Old and young

Perfect graphs and their cliques

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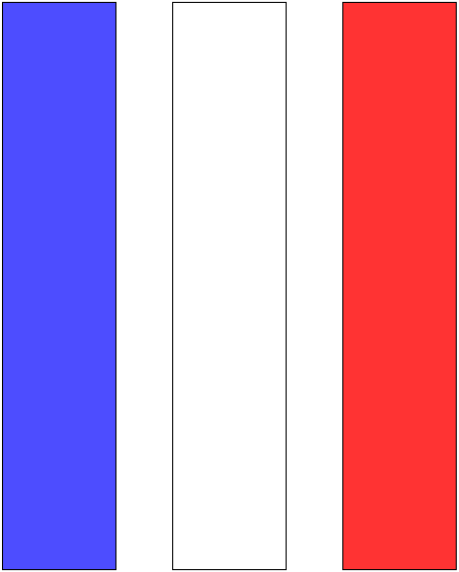
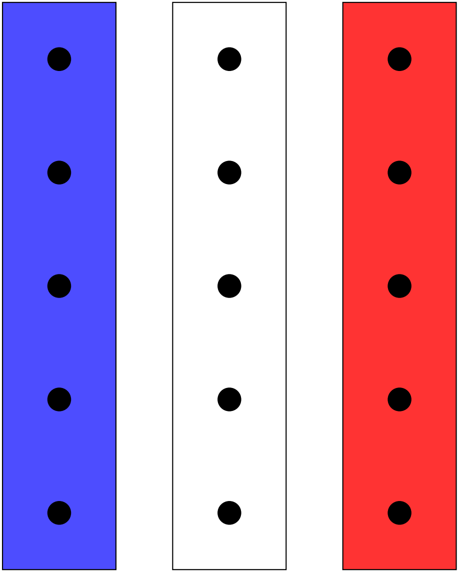
Section 1

First (and last) part: Clique-pairs in uniquely colorable perfect graphs

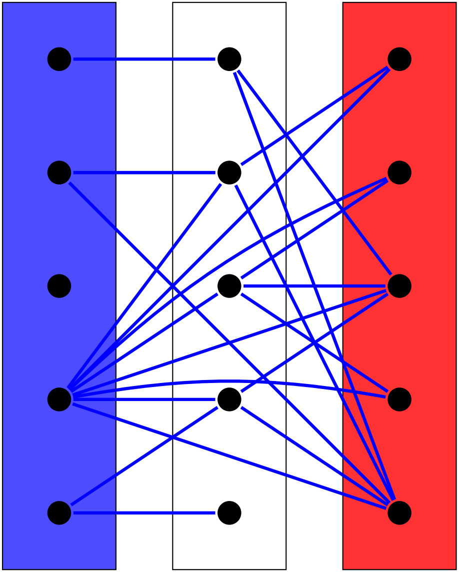
We are here



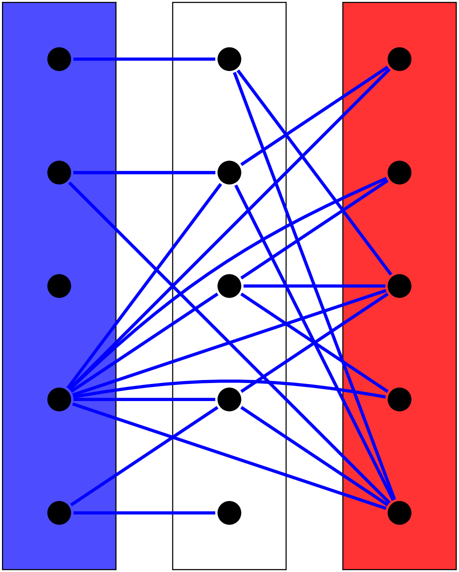
+ vertices



$+$ edges



$\chi(G) = \omega(G) = 3$



Definition (Uniquely colourable graph)

Every minimum colouring yields the same partition of the vertex set.

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(That is: all the minimum colourings can be obtained by colour permutations from one another.)

Definition

R is a rectangle graph:

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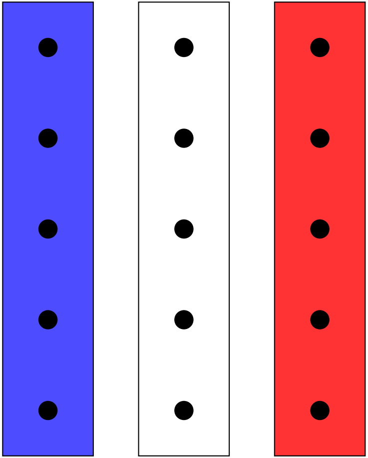
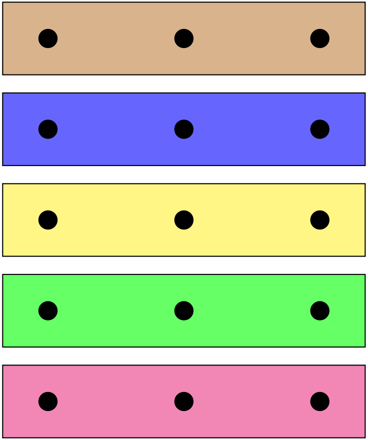
Columns: cliques



Definition

R is a rectangle graph:

Rows: stable sets



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The graph P is partitionable if for every vertex v, $P - v$ is a rectangle graph.

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Theorem (Lovász)

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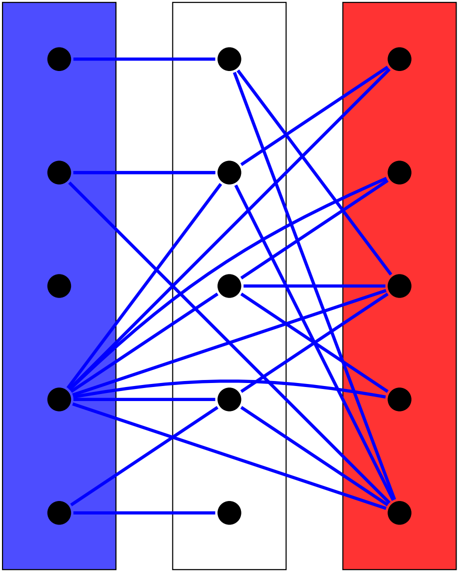
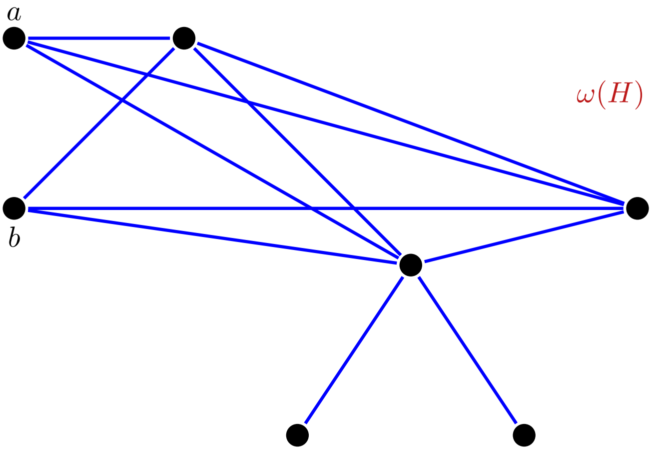
Theorem (Padberg)

For every partitionable graph, the remaining rectangles are uniquely colorable.

$\chi(G) = \omega(G) = 3$



Another graph H



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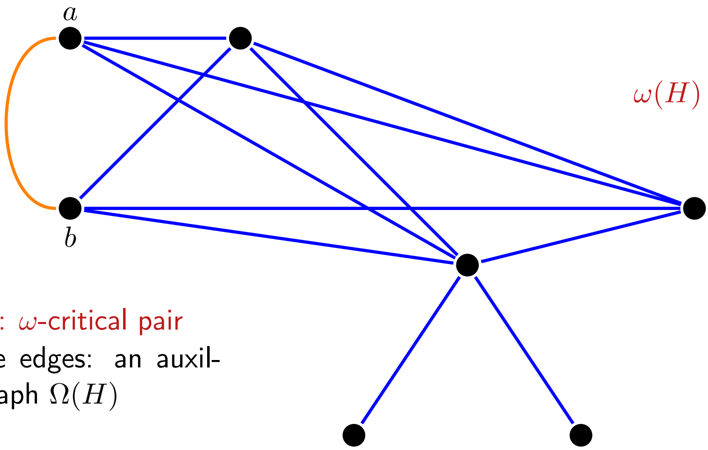

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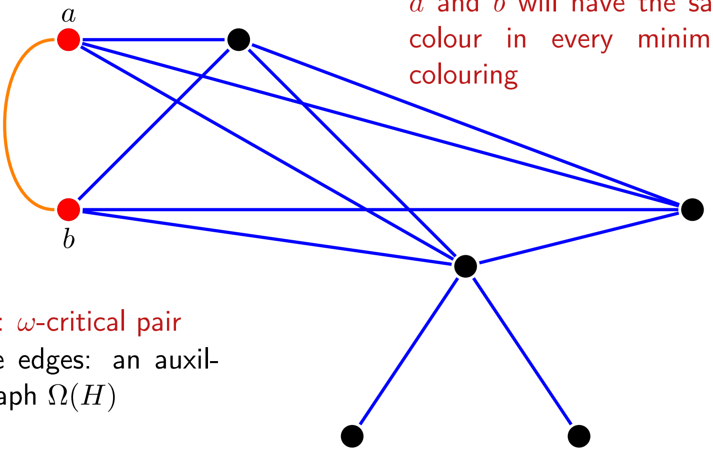
a, b: ω-critical pair

Orange edges: an auxiliary graph $\Omega(H)$

Another graph H

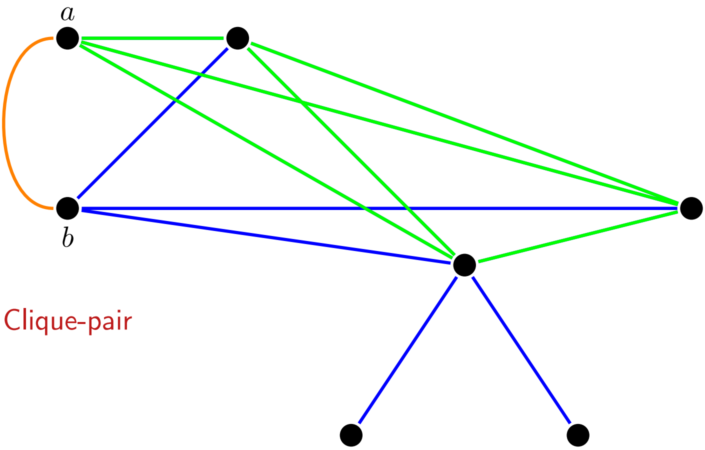


a and b will have the same colour in every minimum colouring

a, b: ω-critical pair

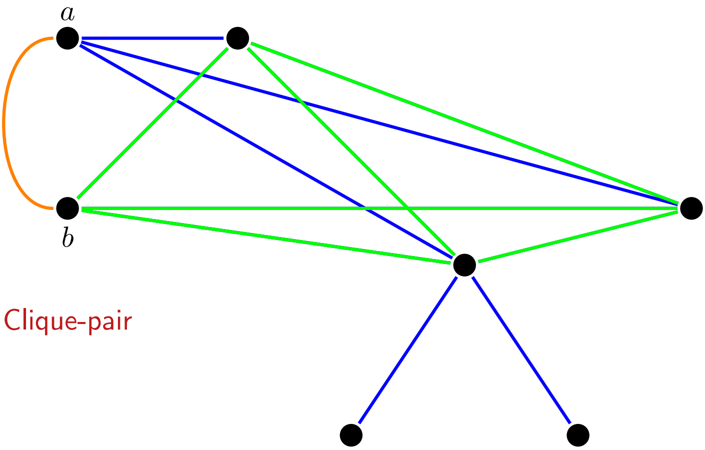
Orange edges: an auxiliary graph $\Omega(H)$

Another graph H



Clique-pair

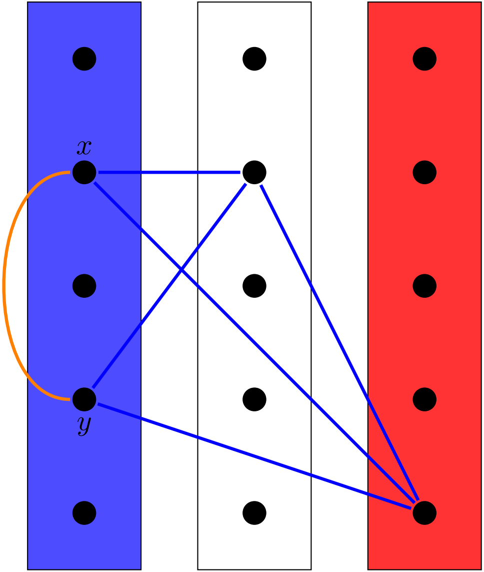
Another graph H



Clique-pair

Extracted from G above

x, y: an ω-critical pair



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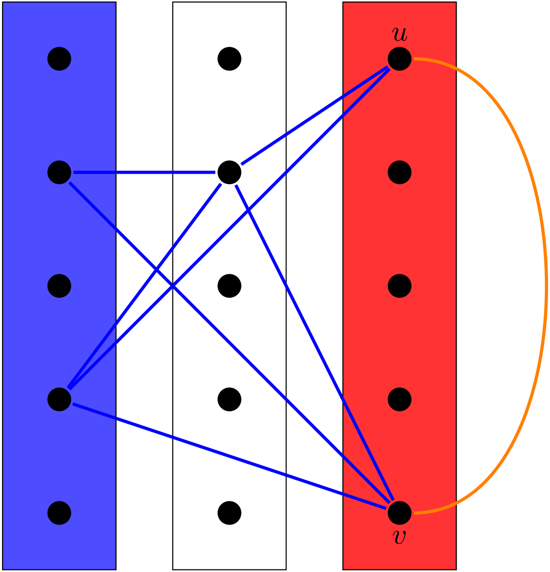
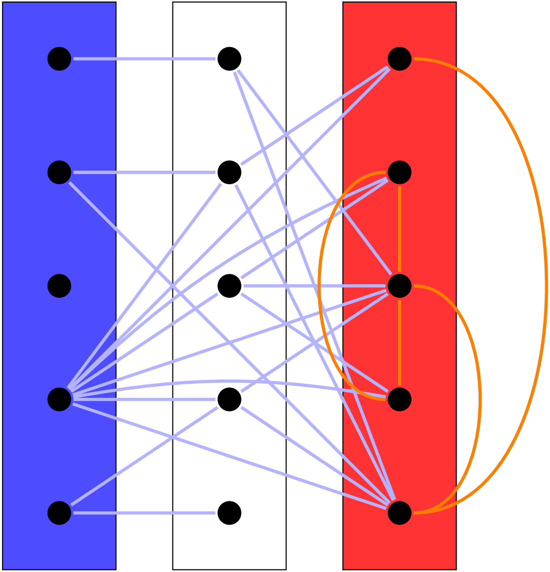


u, v: another ω-critical pair

Extracted from G above



The ω-critical pairs within the red colour class

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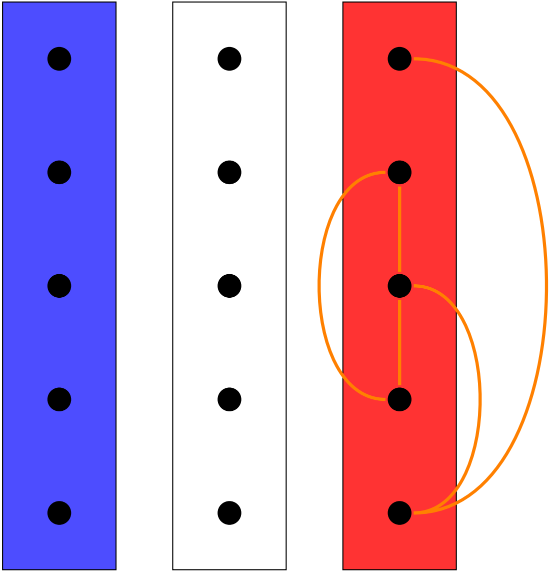

The ω-critical pairs within the red colour class

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A class of graphs C is nice if in every uniquely colourable graph G in C with $\chi(G) = \omega(G)$, there exists a colour class where the auxiliary (orange) graph $\Omega(G)$ is connected.

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Proposition

If $\Omega(G)$ is connected in a colour class D then all the vertices in D necessarily have the same colour. "D can be omitted."

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A graph is Meyniel if every odd cycle of size at least 5 has at least two chords.

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An example for a nice class of perfect graphs: co-Meyniel graphs.

Theorem

In fact, if a co-Meyniel graph G is uniquely colorable then every color class is complete in $\Omega(G)$.

Clique-pair Conjecture

If G is uniquely colourable and perfect then it has at least one clique-pair (that is, at least one ω-critical pair of vertices).

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The Clique-pair Conjecture is true for co-Meyniel graphs (see above), Meyniel graphs, comparability graphs and their complements, for claw-free graphs and their complements, finally for K_4-free graphs.

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The Clique-pair Conjecture is true for co-Meyniel graphs (see above), Meyniel graphs, comparability graphs and their complements, for claw-free graphs and their complements, finally for K_4-free graphs.

(The proof of the latter is the most difficult [J. Fonlupt–A. Sebő; Zemirline])

Section 2

(Maximal cliques in perfect graphs)

Section 3

References

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- G. Bacsó, Zs. Tuza: Clique-transversal sets and weak 2-colorings in graps of small maximum degree, Discrete Mathematics and Theoretical Computer Science 11:2(2009 15-24

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Thanks for your attention!