On 2-vertex-connected orientations of graphs

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Joint work with :
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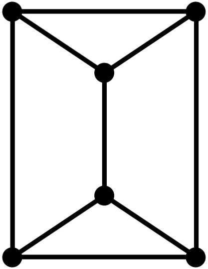
Outline

- Definitions
- Orientations with arc-connectivity constraints
- Orientations with vertex-connectivity constraints
    - k-vertex-connected orientations
    - 2-vertex-connected orientations
- Packing of special spanning subgraphs
- Matroid theory

Orientation



G

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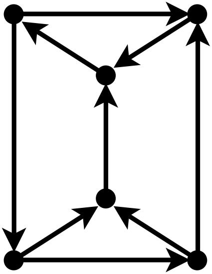

$\vec{G}$

Connectivity

Definitions

An **undirected** graph $G = (V, E)$ is

- connected if there exists a (u, v)-path $\forall u, v \in V$,
- k-edge-connected if $D - X$ is connected $\forall X \subset E, |X| \leq k - 1$,
- k-vertex-connected if $D - X$ is connected $\forall X \subset V, |X| = k - 1$ and $|V| > k$.

A **directed** graph $D = (V, A)$ is

- strongly connected if $\exists$ a directed (u, v)-path $\forall (u, v) \in V \times V$,
- k-arc-connected if $D - X$ is strongly connected $\forall X \subset A, |X| \leq k - 1$,
- k-vertex-conn. if $D - X$ is strongly connected $\forall X \subset V, |X| = k - 1$ and $|V| > k$.

Theorem (Nash-Williams 1960)

Given an undirected graph G,

- *there exists a k-arc-connected orientation of G* $\Longleftrightarrow$
- *G is $2k$-edge-connected.*

k-arc-connected orientation

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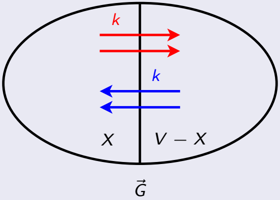

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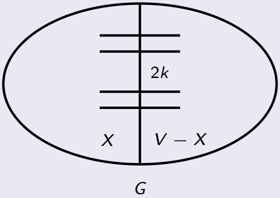

k-vertex-connected orientation

Conjecture (Thomassen 1989)

There exists a function $f(k)$ such that every $f(k)$-vertex-connected graph has a k-vertex-connected orientation.

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Conjecture (Frank 1995)

Given an undirected graph $G = (V, E)$ with $|V| > k$,

- there exists a k-vertex-connected orientation of G $\Longleftrightarrow$
- $G - X$ is $(2k - 2|X|)$-edge-connected for all $X \subseteq V$ with $|X| < k$.

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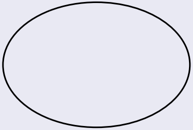

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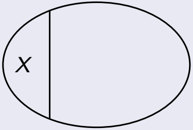

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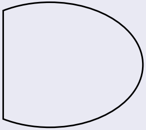

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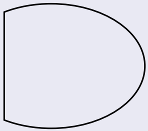

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Remark

Frank's conjecture would imply that $f(k) \leq 2k$.

Conjecture (Frank 1995)

Given an undirected graph $G = (V, E)$ with $|V| > 2$,

- there exists a 2-vertex-connected orientation of G $\Longleftrightarrow$
- G is 4-edge-connected and $G - v$ is 2-edge-connected for all $v \in V$.

2-vertex-connected orientation : Frank's Conjecture

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Theorem (Berg-Jordán 2006)

Given an Eulerian graph $G = (V, E)$ with $|V| > 2$,

- *there exists a 2-vertex-connected Eulerian orientation of G* $\Longleftrightarrow$
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2-vertex-connected orientation : Thomassen's Conjecture

Theorem (Jordán 2005)

Every 18-vertex-connected graph has a 2-vertex-connected orientation :

$$f(2) \leq 18.$$

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Theorem (Cheriyan, Durand de Gevigney, Szigeti 2011)

Every 14-vertex-connected graph has a 2-vertex-connected orientation :

$$f(2) \leq 14.$$

2-vertex-connected orientation : Thomassen's Conjecture

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Frank's conjecture would imply that $f(2) \leq 4$.

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Find a spanning subgraph G such that

- $G - v$ is 2-edge-connected for all $v \in V$, $\Longleftarrow$
- $G - v$ contains 2 edge-disjoint connected spanning subgraphs for all $v \in V$, $\Longleftarrow$
- G contains 2 edge-disjoint 2-vertex-connected spanning subgraphs.

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Theorem (Jordán 2005) (Tool 2)

Every $6k$*-vertex-connected graph contains* k *edge-disjoint 2-vertex-connected spanning subgraphs.*

Proof

(Jordán 2005)

- Let H be a 18-vertex-connected graph.
- By **Tool 2**, H contains 3 edge-disjoint 2-vertex-connected spanning subgraphs : G_1, G_2, and G_3.
- Let $G' := G_1 \cup G_2$. Then $G' - v$ is 2-edge-connected for every vertex v.
- Let T be the set of odd degree vertices in G'.
- By **Tool 1**, G, and hence H, has a 2-vertex-connected orientation.

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Theorem (Cheriyan, Durand de Gevigney, Szigeti 2011) (Tool 2')

Every $(6k + 2\ell)$-vertex-connected graph contains k 2-vertex-connected and ℓ connected edge-disjoint spanning subgraphs.

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- Let H be a 14-vertex-connected graph.
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How to prove them ?

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Theorem (Tool 2')

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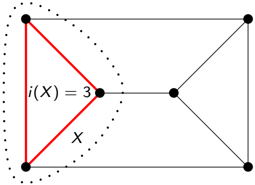
Rigidity Matroid

Definition

Given a graph $G = (V, E)$ with $n = |V|$.

Rigidity Matroid :

- independent sets : $\mathcal{R}(G) = \{F \subseteq E : i_F(X) \leq 2|X| - 3 \quad \forall\, X \subseteq V\}$ (Crapo 1979).
- rank function $r_{\mathcal{R}}(F) = \min\{\sum_{X \in \mathcal{H}}(2|X| - 3) : \mathcal{H}$ set of subsets of V covering $F\}$ (Lovász-Yemini 1982).
- G is rigid if $r_{\mathcal{R}}(E) = 2n - 3$ (Laman 1970).



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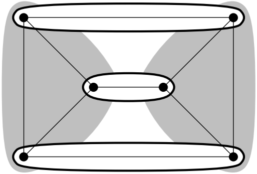

a covering $\mathcal{H}$ of E

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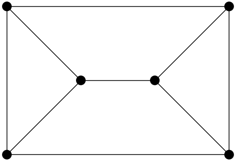

a rigid graph

Packing of rigid spanning subgraphs

Remark

Every rigid graph is 2-vertex-connected.

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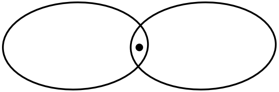
Proof

- Suppose G is not 2-vertex-connected.
- Then there exists a covering $\{X, Y\}$ of E such that $|X \cap Y| \leq 1$.
- $r_{\mathcal{R}}(E) \leq 2|X| - 3 + 2|Y| - 3 = 2|X \cup Y| + 2|X \cap Y| - 6 \leq 2n - 4$.
- G is not rigid.



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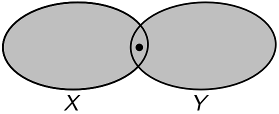

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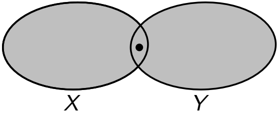

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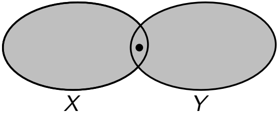

Packing of rigid spanning subgraphs

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Theorem (Lovász-Yemini 1982)

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Theorem (Jordán 2005)

Every $6k$-vertex-connected graph contains k rigid edge-disjoint spanning subgraphs.

Circuit Matroid

Definition

Given a graph $G = (V, E)$ with $n = |V|$.

Circuit Matroid :

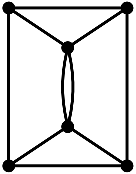
- independent sets : $\mathcal{C}(G)$ = the edge sets of the forests of G.
- rank function $r_{\mathcal{C}}(F) = n - c(F)$.
- G is connected if $\exists$ a spanning tree $(r_{\mathcal{C}}(E) = n - 1)$.

Packing of connected spanning subgraphs

Theorem (Tutte 1961)

Given an undirected graph G and an integer $\ell \geq 1$,

- *there exist ℓ edge-disjoint spanning trees of G* $\iff$
- *for every partition $\mathcal{P}$ of V,*



$G, \ell = 2$

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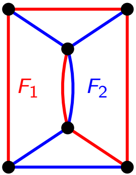

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Given an undirected graph G and an integer $\ell \geq 1$,

- *there exist ℓ edge-disjoint spanning trees of G* $\iff$
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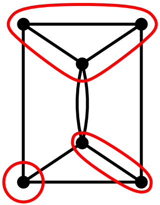

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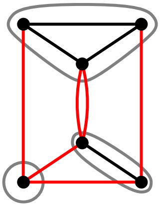

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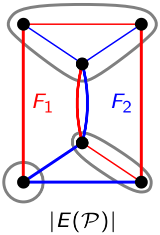

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Remark

Every 2ℓ-edge-connected graph contains ℓ edge-disjoint spanning trees.
$|E(\mathcal{P})| = \frac{1}{2}\sum_{P\in\mathcal{P}} d(P) \geq \frac{1}{2}2\ell|\mathcal{P}| > \ell(|\mathcal{P}| - 1)$.

Packing of rigid and connected spanning subgraphs

Theorem (Cheriyan, Durand de Gevigney, Szigeti 2011)

Every $(6k + 2\ell)$-vertex-connected graph contains k rigid and ℓ connected edge-disjoint spanning subgraphs.

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Tool

$\mathcal{M}_{k,\ell}(G)$ = matroid union of k copies of $\mathcal{R}(G)$ and ℓ copies of $\mathcal{C}(G)$.

- independent sets are the union of k independent sets of $\mathcal{R}(G)$ and ℓ independent sets of $\mathcal{C}(G)$.
- rank $r_{\mathcal{M}_{k,\ell}}(E) = \min_{F\subseteq E} kr_{\mathcal{R}}(F) + \ell r_{\mathcal{C}}(F) + |E \setminus F|$. (Edmonds 1968)
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Conjecture (Frank 1995)

Given an undirected graph $G = (V, E)$ with $|V| > 2$,

- there exists a 2-vertex-connected orientation of G $\Longleftrightarrow$
- G is $(4, 2)$-connected.

Main result

Theorem (Lovász-Yemini 1982)

Every 6-vertex-connected graph is rigid.

Theorem (Jordán 2005)

Every $6k$-vertex-connected graph contains k rigid edge-disjoint spanning subgraphs.

Theorem (Jackson et Jordán 2009)

Every simple $(6,2)$-connected graph is rigid.

Theorem (Cheriyan, Durand de Gevigney, Szigeti 2011)

Every simple $(6k+2\ell, 2k)$-connected graph contains k (≥ 1) rigid (2-vertex-connected) and ℓ connected edge-disjoint spanning subgraphs.

Thank you for your attention !