Boxicity

Henning Bruhn

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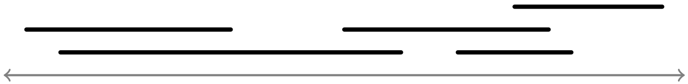
Morgan Chopin, Felix Joos, Oliver Schaudt

Interval graphs



- attractive (algorithmic) properties
- rather limited class
- no graph with induced C_4 is interval

Interval graphs

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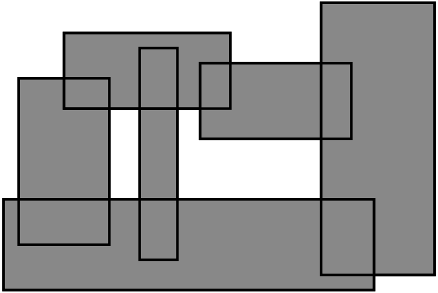
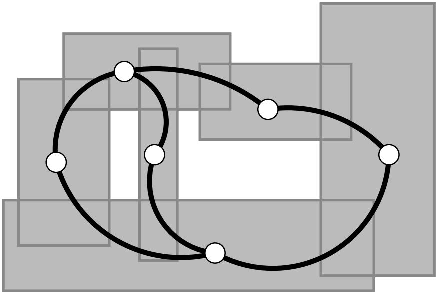
Box intersection graphs



- intersection graph of axis-parallel boxes in some $\mathbb{R}^d$
- some attractive algorithmic properties

Box intersection graphs



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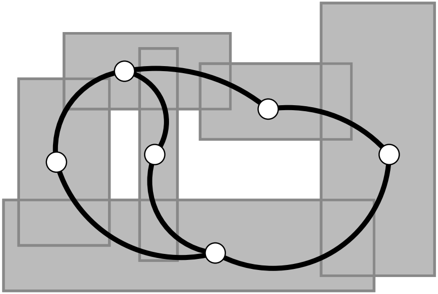
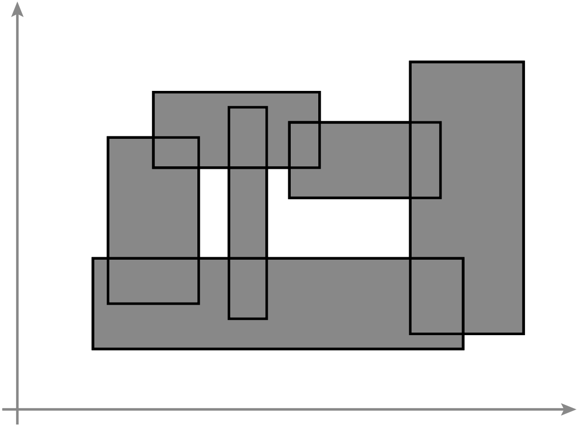
- some attractive algorithmic properties

Two views



G has box-representation in $\mathbb{R}^d$ $\iff$ G is intersection of d interval graphs

Two views



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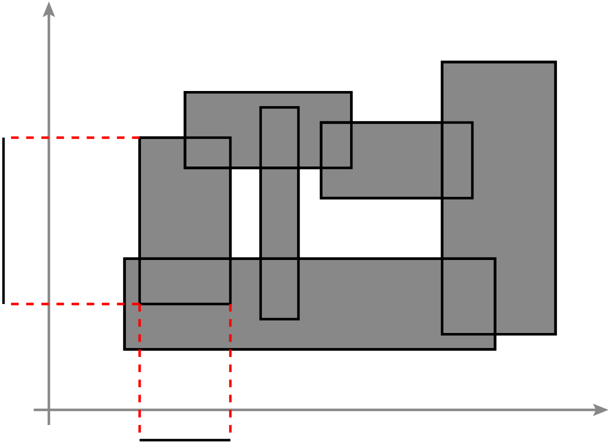

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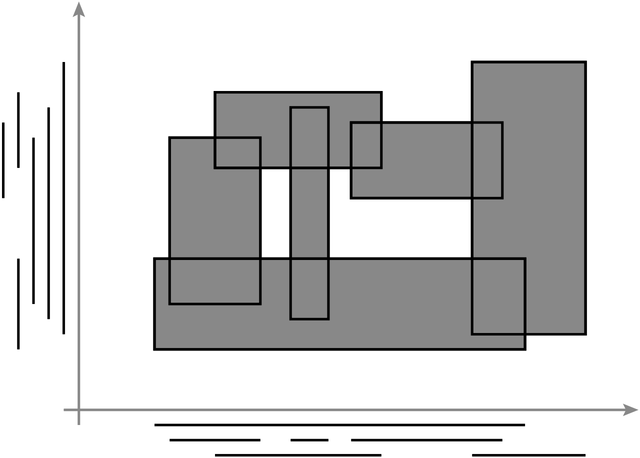

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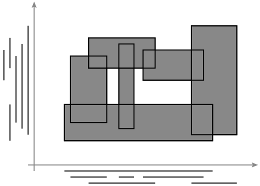
Boxicity



FACT every graph has box representation in some $\mathbb{R}^d$

Roberts 1969: even $d \leq \frac{|V|}{2}$.

DEF box(G): smallest d s.t. G has box representation in $\mathbb{R}^d$

Some properties

- $\text{box}(G) \leq \Delta(G)^2 + 2$ (Esperet)
- $\text{box}(G) \leq O(\Delta(G)\log(\Delta(G))^2)$ (Adiga, Bhowmick,Chandran)
- $\text{box}(G) \leq tw(G) + 2$ (Chandran and Sivadasan)

- $\text{box}(G) = 1 \Leftrightarrow G$ interval graph
- $\text{box}(G) \leq 2$ for any outer-planar graph (Scheinerman)
- $\text{box}(G) \leq 3$ for any planar graph (Thomassen)

Applications?

ECOLOGY LETTERS

Ecology Letters, (2013) **16**: 577–583 doi: 10.1111/ele.12081

LETTER

The dimensionality of ecological networks

Anna Eklöf,[1†*] Ute Jacob,[2] Jason Kopp,[1] Jordi Bosch,[3] Rocío Castro-Urgal,[4] Natacha P. Chacoff,[5] Bo Dalsgaard,[6] Claudio de Sassi,[7] Mauro Galetti,[8] Paulo R. Guimarães,[9] Silvia Beatriz Lomáscolo,[5,10] Ana M. Martín González,[6,11] Marco Aurelio Pizo,[12] Romina Rader,[13] Anselm Rodrigo,[3] Jason M. Tylianakis,[7] Diego P. Vázquez,[5,10] and Stefano Allesina,[1,14]

Abstract

How many dimensions (trait-axes) are required to predict whether two species interact? This unanswered question originated with the idea of ecological niches, and yet bears relevance today for understanding what determines network structure. Here, we analyse a set of 200 ecological networks, including food webs, antagonistic and mutualistic networks, and find that the number of dimensions needed to completely explain all interactions is small (< 10), with model selection favouring less than five. Using 18 high-quality webs including several species traits, we identify which traits contribute the most to explaining network structure. We show that accounting for a few traits dramatically improves our understanding of the structure of ecological networks. Matching traits for resources and consumers, for example, fruit size and bill gape, are the most successful combinations. These results link ecologically important species attributes to large-scale community structure.

Keywords

Ecological networks, food web structure, intervality, niche space, species traits, scaling.

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INTRODUCTION

Will two individuals of different species interact if given the opportunity? If the two species have matching traits, then an interaction

Knowing the maximum number of dimensions needed to fully describe complex ecological networks is important, as current ecological theory implicitly relies on the assumption that few dimensions are needed: species compete for few limiting factors in

Applications?

Boxicity and the social context of Swedish literary criticism, 1881–1883

Linton C. Freeman

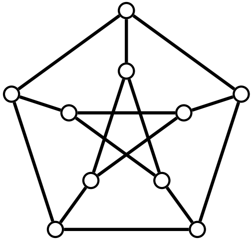
School of Social Sciences, University of California, Irvine, CA 92717, USA

This paper attempts to case new light on the results of an earlier study of the social context of Swedish literary criticism. Data from a study of persons mentioned by literary critics are re-analysed using new ideas and techniques from biology, mathematics and social network analysis. The results of this new analysis suggest that there may be previously unrecognized structural dimensions of the 'frame of reference' within which new literary works are evaluated.

Introduction

This is a paper on sociology, but it will draw on several other fields and try to show how ideas and tools from those fields may help to cast some light on a sociological problem.

Calculating the boxicity...



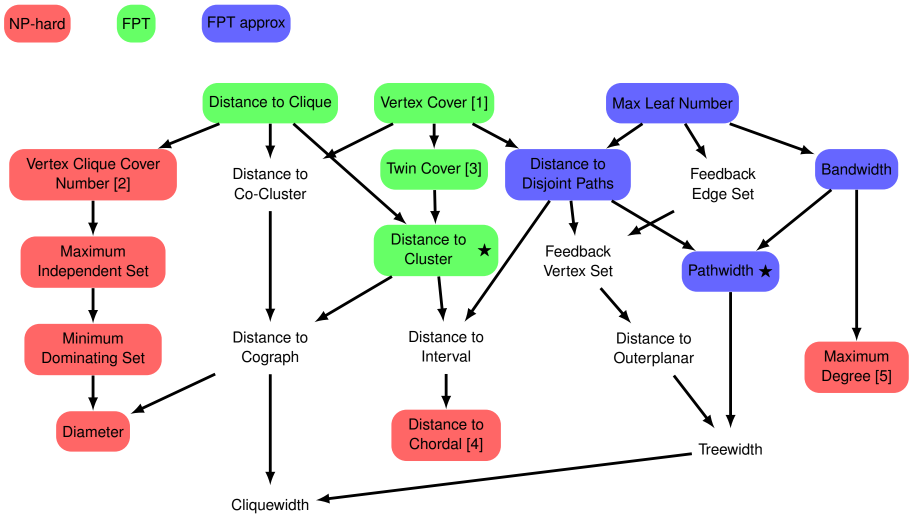
- ...not an easy task: try Petersen graph
- ...NP-complete, even for constant $d \geq 2$ (Cozzens, Yannakakis, Kratochvíl)

instead: try **FPT-**algorithm, that is

algorithm for box(G) of $f(k) \cdot n^{O(1)}$-time, where

- n : number of vertices
- k : some parameter of G (treewidth, girth, max degree...)

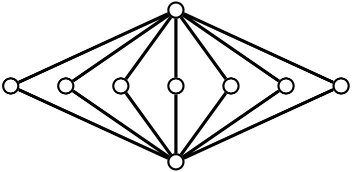
Parameter map

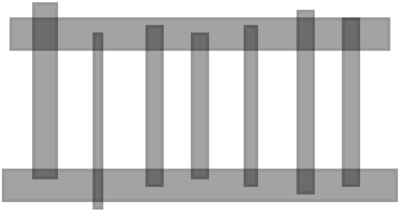


[1] Adiga, Chitnis and Saurabh 2010
[2] Yannakakis 1982
[3] Ganian 2011
[4] Adiga, Bhowmick and Chandran 2010
[5] Kratochvíl 1994

★ our results

Bounded treewidth probably doesn't help





G bounded treewidth $\longrightarrow$ one dimension unbounded tw

Our results

Theorem (B, Chopin, Joos, Schaudt)

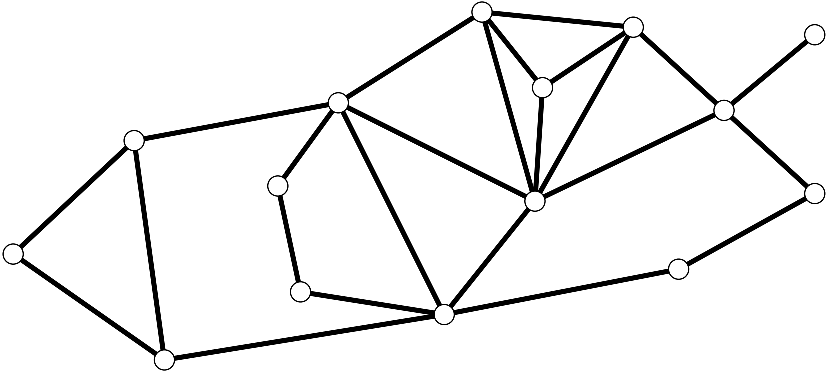
Boxicity is FPT when parameterised by distance to cluster

Theorem (B, Chopin, Joos, Schaudt)

$+1$*-approximation algorithm that is FPT when parameterised by pathwidth*

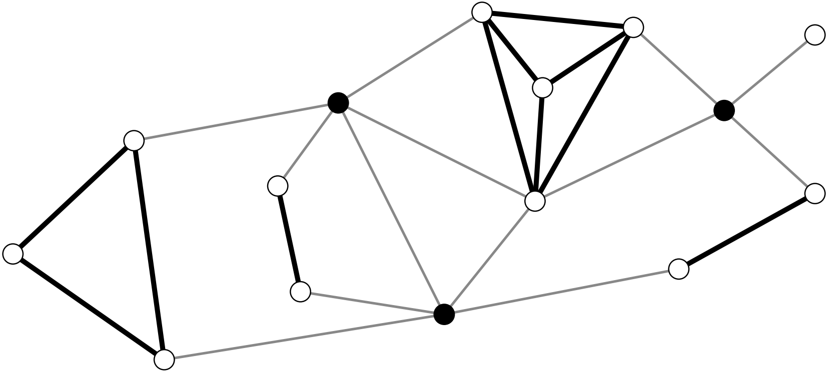
+a hint of hardness for bandwidth as parameter.

Distance to cluster



- cluster graph: disjoint union of cliques
- distance to cluster: smallest $|X|$, so that $G - X$ cluster graph

Distance to cluster



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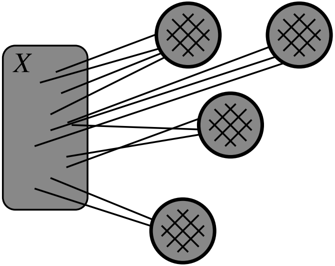
Set up

Theorem

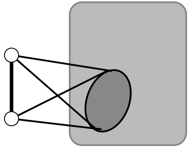
Algorithm of running time $f(k)n^{O(1)}$ for boxicity, where k is the distance to cluster



- Parameter k
- Compute X with $|X| \leq 3k$ s.t. $G - X$ cluster graph

AIM delete all but $f(k)$ vertices without changing the boxicity

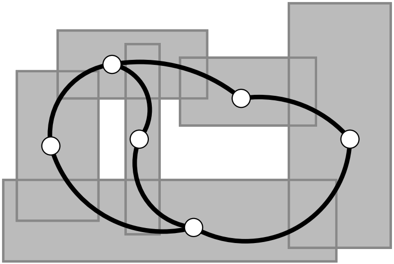
True twin reduction



True twins:

- adjacent
- same neighbourhood

FACT Deleting true twins does not change boxicity



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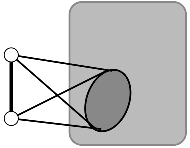

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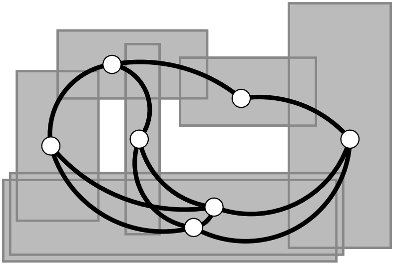
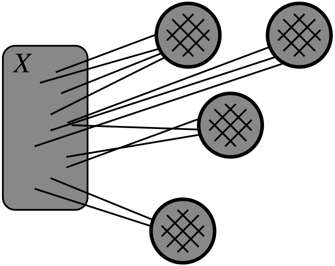


Strategy

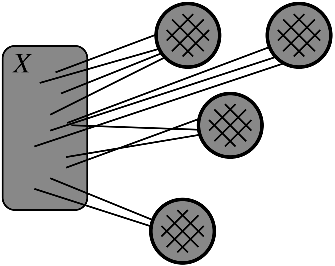


AIM Mimick true twin reduction:

- find two clusters K, K' that are the same in G
- delete K'
- get box representation of $G - K'$
- add boxes for K' that are copies of the boxes of K

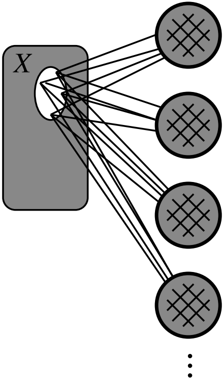
$\rightarrow$ for this to work we need many identical clusters

Clusters have bounded size



- Cluster with many vertices $\rightarrow$ true twins!
- Delete one twin, reduce size
- may assume: each cluster of bounded size

Many clusters are the same



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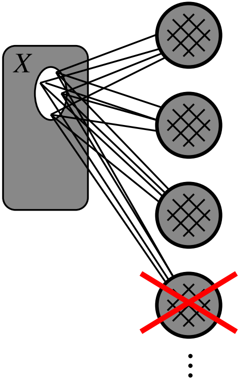

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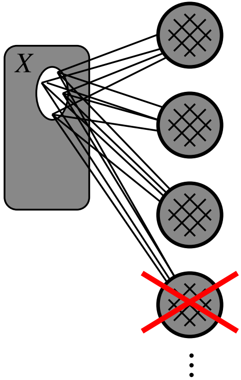

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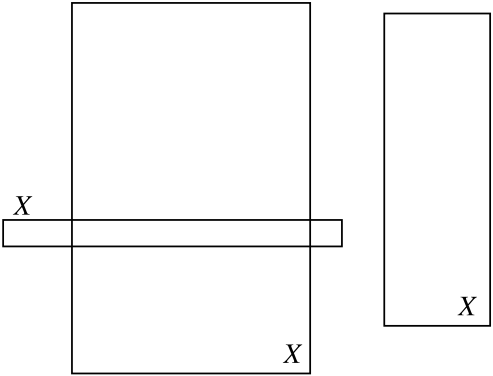
CLAIM Boxicity does not change

$\rightarrow$ Proof of claim enough for kernel!

Squeeze in



- many identical clusters
- many of them “same” box representation

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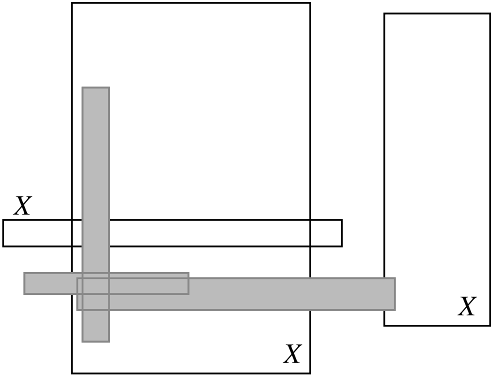

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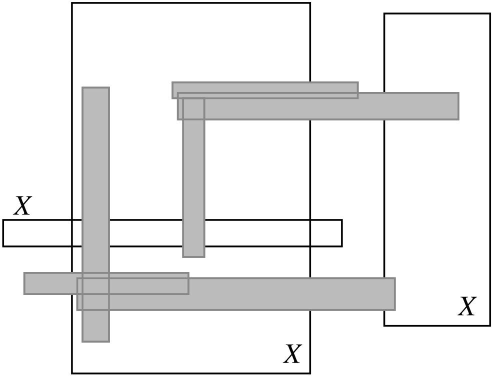

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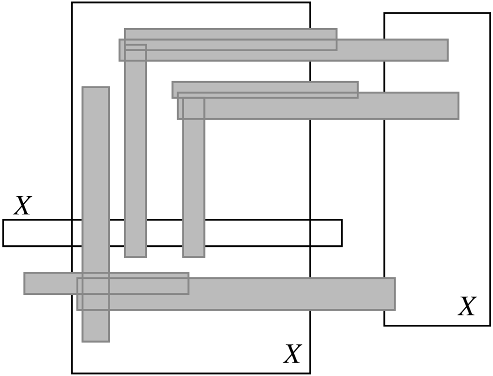

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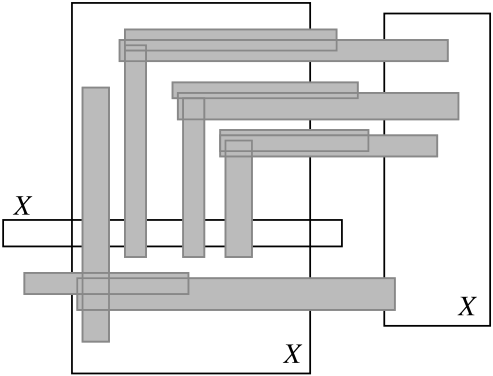

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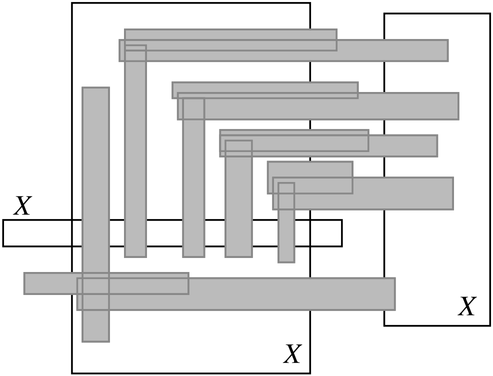

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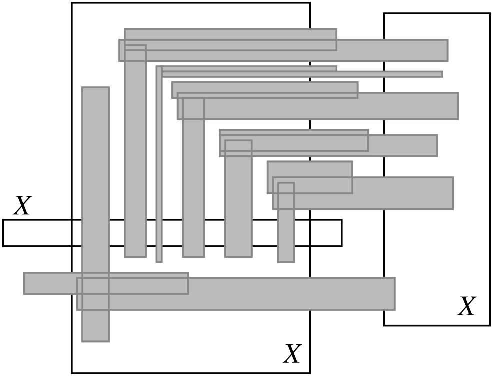

- many identical clusters
- many of them “same” box representation
- then: place for one more cluster

Extensions?

- distance to stars (needs false twin reduction)
- distance to bounded size?
- distance to disjoint paths?? (needs new ideas!)

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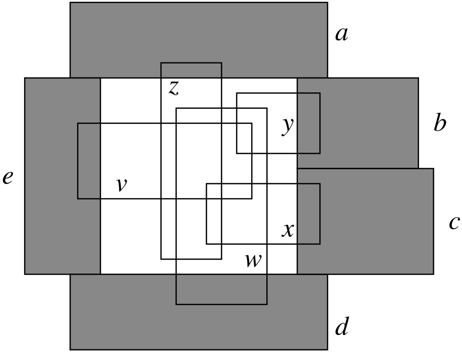
- distance to bounded size?
- distance to disjoint paths?? (needs new ideas!)

- prove hardness result for bounded treewidth!

A last box

Thanks for your attention!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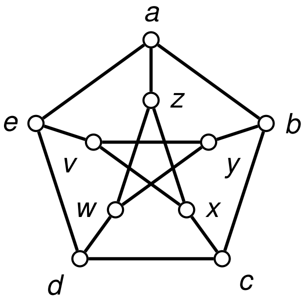
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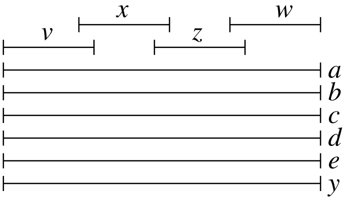
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