ON A MATROID DEFINED BY EAR-DECOMPOSITIONS OF GRAPHS

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A.Frank described in [1] an algorithm to determine the minimum number of edges in a graph G whose contraction leaves a factor-critical graph and he asked if there was an algorithm for the weighted version of the problem. We prove that the minimal critical–making edge–sets form the bases of a matroid and hence the matroid greedy algorithm gives rise to the desired algorithm.

1. Introduction

Given a connected graph G, what is the minimum number of edges whose contraction leaves a factor-critical graph? A.Frank [1] noticed that for 2-edge–connected graphs this value equals the minimum number $\varphi(G)$ of even ears in ear–decompositions of G, and he proved a minimax formula for $\varphi(G)$. In the same paper he proposed the problem of describing the structure of the edge–sets above. The aim of this note is to prove that minimal critical–making sets form the bases of a matroid. We refer the reader to [3] for basic concepts of matroids.

For a connected graph G, an edge–set is called *critical–making* if its contraction leaves a factor–critical graph. A graph G is *factor–critical* if for every $v \in V(G)$ $G - v$ has a perfect matching. Since factor–critical graphs are 2-edge–connected, every cut edge of G is contained in any critical–making edge–set. Thus we may assume that G is 2-edge–connected.

Let $G = (V, E)$ be an undirected, 2-edge–connected graph. An *ear–decomposition* of G is a sequence $G_0, G_1, ..., G_n = G$ of subgraphs of G where G_0 is a vertex and each G_i arises from G_{i-1} by adding a path P_i for which the two end–vertices (they are not necessarily distinct) belong to G_{i-1} while the inner vertices of P_i do not. This means the graph G can be written in the following form: $G = P_1 + P_2 + ... + P_n$ where the paths P_i are called the *ears* of this decomposition. An ear is *odd* (resp. *even*) if its length is odd (resp. even). Let us consider an ear–decomposition of G which has as few even ears as possible. Let $\varphi(G)$ denote this minimum number of even ears.

The *contraction* of an edge e of a graph G is defined in the usual way. We will denote the contracted graph by G/e. Note that the contraction of an edge can produce parallel edges. By the contraction of an edge–set F of G we mean the graph $G' = G/F$ arising from G by contracting each edge of F. When we contract an edge-set then we will always assume (without loss of generality) that this edge-set is circuit-free, that is, it is a forest.

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By contracting a connected subgraph H of G we mean the contraction of a spanning tree of H.

The *subdivision* of an edge–set F of a graph G means that we subdivide each edge e of F by a new vertex. The resulting graph is denoted by $G \times F$. The following lemma gives the relation between contraction and subdivision.

Lemma 1.1. *Let G be a 2–edge–connected graph and let k be a positive integer. Then the following are equivalent:*

a.) the minimum number of edges whose contraction leaves a factor–critical graph is k,
b.) the minimum number of edges whose subdivision leaves a factor–critical graph is k,
c.) the minimum number of even ears in an ear–decomposition of G is k, i.e. $\varphi(G) = k$.

The proof of this lemma is given in Section 2 and it implies that $\varphi(G/F) = \varphi(G \times F)$ for any forest F. In the view of this fact we shall use the subdivision of an edge–set rather than the contraction. (It is easier to deal with subdivision than with contraction.) We would like to emphasize that Lemma 1.1 is not completely trivial. It is not true that the ear–decomposition of G/e can always be extended to an ear–decomposition of G. (see Figure 1.) For more details see [6, Lemma 9.2].

Clearly, the subdivision of any edge in a graph either decreases or increases $\varphi(G)$ by one. Thus $\varphi(G \times F) \geq \varphi(G) - |F|$ for any edge–set of G. An edge–set F of a graph G is called *ear–extreme* if $\varphi(G \times F) = \varphi(G) - |F|$. Note that every ear–extreme edge–set is a forest. Our purpose is to prove that the ear–extreme edge–sets form the independent sets of a matroid. Clearly, the ear–extreme edge–sets of maximum size and the critical–making sets of minimum size are the same. Furthermore, by Lemma 3.1 and Theorem 3.4 it follows that the maximal ear–extreme edge–sets and the minimal critical–making sets are the same, as well.

The properties of ear–decompositions of graphs are closely related to matching theory so we need some basic definitions and theorems from this area.

A graph G is said to be *bicritical* if it has at least one edge and for every pair of vertices $u, v \in V(G)$ $G - u - v$ has a perfect matching. Recall that a graph G is *factor–critical* if for every $v \in V(G)$ $G - v$ has a perfect matching. Factor–critical graphs have a very useful characterization in the language of ear–decompositions [2]. An ear–decomposition is said to be *odd* if every ear is odd in it. A subgraph G' of a graph G is *nice* if $G - V(G')$ has a perfect matching.

Theorem 1.2. *[Lovász] A graph G is factor–critical if and only if it has an odd ear–decomposition, that is, $\varphi(G) = 0$. Moreover, if G is a factor–critical graph, then any odd ear–decomposition of a nice factor–critical subgraph extends to an odd ear–decomposition of G.* □

If for a connected graph G, all of its edges lie in some perfect matching then G is said to be *1–extendable*. The following important property of 1–extendable graphs can be found in [2, Theorem 5.4.1, Theorem 5.4.4].

Theorem 1.3. *If G is 1–extendable and e_1 and e_2 are any two non–parallel edges of G, then G has an ear–decomposition such that only the first ear is even and it contains e_1 and e_2.* □

The following theorem is well–known and it follows from Theorem 1.2.

Theorem 1.4. *Let H be a subgraph of G. If both H and G/H are factor–critical then so is G.* □

We need a similar theorem, which will be proved in Section 2.

Lemma 1.5. *Let $G = (V, E)$ be a 2–edge–connected graph and assume that V is partitioned into V_1 and V_2 such that $G - V_1$ is connected. Denote G_i the graph obtained from G by deleting all the edges with both ends in V_i and identifying all the vertices of V_i $(i = 1, 2)$.*

If G_1 is factor–critical and G_2 is 1–extendable then G is a factor–critical graph.

For a graph $G = (V, E)$, let $c_o(X)$ denote the number of odd components of $G - X$. A set of vertices X is a *barrier* if $c_o(X) - |X|$ =max$\{c_o(Y) - |Y| : Y \subseteq V\}$. Let G be any graph with a perfect matching. Then by Tutte's theorem $c_o(X) = |X|$ for any barrier X. A barrier X of G is said to be a *strong barrier* if $G - X$ has no even components, each of the odd components is factor–critical and the bipartite graph, obtained from G by deleting the edges induced by X and by contracting each factor–critical component to a single vertex, is 1-extendable. If G has a strong barrier then G is called *half–elementary.* Let the subgraph H of G induced by $U \subseteq V$ be half–elementary with a strong barrier $X \subseteq U$, then H is said to be a *strong subgraph* of G attached at X if X separates $U - X$ from $V - U$ or if $U = V$.

The definition of strong subgraph was introduced by A.Frank [1] (he called it strong–end) and he proved the following theorems.

Theorem 1.6. *G is factor–critical if and only if G has no strong subgraphs.* □

Theorem 1.7. *For a strong subgraph H of G, $\varphi(G/H) = \varphi(G) - 1$.* □

The following lemma is not explicitly stated in Theorem 2.4. in [1] but in fact A.Frank proved this statement.

Lemma 1.8. *Let G be a connected graph. Suppose for a vertex–set X, $G - X$ has at least $|X|$ factor–critical components. Then there exists a strong subgraph H attached at some $Y \subseteq X$ such that all the factor–critical components of $G - Y$ are components of $G - X$ as well.* □

Let us recall briefly the Gallai–Edmonds Structure Theorem (see [2]). Let N be an arbitrary graph. Let $D(N)$ denote the set of those vertices in N which are not covered by at least one maximum matching of N. Let $A(N)$ be the set of vertices in $V(N) - D(N)$ adjacent to at least one vertex in $D(N)$. Then by the Gallai-Edmonds Structure Theorem, $A(N)$ is a barrier of N, the odd components of $N - A(N)$ are factor-critical and their union is exactly $D(N)$. Furthermore if we assume that N has no perfect matching, then $A(N) = \emptyset$ if and only if N is factor–critical.

2. Proofs of Lemmas

Proof. (of Lemma 1.1) First we show the equivalence of a.) and b.).

i.) We prove by induction on k that if $F \subseteq E(G)$ is a forest, $|F| = k$ and G/F is factor–critical, then $G \times F$ is factor–critical, as well.

Assume that for some edge f G/f is factor–critical but $G \times f$ is not. Since $G \times f$ has an odd number of vertices and it is not factor–critical, by Gallai–Edmonds Theorem $A(G \times f) \neq \emptyset$ and $G \times f - A(G \times f)$ has at least $|A(G \times f)| + 1$ odd components.

It is easy to see using that G is 2-edge–connected that wherever lie the two end–vertices of f in $G \times f$, there exists a vertex set $X \neq \emptyset$ in G/f so that $(G/f) - X$ has at least $|X|$ odd components. Thus for $x \in X$, $(G/f) - x$ has no perfect matching, which contradicts the assumption that G/f is factor–critical.

Now assume that the statement is true for $|F| \leq k-1$. Let $F \subseteq E(G)$ be a forest so that G/F is factor–critical and $|F| = k$. Let $f \in F$. Consider the graph $G' := G/f$ and let $F' := F - \{f\}$. Then G'/F' is factor–critical, F' is a forest and $|F'| = k-1$, thus by induction $G' \times F'$ is factor–critical. By the induction hypothesis for the graph $G \times F'$ and the edge f, $(G \times F') \times f = G \times F$ is factor–critical. Thus we have proved that if for a forest F, the contraction of F leaves a factor–critical graph, then so does its subdivision.

ii.) Assume that $G \times F$ is factor–critical. Then by Theorem 1.2 it has an odd ear–decomposition. The corresponding odd ear–decomposition of G/F shows that G/F is factor–critical.

Finally, we show the equivalence of b.) and c.).

iii.) If $G \times F$ is factor–critical, then it has an odd ear–decomposition by Theorem 1.2. The corresponding ear–decomposition of G has exactly $|F| = k$ even ears, thus $\varphi(G) \leq k$.

iv.) Let $P_0 + ... + P_l$ be an ear–decomposition of G with $\varphi(G)$ even ears. Choose an edge from each even ear. Then the subdivision of these edges leaves a factor–critical graph, thus $\varphi(G) \geq k$, which completes the proof. □

We need two propositions to prove Lemma 1.5.

Proposition 2.1. *Let v be a vertex of a factor–critical graph H so that $H - v$ is connected. Then there exists an odd ear–decomposition of H such that*

($*$) *the first ear contains v and the other ears contain at most one edge incident to v.*

Proof. We have to slightly modify the algorithm which proves Theorem 1.2 (see [2]).

The first ear is defined by two edges incident to v like in [2]. Now the next ear will be defined by an edge *not* incident to v. Since v is not a cut–vertex in H, we can build up the odd ear–decomposition with this restriction as well. □

The following proposition can be proved similarly as Theorem 5.4.1 in [2] using Theorem 5.4.4 from [2].

Proposition 2.2. *Let H be a 1–extendable graph. Let $g_0, g_1, ..., g_k$ denote the edges incident to a vertex v of H, so that g_0 and g_1 are not parallel. Then there exists an ear–decomposition $Q_1 + Q_2 + ... + Q_l$ of H such that*

($**$) *Q_1 is an even cycle, all the other ears Q_i $(2 \leq i \leq l)$ are paths of odd length, Q_1 contains g_0, g_1 and Q_i contains g_i $2 \leq i \leq k$.* □

Let us turn to the proof of Lemma 1.5.

Proof. (of Lemma 1.5) Let v_i denote the vertex of G_i corresponding to V_i.

By assumption, G_1 and v_1 satisfy the conditions of Proposition 2.1, thus there is an odd ear–decomposition $P_1 + ... + P_h$ of G_1 with property $(*)$. Let e_0 and e_1 be the edges of P_1 incident to v_1. Let $e_2, ..., e_k$ denote the other edges incident to v_1. For $2 \leq i \leq k$ there is an ear P_{π_i} containing e_i in G_1 and by $(*)$ $P_{\pi_i} \neq P_{\pi_j}$ if $i \neq j$. We may assume that $\pi_i < \pi_j$ if $i < j$. Let g_i denote the edge in G_2 corresponding to e_i $(i = 0, 1, ..., k)$. Note that the edges between V_1 and V_2 correspond to $e_0, e_1, ..., e_k$ in G_1 and to $g_0, g_1, ..., g_k$ in G_2.

a.) First assume that g_0 and g_1 are non–parallel in G_2.

By assumption G_2 is a 1–extendable graph, thus by Proposition 2.2 there is an ear–decomposition $Q_1 + ... + Q_l$ of G_2 with property $(**)$.

We show how to build up an odd ear–decomposition of G from these two ear–decompositions. Consider the following ear–decomposition of G. We take the ears $P'_1, ..., P'_h$ of G corresponding to $P_1, ..., P_h$, extending each ear P'_{π_i} by $Q_i - g_i$ and finally we add the remaining ears of G_2. Thus

$$G = P'_1 \cup (Q_1 - g_0 - g_1) + P'_2 + ... + P'_{\pi_2 - 1} + P'_{\pi_2} \cup (Q_2 - g_2) + P'_{\pi_2+1} + ... + P'_{\pi_i} \cup (Q_i - g_i) + ... + P'_{\pi_k - 1} + P'_{\pi_k} \cup (Q_k - g_k) + ... + P'_h + Q_{k+1} + ... + Q_l.$$

It is easily seen that each ear has an odd length, thus by Theorem 1.2 G is factor–critical.

b.) Now consider that case when g_0 and g_1 are parallel in G_2.

Let j be the least index so that g_0 and g_j are not parallel in G_2. Clearly, we may assume that such an edge g_j exists. Let us change the indices 1 and j. Then the graph G_2 and the edges g_0, g_1 satisty the conditions of Proposition 2.2, thus there is an ear–decomposition $Q_1 + ... + Q_l$ of G_2 with property $(**)$. Note that the ears $Q_2, ..., Q_j$ contain single edges $g_2, ..., g_j \neq g_1$.

Consider the following ear–decomposition of G. We take the ears $P'_1, ..., P'_h$ of G corresponding to $P_1, ..., P_h$, extending the ear P'_{π_j} by $Q_1 - g_0 - g_1$ and P'_{π_i} by $Q_i - g_i$ for $j + 1 \leq i \leq k$ and finally we add the remaining ears of G_2. Thus

$$G = P'_1 + ... + P'_{\pi_j - 1} + P'_{\pi_j} \cup (Q_1 - g_0 - g_1) + P'_{\pi_j+1} + ... + P'_{\pi_{j+1}-1} + P'_{\pi_{j+1}} \cup (Q_{j+1} - g_{j+1}) + P'_{\pi_{j+1}+1} + ... + P'_{\pi_i} \cup (Q_i - g_i) + ... + P'_{\pi_k - 1} + P'_{\pi_k} \cup (Q_k - g_k) + ... + P'_h + Q_{k+1} + ... + Q_l.$$

It is easy to see that this is an odd ear–decomposition of G, thus by Theorem 1.2 G is factor–critical. This completes the proof. □□□

3. The matroid property

In this section we prove that the ear–extreme edge–sets are the independent sets of some matroid. First of all we show that the maximal ear–extreme edge–sets have the same cardinality.

Lemma 3.1. *Any ear–extreme edge–set of G can be extended to an ear–extreme edge–set of size $\varphi(G)$.*

Proof. Let Y be an ear–extreme edge–set of G. Let Z be an ear–extreme edge–set of $G \times Y$ for which $\varphi((G \times Y) \times Z) = 0$. Then $Y \cup Z$ is an ear–extreme edge–set of G with size $\varphi(G)$. □

For graphs with $\varphi(G) = 1$ the ear–extreme edge–sets trivially form the independent sets of a matroid. In this case the set of ear–extreme edges can be characterized and a structure theorem can be given for these graphs similar to the Cathedral Theorem for saturated graphs due to L.Lovász [2]. This result can be found in [5].

Theorem 3.2 contains our basic observation.

Theorem 3.2. *Let G be a graph with $\varphi(G) = 1$, and let $F \subseteq E(G)$ be any forest of G such that $\varphi(G \times F) = 0$. Then F contains an edge e for which $\varphi(G \times e) = 0$.*

Proof. Let G be a counterexample to the theorem with minimum number of vertices. The theorem is trivially true for 1–extendable graphs by Theorem 1.3. Thus G is not bicritical.

From the assumption that $\varphi(G) = 1$ it follows easily that G has a perfect matching. Let X be a maximal barrier of G. Then, clearly, all the components of $G - X$ are factor–critical. Since G is not bicritical, $|X| \geq 2$ by [2, Theorem 5.2.5]. By Lemma 1.8 there exists a strong subgraph H attached at $Y \subseteq X$ such that all the factor–critical components of $G - Y$ are components of $G - X$. Then $G - Y$ has at least two connected components. Let $C_1, ..., C_{|Y|}$ be the factor-critical components of $G - Y$. Let $C_i^* = G/(G - C_i)$ and $F_i = E(C_i) \cap F$. We may assume that F does not contain any edge connecting Y with some C_i, for otherwise this edge is ear–extreme in G by [1, Theorem 4.3/b] and by Theorem 1.7. Thus F_i is a forest in C_i^*.

Lemma 3.3. *For some component C_i $\varphi(C_i^* \times F_i) = 0$.*

Proof. Let s_i denote the vertex in C_i^* corresponding to $G - C_i$.

Since $G \times F$ is factor–critical, $G \times F - u$ has a perfect matching for any $u \in Y$. Therefore $C_i^* \times F_i - s_i$ has a perfect matching for some component C_i. We show that $\varphi(C_i^* \times F_i) = 0$. Assume that $N := C_i^* \times F_i$ is not factor–critical. Clearly, N has no perfect matching and since N is not factor-critical, $A(N) \neq \emptyset$. Since $N - s_i$ has a perfect matching, the vertex s_i is in $D(N)$. Let L denote the factor–critical component of $D(N)$ containing s_i. Then there is a strong subgraph in the graph induced by the vertices $A(N) \cup (D(N) - V(L))$ by Lemma 1.8. This is a strong subgraph in $G \times F$ as well which contradicts the fact that $G \times F$ is factor–critical (by Theorem 1.6). □

Since C_i is factor–critical, $\varphi(C_i^*) = 1$. $\varphi(C_i^* \times F_i) = 0$, and $|V(C_i^*)| < |V(G)|$ because $G - Y$ has at least two connected components. Therefore, the minimality of G implies that there exists an edge $e \in F_i$ for which $\varphi(C_i^* \times e) = 0$, i.e. $C_i^* \times e$ is factor–critical.

We show that $\varphi(G \times e) = 0$.

By definition, the graph H', obtained from H by deleting the edges spanned by Y, and contracting each factor–critical component of $H - Y$ to a single vertex, is a 1–extendable bipartite graph. Thus by Lemma 1.5 the graph obtained from H' by "blowing up" $C_i^* \times e$ is factor–critical. (In other words, we replaced the vertex corresponding to C_i by $C_i^* \times e$.) This implies that $H \times e$ is factor–critical by Theorem 1.4 and by Theorem 1.7 G/H is factor–critical as well, whence by Theorem 1.4 $G \times e$ is factor–critical. This completes the proof of Theorem 3.2. □□□

Theorem 3.4. *Let G be a graph with $\varphi(G) = \varphi$, and let $F \subseteq E(G)$ be a forest of G such that $\varphi(G \times F) = 0$. Then there exists an edge–set $F' \subseteq F$ for which $|F'| = \varphi$ and $\varphi(G \times F') = 0$.*

Proof. We prove by induction on φ. For $\varphi = 1$ it is true by the previous theorem. Assume that the theorem is true for all graphs G^* with $\varphi(G^*) = \varphi - 1$. Let G be a graph with $\varphi(G) = \varphi$.

Let e be an ear–extreme edge of G. If $e \in F$, then we are done by induction for $G \times e$. Thus we may assume that $e \notin F$. Let $G' = G \times e$ and let e_1 and e_2 be the two new edges in G'. Then $\varphi(G') = \varphi - 1$ and $\varphi(G' \times (F \cup \{e_1\})) = 0$. By the induction hypothesis there exists an edge–set $F_1 \subseteq F \cup \{e_1\}$ such that $|F_1| = \varphi - 1$ and $\varphi(G' \times F_1) = 0$. Furthermore, $F_1 \subseteq F$, otherwise $\varphi(G) \leq \varphi - 2$. Thus $F_1 \subset F, |F_1| = \varphi - 1, \varphi(G \times F_1) = 1$. Using the previous theorem for the graph $G \times F_1$ and the edge–set $F - F_1$, $\varphi(G \times F_1) = 1, \varphi((G \times F_1) \times (F - F_1)) = 0$, we get that there exists an edge $f \in F - F_1$ such that $\varphi((G \times F_1) \times f) = 0$. Let $F' = F_1 \cup \{f\}$. Then $F' \subseteq F, |F'| = \varphi, \varphi(G \times F') = 0$. This was to be proved. □

Now we are ready to prove our main result. It is interesting that Theorem 3.2 enables us to prove this theorem and we do not need Theorem 3.4.

Main Theorem. *The ear–extreme edge–sets of any 2-edge–connected graph G form the independent sets of a matroid.*

Proof. Let G be a counterexample with minimum $\varphi(G)$. For $\varphi = 1$ there is nothing to prove. Let $M = \{F \subseteq E : \varphi(G \times F) = \varphi(G) - |F|\}$. Our assumption for G means that there are two sets $F_1, F_2 \in M$, with $|F_1| = |F_2| = \varphi(G)$, and $f \in F_1 \setminus F_2$ such that for every edge $e \in F_2 \setminus F_1$ $(F_1 \setminus \{f\}) \cup \{e\} \notin M$, and the theorem is true for graphs with smaller value of φ.

First we show that $F_1 \cap F_2 = \emptyset$. Suppose there is an edge $e \in F_1 \cap F_2$. Let $G' = G \times e$. Then $\varphi(G') = \varphi(G) - 1$ and $F_1 \setminus \{e\}$ and $F_2 \setminus \{e\}$ are ear–extreme edge–sets in G'. From the minimality of G follows that there exists $f' \in F_2 \setminus F_1$ such that $(F_1 \setminus \{e\} \setminus \{f\}) \cup \{f'\}$ is an ear–extreme edge–set with maximum size in G', that is $(F_1 \setminus \{f\}) \cup \{f'\} \in M$. This contradicts our assumption.

Let $G^* = G \times (F_1 \setminus \{f\})$ and for every edge $g \in F_1 \setminus \{f\}$ let g_1 and g_2 denote the two edges in G^* corresponding to g. Let $T = \{g_1 : g \in F_1 \setminus \{f\}\}$. Then $\varphi(G^*) = \varphi(G \times (F_1 \setminus \{f\})) = 1$ and $\varphi(G^* \times (F_2 \cup T)) = 0$. Theorem 3.2 implies the existence of an edge $e \in F_2 \cup T$ such that $\varphi(G^* \times e) = 0$. Furthermore, $e \in F_2 \setminus F_1$, for otherwise, if $e = g_1$ for some edge $g \in F_1 \setminus \{f\}$, then $\varphi(G \times (F_1 \setminus \{f\} \setminus \{g\})) = 0$ contradicts $\varphi(G) = \varphi$.

Therefore, there exists an edge $e \in F_2 \setminus F_1$ such that $\varphi(G \times ((F_1 \setminus \{f\}) \cup \{e\})) = 0$, that is, $(F_1 \setminus \{f\}) \cup \{e\} \in M$, contradicting our assumption. □□□

If $\varphi(G) = 2$ then we can say more, in this case the matroid is a partitional matroid, but we omit the proof of this theorem. (see [4] or [6]) When $\varphi(G) \geq 3$ then this matroid is not a partitional matroid in general.

Theorem 3.5. *Let G be a graph with $\varphi(G) = 2$. In this case the ear–extreme edge–sets of G form the independent sets of a partitional matroid.* □

We conclude this paper with some algorithmic aspects. Let us recall the problem we mentioned in the abstract. Given a connected graph G and a non-negative weighting w on its edges, *what is the minimum weight of an edge-set whose contraction leaves a factor-critical graph?* Since any minimum weight edge-set J with this property is trivially a forest, by Theorem 3.4 J includes a critical-making set with size $\varphi(G)$ and of weight not more than $w(J)$. By the Main Theorem the ear–extreme edge-sets form the independent sets of a matroid, thus the greedy algorithm gives rise to the desired algorithm, using as an independence oracle the polynomial time algorithm developed by A.Frank in [1] for computing the value $\varphi(H)$ for any 2-edge-connected graph H.

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