On packing of arborescences with matroid constraints

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Joint work with :
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Outline

- Motivations
  - Undirected = Orientation + Directed
  - Rigidity
- Results
  - Undirected : Matroid-based packing of rooted-trees
  - Directed : Matroid-based packing of rooted-arborescences
  - Orientation : Supermodular function
- Further results
  - Algorithmic aspects
  - Generalization
- Conclusion

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Theorem (Tutte, Nash-Williams 1961)

Let G be an undirected graph and k a positive integer.

- *There exists a packing of k spanning trees in G* $\Longleftrightarrow$
- *G is k-partition-connected.*

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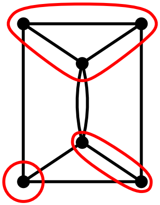

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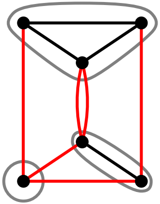

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Let D be a directed graph, r a vertex of D and k a positive integer.

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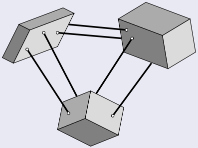
Motivation 2 : Rigidity

Body-Bar Framework



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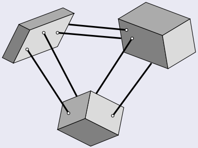

Theorem (Tay 1984)

"Rigidity" of a Body-Bar Framework can be characterized by the existence of a spanning tree decomposition.

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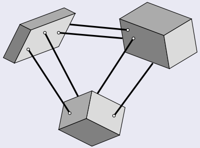
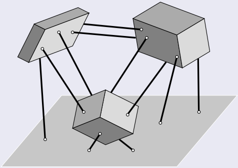


Body-Bar Framework with Bar-Boundary



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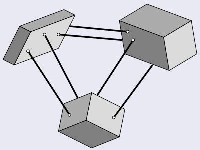

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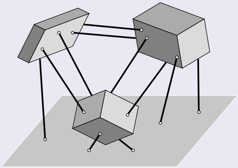

Theorem (Katoh, Tanigawa 2012)

"Rigidity" of a Body-Bar Framework with Bar-Boundary can be characterized by the existence of a matroid-based rooted-tree decomposition.

Matroids

Definition

For $\mathcal{I} \subseteq 2^{\mathsf{S}}$, $\mathcal{M} = (\mathsf{S}, \mathcal{I})$ is a matroid if

1. $\mathcal{I} \neq \emptyset$,
2. If $X \subseteq Y \in \mathcal{I}$ then $X \in \mathcal{I}$,
3. If $X, Y \in \mathcal{I}$ with $|X| < |Y|$ then $\exists\ y \in Y \setminus X$ such that $X \cup y \in \mathcal{I}$.

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2. Edge-sets of forests of a graph,
3. $U_{n,k} = \{X \subseteq S : |X| \leq k\}$ where $|S| = n$,

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3. $U_{n,k} = \{X \subseteq \mathrm{S} : |X| \leq k\}$ where $|\mathrm{S}| = n$, free matroid $= U_{n,n}$.

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3. rank function : $r(X) = \max\{|Y| : Y \in \mathcal{I}, Y \subseteq X\}$.
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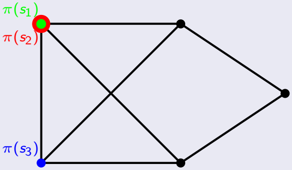
A matroid-based rooted-graph is a quadruple $(G, \mathcal{M}, S, \pi)$:

1. $G = (V, E)$ undirected graph,
2. $\mathcal{M}$ a matroid on a set $S = \{s_1, \dots, s_t\}$.
3. π a placement of the elements of S at vertices of V.

G

$S = \{s_1, s_2, s_3\}$

$\mathcal{M} = U_{3,2}$



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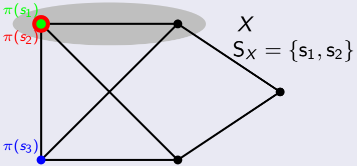

Notation

- S_X = the elements of S placed at X $(= \pi^{-1}(X))$.

$\mathcal{M}$-based packing of rooted-trees

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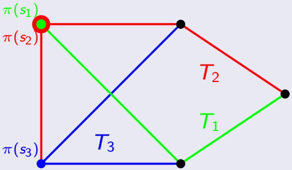

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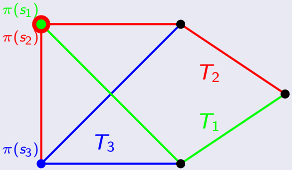

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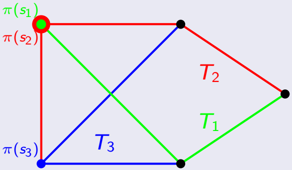

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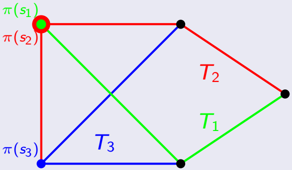

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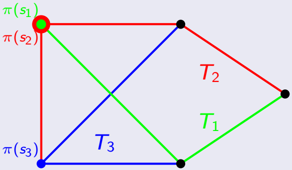

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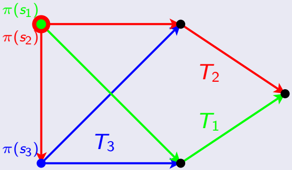

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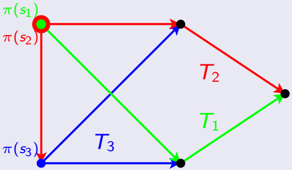

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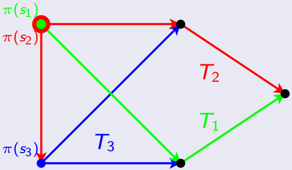

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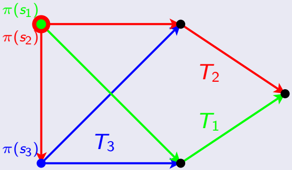

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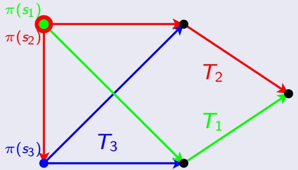

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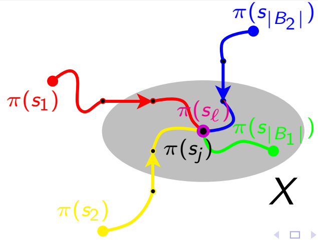

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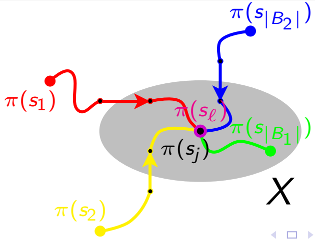

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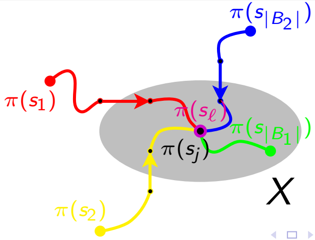

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Theorem (Frank 1980)

Let $G = (V, E)$ be an undirected graph and $h : 2^V \to \mathbb{Z}_+$ an intersecting supermodular non-increasing set-function.

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Theorem (Durand de Gevigney, Nguyen, Szigeti 2012)

1. A matroid-based packing of rooted-arborescences can be found in polynomial time,
2. We have a complete description of the convex hull of the incidence vectors of the matroid-based packings of rooted-arborescences,
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Open problem

Combinatorial algorithm for finding a matroid-based packing of rooted-arborescences of minimum weight ?

Thank you for your attention !