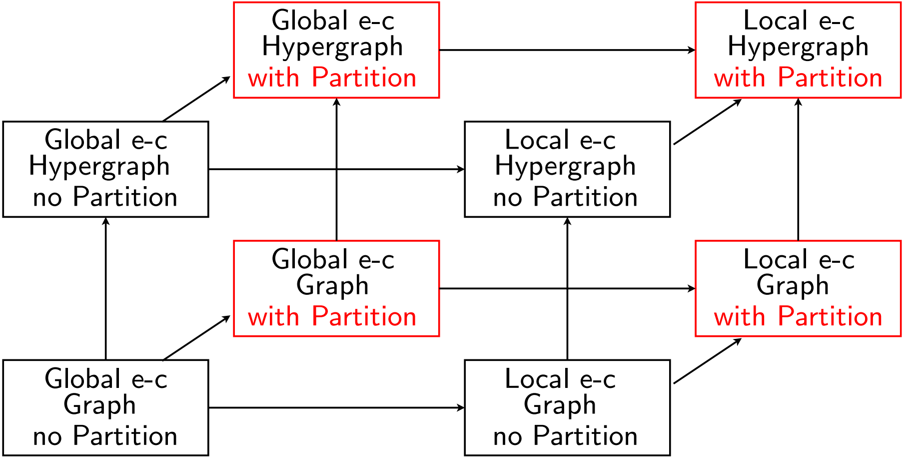
Local edge-connectivity



No partition constraint



With partition constraint



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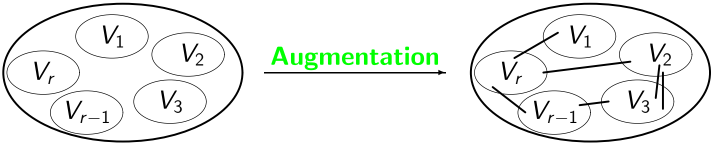
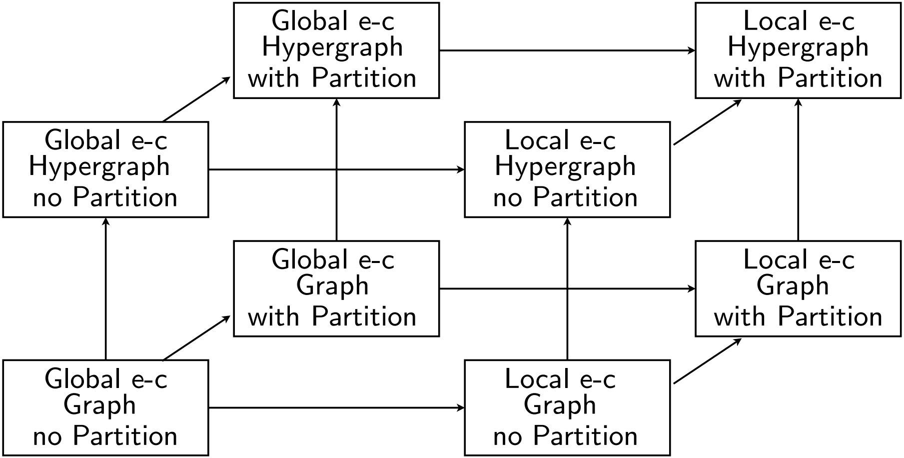


Graph $G = (V, E)$
Partition $\mathcal{P}$ of V

G' k-E-C

Problems to be considered



Definitions : global and local edge-connectivity

Global edge-conn. : k-edge-connected graph

Given a graph $G = (V, E)$ and an integer k, G is called k-edge-connected if each cut contains at least k edges.

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Given a graph $G = (V, E)$ and $u, v \in V$, the local edge-connectivity between u and v is defined as follows :

$$\lambda(u, v) = \min\{d_G(X) : u \in X, v \in V - X\}.$$

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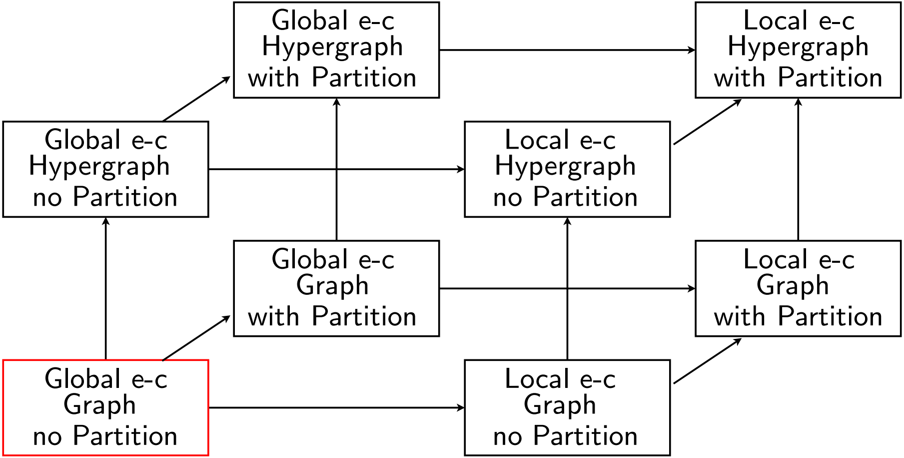
Local edge-conn. : **r**-edge-connected graph

Given a graph $G = (V, E)$ and a function $r : V \times V \to \mathbb{Z}_+$, we say that G is **r**-edge-connected if

$$\lambda(u, v) \geq r(u, v) \quad \forall\ u, v \in V.$$

Graphs : global edge-connectivity



Graphs : global edge-connectivity

Global edge-connectivity augmentation of a graph

Given a graph G and an integer $k \geq 2$, what is the minimum number γ of new edges whose addition results in a k-edge-connected graph ?

1. Minimax theorem (Watanabe, Nakamura (1987))

   $\gamma = \alpha := \max\{\lceil \frac{1}{2} \sum_{X \in \mathcal{X}} (k - d(X)) \rceil : \mathcal{X} \text{ subpartition of } V(G)\}.$

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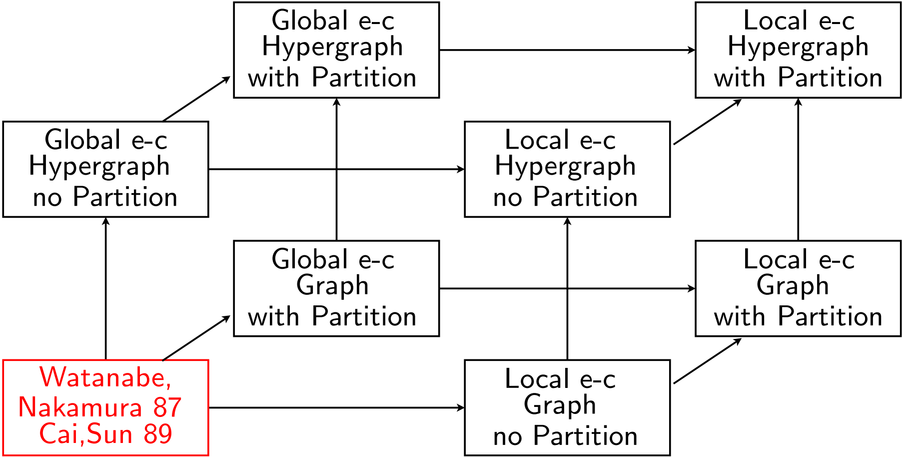
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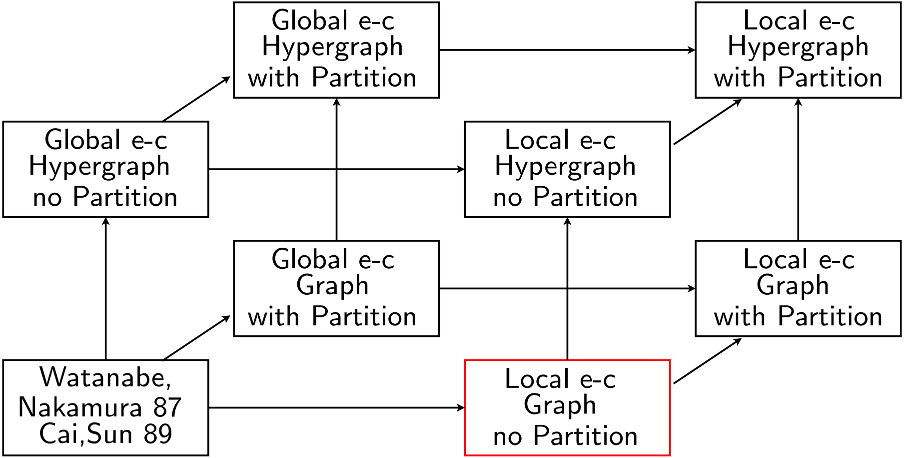

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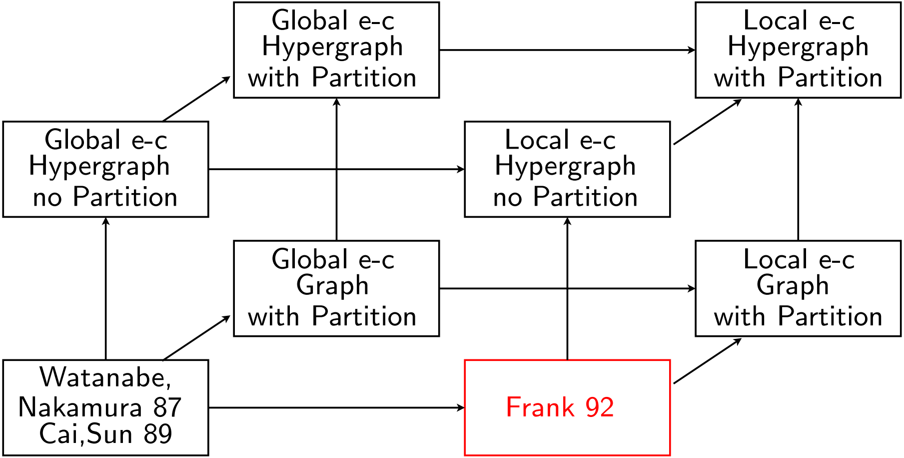
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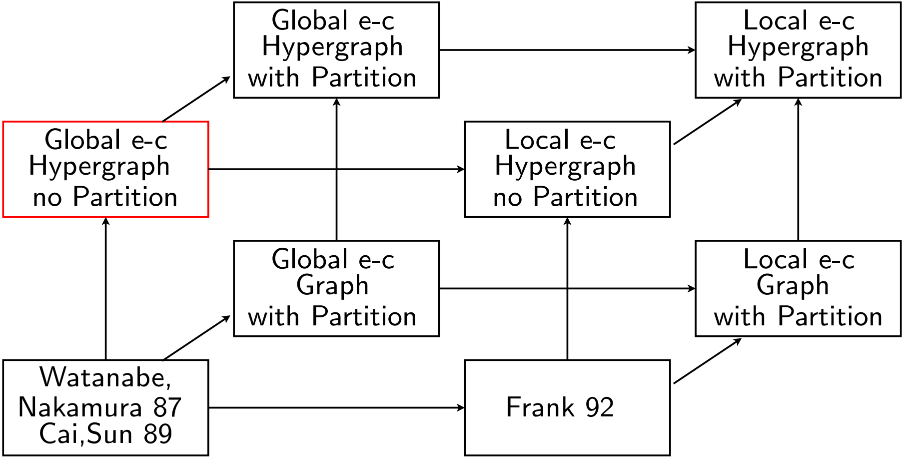

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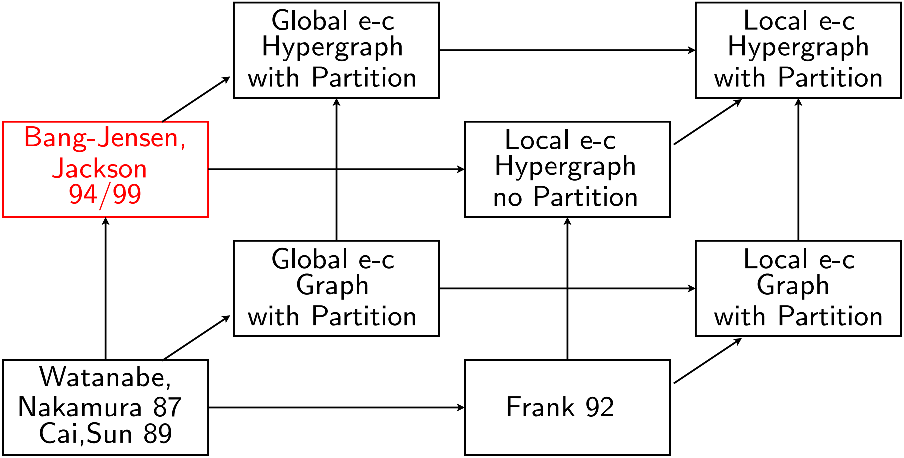
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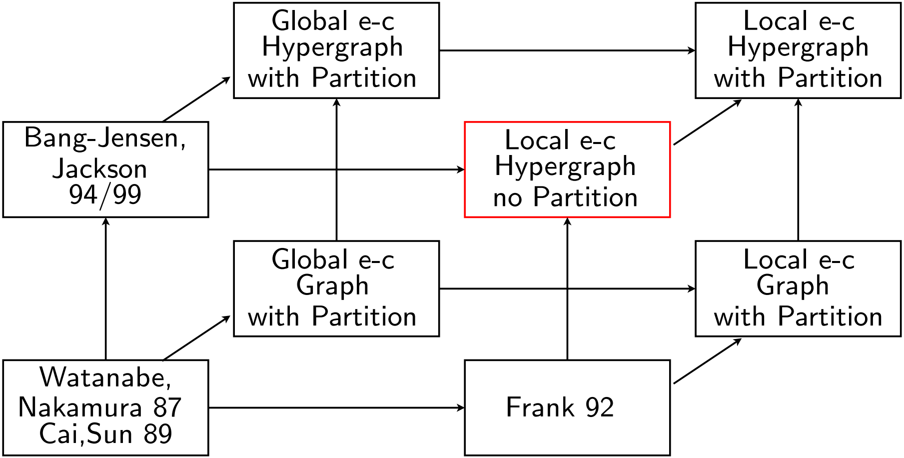
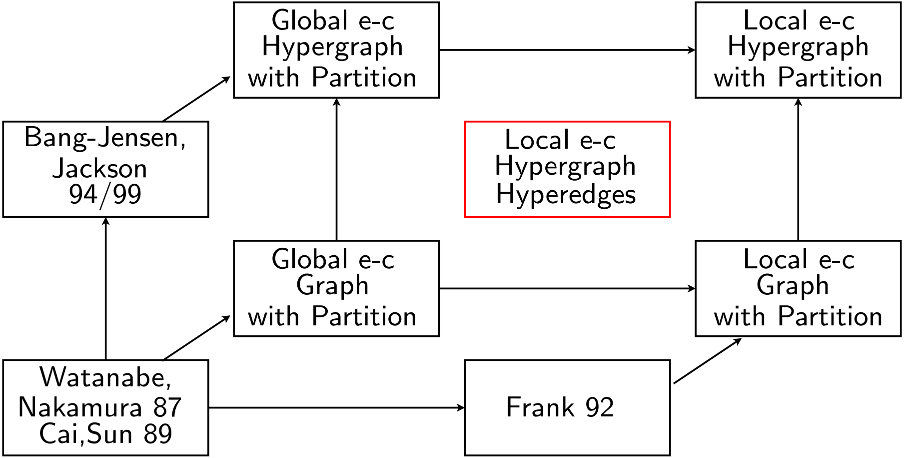


Hypergraphs : local edge-connectivity, adding hyperedges



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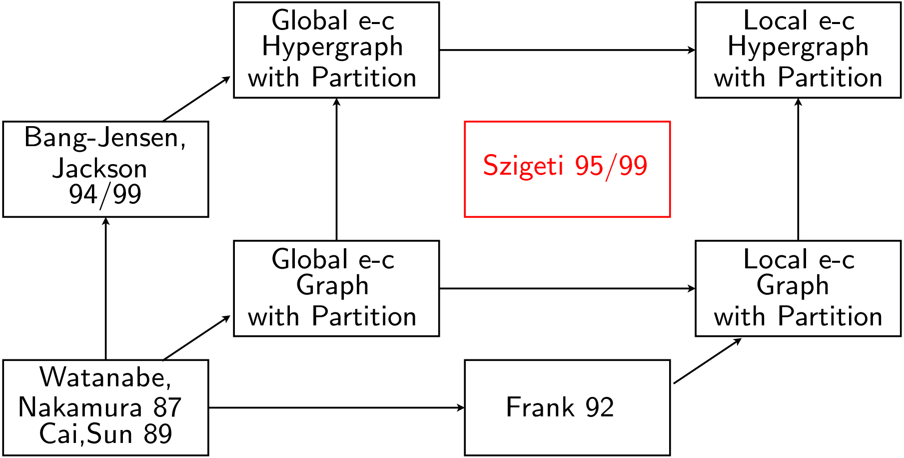
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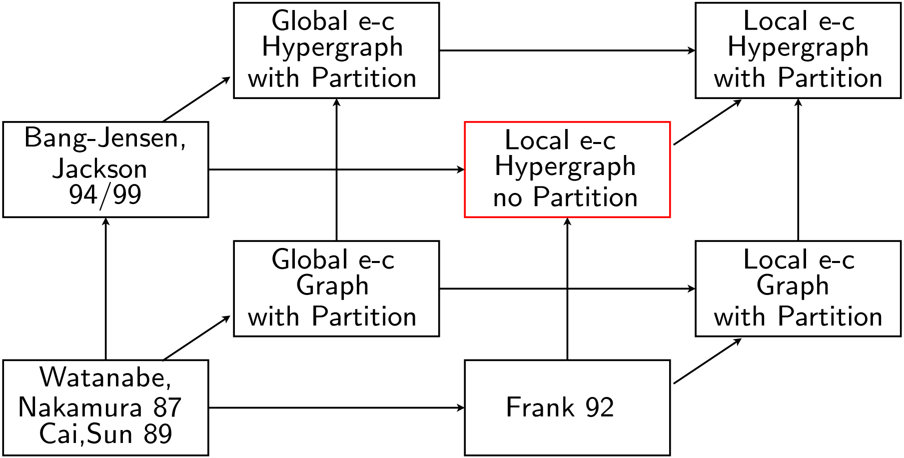

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Towards the boundary of NP-completeness

Remark

In the reduction of Cosh, Jackson, Z. Király : the hypergraph

1. contains only one hyperedge that is not a graph edge,
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Question

What about hypergraphs of low rank ?

1. rank $= 2$: Polynomial ($=$ local e-c augm. of a graph),
2. rank $= 3$: Polynomial (reduced to previous one by $\Delta - Y$ operation),
3. rank $= 4$: Open.

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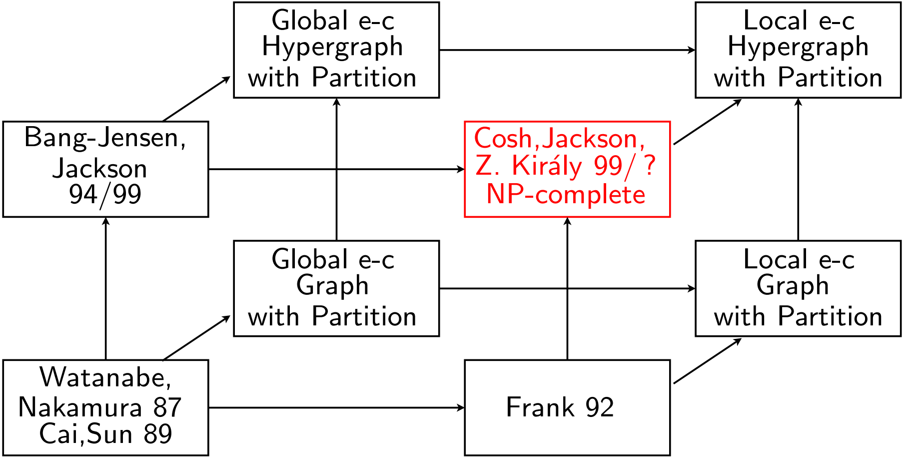

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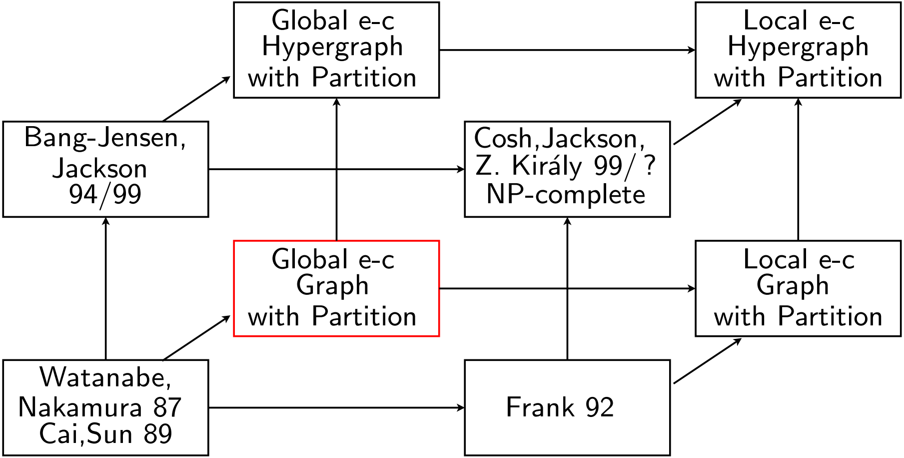
Graphs with partition constraints : global edge-connectivity



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Partition constrained global edge-conn. augmentation of a graph

Given a graph G, a partition $\mathcal{P}$ of $V(G)$ and an integer $k \geq 2$, what is the minimum number γ of new edges, between different members of $\mathcal{P}$, whose addition results in a k-edge-connected graph ?

1. Minimax theorem (Bang-Jensen, Gabow, Jordán, Szigeti (1999))

$$\gamma = \begin{cases} \Phi & \text{if } G \text{ contains no } C_4\text{- and no } C_6\text{-configuration,} \\ \Phi + 1 & \text{otherwise,} \end{cases}$$

where $\Phi := \max\{\alpha, \beta_1, \ldots, \beta_r\}$ and
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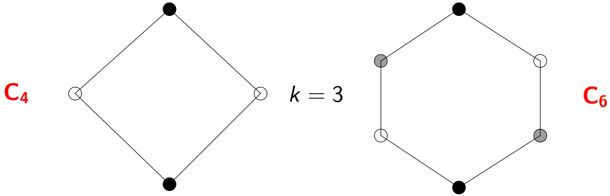
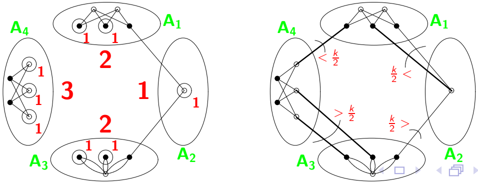


C_4-configuration

A partition $\{A_1, A_2, A_3, A_4\}$ of V is a C_4-configuration of G if k is odd and

$$
\begin{array}{rcll}
k - d(A_i) & > & 0 & \forall 1 \leq i \leq 4, \\
d(A_i, A_{i+2}) & = & 0 & \forall 1 \leq i \leq 2, \\
\sum_{X \in \mathcal{X}_i} (k - d(X)) & = & k - d(A_i) & \exists \mathcal{X}_i \in \mathcal{S}(A_i)\ \forall 1 \leq i \leq 4, \\
\mathcal{X}_j \cup \mathcal{X}_{j+2} & \in & \mathcal{S}(V_l) & \exists 1 \leq l \leq r\ \exists 1 \leq j \leq 2, \\
k - d(A_i) + k - d(A_{i+2}) & = & \Phi & \forall 1 \leq i \leq 2.
\end{array}
$$

C_4-configuration

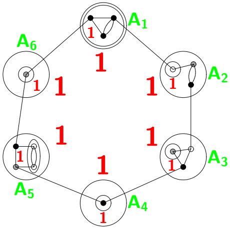


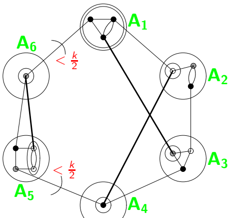
C_6-configuration

A partition $\{A_1, A_2, \dots, A_6\}$ of V is a C_6-configuration of G if k is odd,

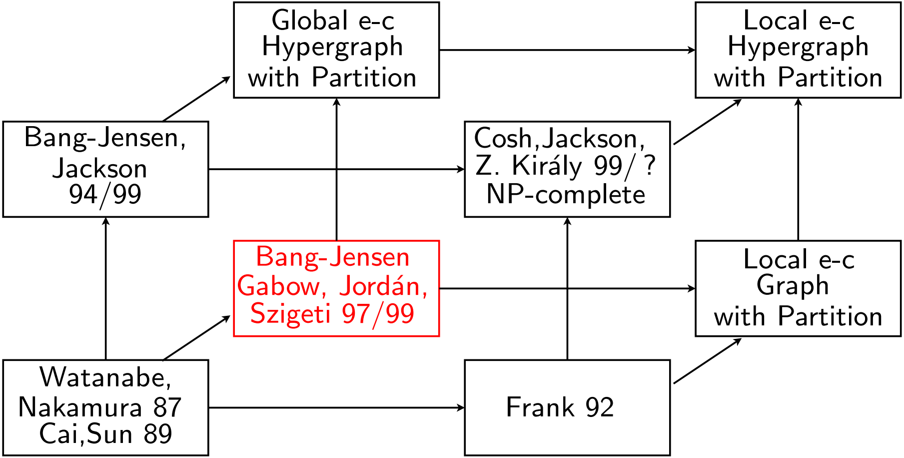
$$\begin{aligned} k - d(A_i) &= 1 \quad \forall 1 \leq i \leq 6, \\ k - d(A_i \cup A_{i+1}) &= 1 \quad \forall 1 \leq i \leq 6, (A_7 = A_1) \\ \Phi &= 3, \\ k - d(A'_i) &= 1 \quad \exists 1 \leq j_1, j_2, j_3 \leq r, \ \forall 1 \leq i \leq 6, \ \exists A'_i \subseteq A_i \cap V_{j_{i-3\lfloor}} \end{aligned}$$

C_6-configuration





Result


Global e-c
Hypergraph
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Local e-c
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Bang-Jensen,
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94/99
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Graph
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Watanabe,
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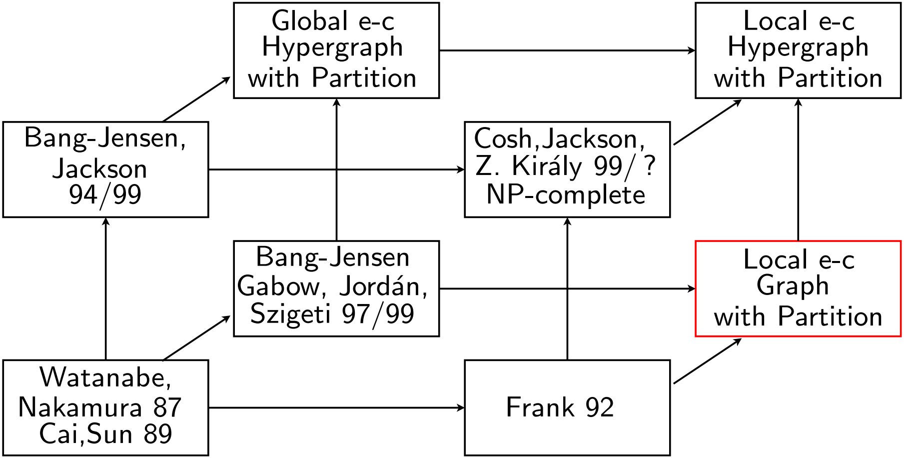
Frank 92

Graphs with partition constraints : local edge-connectivity



Partition constrained local edge-conn. augmentation of a graph

Given a graph $G = (V, E)$, a partition $\mathcal{P}$ of V and a requirement function $r : V \times V \to \mathbb{Z}_+$, what is the minimum number γ of new edges, between different members of $\mathcal{P}$, whose addition results in a **r**-edge-connected graph ?

1. Open problem

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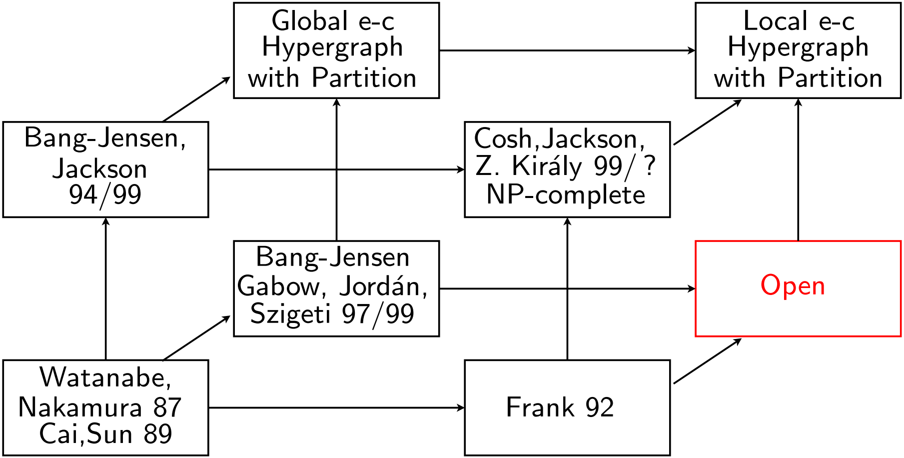
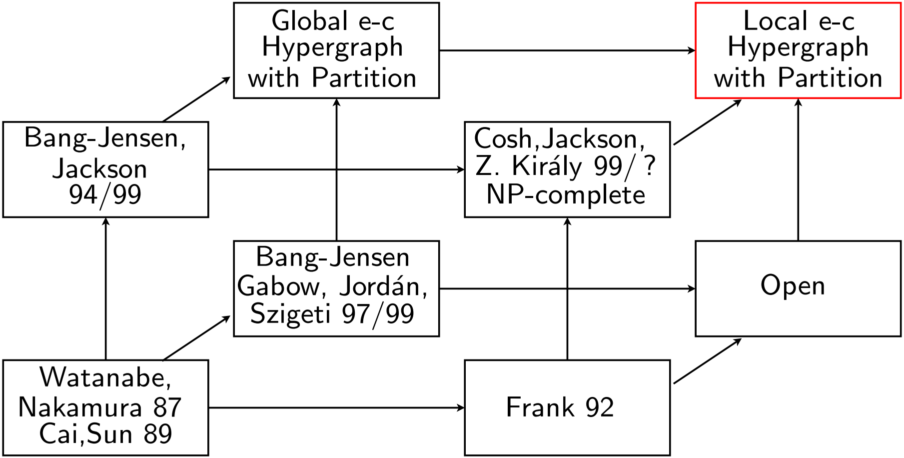


Hypergraphs with partition constraints : local edge-conn.



Partition constrained local edge-conn. augmentation of a hypergraph

Given a hypergraph $\mathcal{G} = (V, E)$, a partition $\mathcal{P}$ of V and a requirement function $r : V \times V \to \mathbb{Z}_+$, what is the minimum number γ of new edges, between different members of $\mathcal{P}$, whose addition results in a **r**-edge-connected hypergraph ?

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Hypergraphs with partition constraints : local edge-conn.

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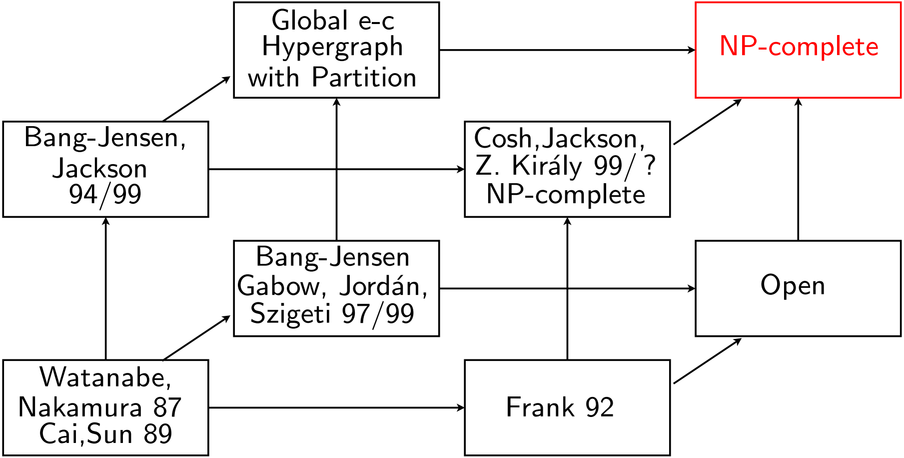
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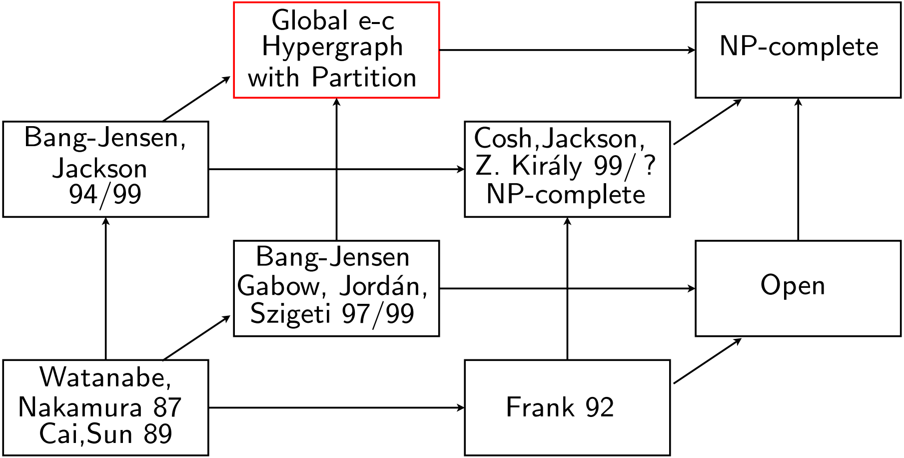
Hypergraphs with partition constraints : global edge-conn.



Hypergraphs with partition constraints : global edge-conn.

Partition constrained global edge-conn. augmentation of a hypergraph

Given a hypergraph $\mathcal{G}$, a partition $\mathcal{P}$ of $V(\mathcal{G})$ and an integer k, what is the minimum number γ of new edges, between different members of $\mathcal{P}$, whose addition results in a k-edge-connected hypergraph ?

1. Minimax theorem (Bernáth, Grappe, Szigeti (2008))

$$\gamma = \begin{cases} \Phi & \text{if } \mathcal{G} \text{ contains no } \mathcal{C}_4\text{- and no } \mathcal{C}_6\text{-configuration,} \\ \Phi + 1 & \text{otherwise,} \end{cases}$$

where $\Phi := \max\{\alpha, c_k(\mathcal{G}) - 1, \beta_1, \ldots, \beta_r\}$.

2. Polynomially solvable (Bernáth, Grappe, Szigeti (2008))

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Ben Cosh (2000) solved the special case of bipartition.

$\mathcal{C}_4$-configuration

A partition $\{A_1, A_2, A_3, A_4\}$ of V is a $\mathcal{C}_4$-configuration of $\mathcal{G}$ if

$$\begin{array}{rcll} k - d(A_i) & > & 0 & \forall 1 \leq i \leq 4, \\ \delta(A_1) \cap \delta(A_3) & = & \delta(A_2) \cap \delta(A_4) =: A, & \\ k - |A| & is & odd, & \\ \sum_{X \in \mathcal{X}_i} (k - d(X)) & = & k - d(A_i) & \exists \mathcal{X}_i \in \mathcal{S}(A_i) \ \forall 1 \leq i \leq 4, \\ \mathcal{X}_j \cup \mathcal{X}_{j+2} & \in & \mathcal{S}(V_l) & \exists 1 \leq l \leq r \ \exists 1 \leq j \leq 2, \\ k - d(A_i) + k - d(A_{i+2}) & = & \Phi & \forall 1 \leq i \leq 2. \end{array}$$

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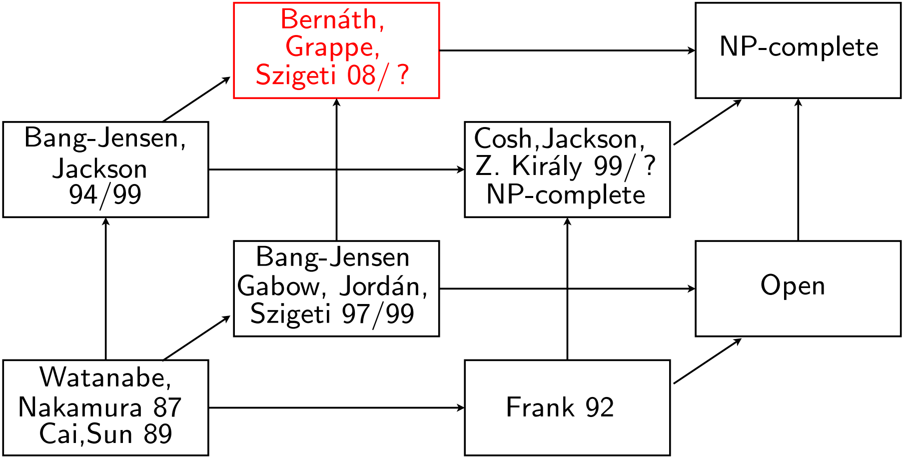
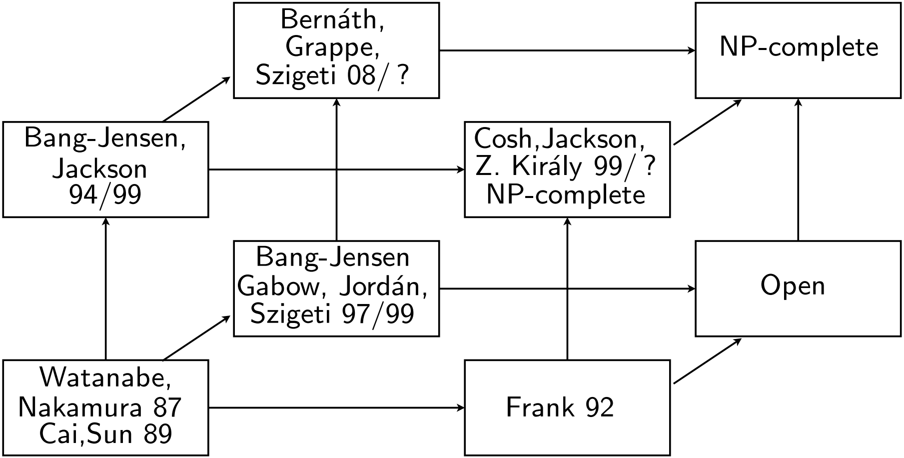
Result



Bernáth, Grappe, Szigeti 08/ ?
NP-complete
Bang-Jensen, Jackson 94/99
Cosh,Jackson, Z. Király 99/ ? NP-complete
Bang-Jensen Gabow, Jordán, Szigeti 97/99
Open
Watanabe, Nakamura 87 Cai,Sun 89
Frank 92

Results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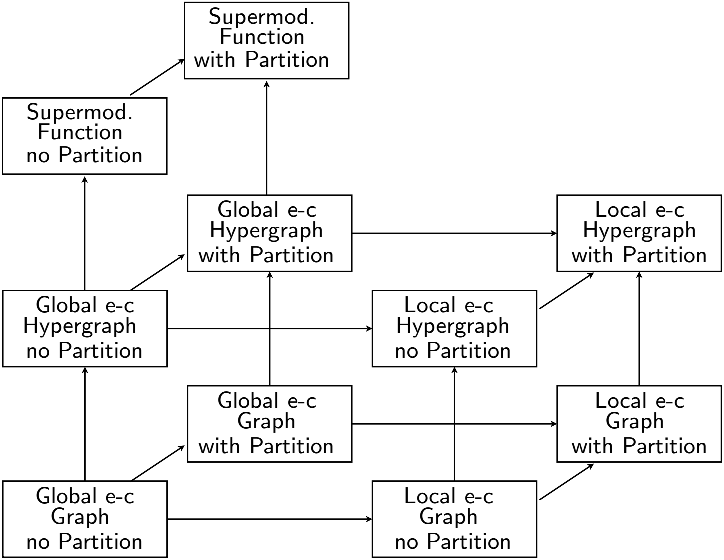
Generalizations



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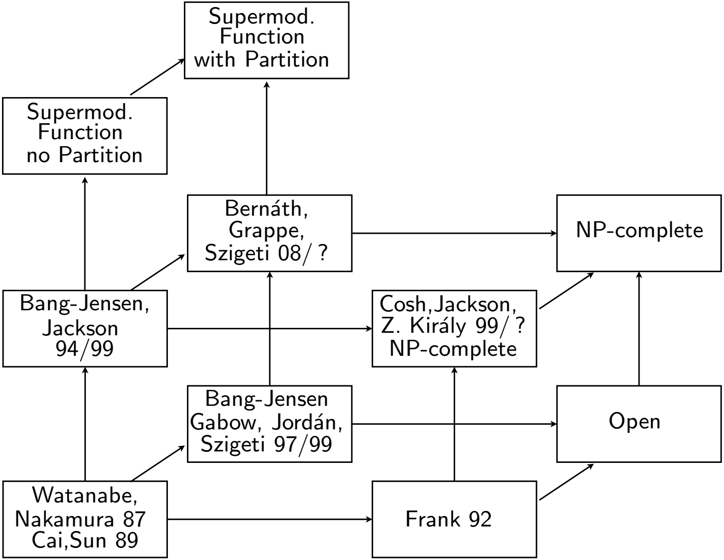
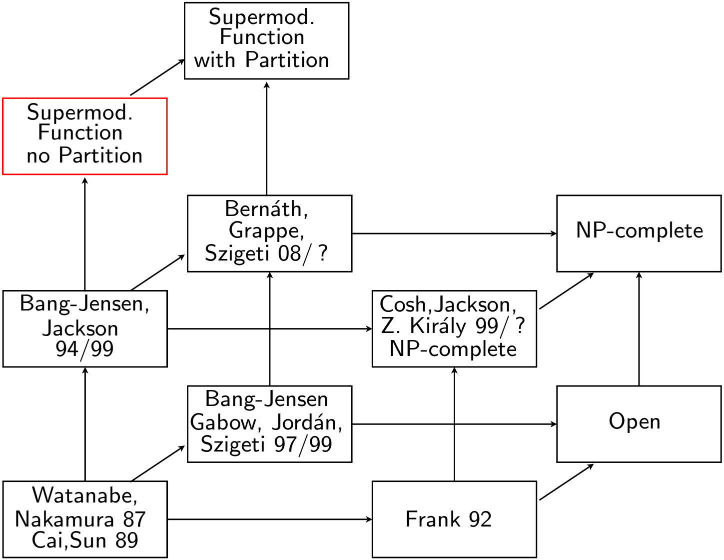


Generalizations



Covering a symmetric, crossing supermodular set function

Covering a symmetric, crossing supermodular set function

Given a symmetric, positively crossing supermodular set function $p : 2^V \to \mathbb{Z}_+$, what is the minimum number γ of edges of a graph on V that covers p? ($d(X) \geq p(X) \ \ \forall X \subset V$)

1. Minimax theorem (Benczúr, Frank (1999))

   $\gamma = \max\{\alpha_p, c_p - 1\}$,

   where $\alpha_p := \max\{\lceil \frac{1}{2} \sum_{X \in \mathcal{X}} p(X) \rceil : \mathcal{X}$ subpartition of $V\}$, and $c_p := \max\{l : p$-full l-partition exists.$\}$

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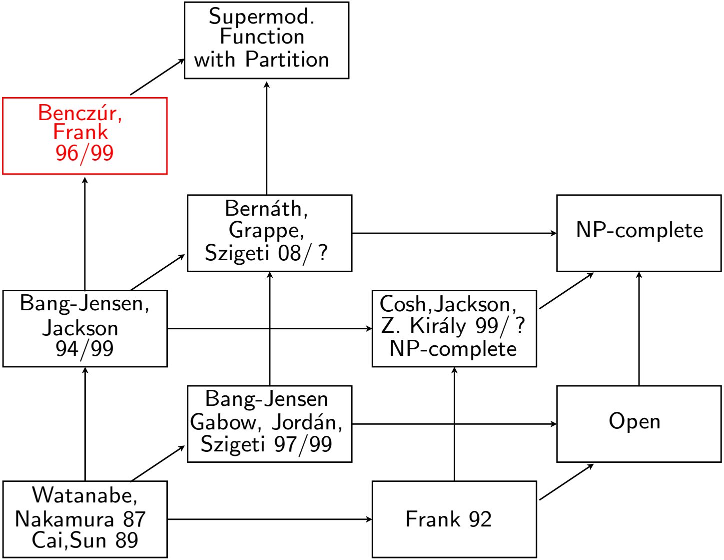
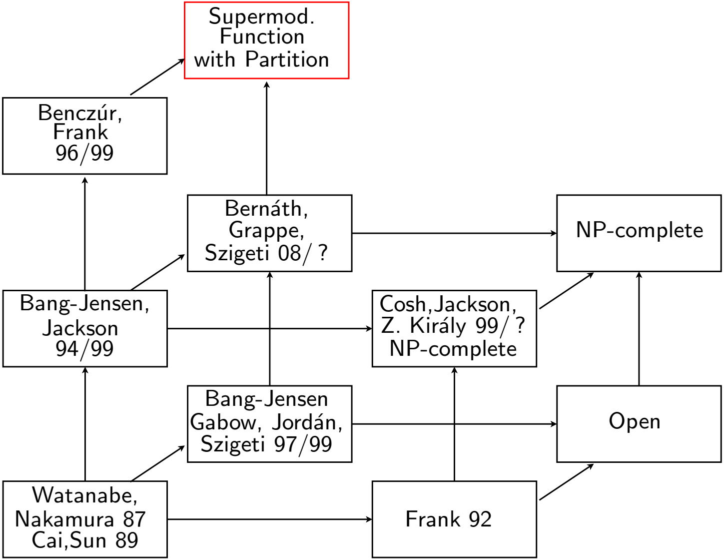


Generalizations



Covering a symmetric, crossing supermodular set function with partition constraints

Covering a symmetric, crossing supermodular set function with partition constraints

Given a symmetric, positively crossing supermodular set function $p : 2^V \to \mathbb{Z}_+$ and a partition $\mathcal{P}$ of V, what is the minimum number γ of edges, between different members of $\mathcal{P}$, of a graph on V that covers p?

1. Minimax theorem (Bernáth, Grappe, Szigeti (2008))

$$\gamma = \begin{cases} \Phi_p & \text{if no } \mathcal{C}_4^*\text{-, no } \mathcal{C}_5^*\text{- and no } \mathcal{C}_6^*\text{-configuration exists,} \\ \Phi_p + 1 & \text{otherwise,} \end{cases}$$

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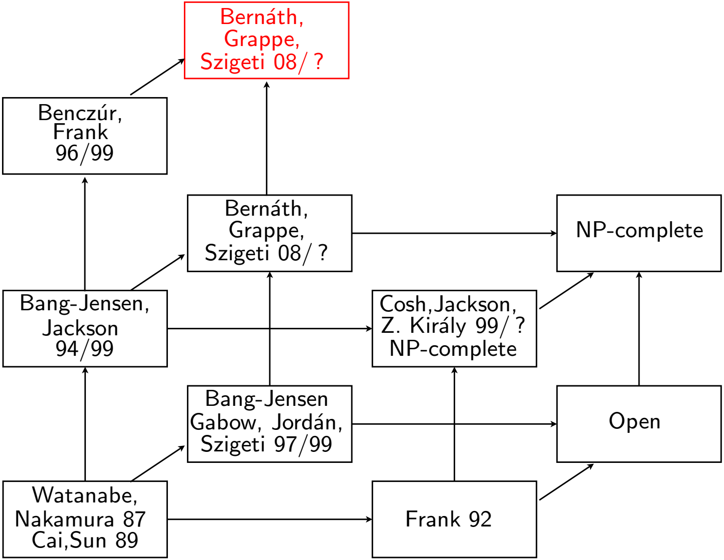
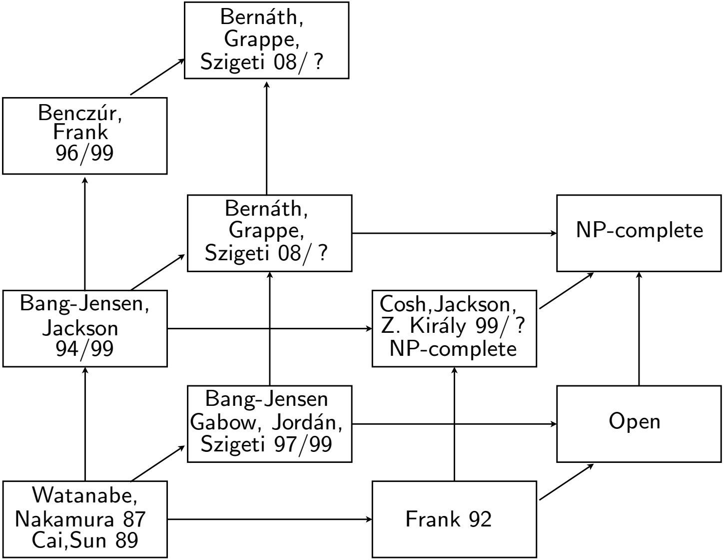
Generalizations



Results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