

# Combinatorial Optimization and Graph Theory ORCO

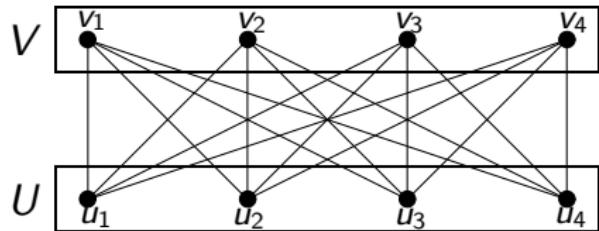
Matchings in bipartite graphs  
Execution of the Hungarian Method

Zoltán Szigeti

# Algorithm Hungarian method (Kuhn)

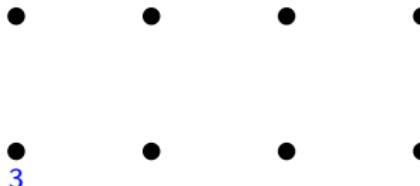
## Example

Execute the Hungarian method to find a minimum weight perfect matching in the following bipartite graph.

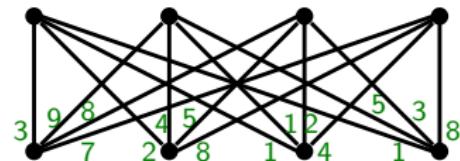


	$u_1$	$u_2$	$u_3$	$u_4$
$v_1$	3	2	1	1
$v_2$	9	4	1	5
$v_3$	8	5	2	3
$v_4$	7	8	4	8

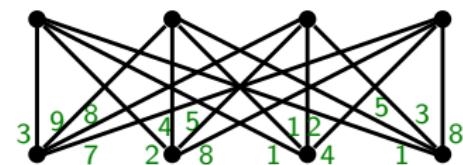
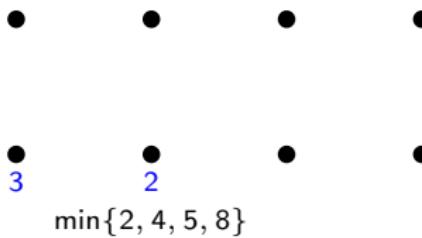
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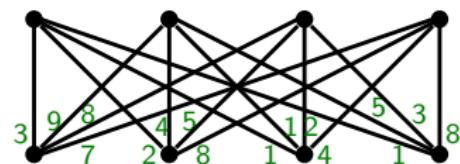
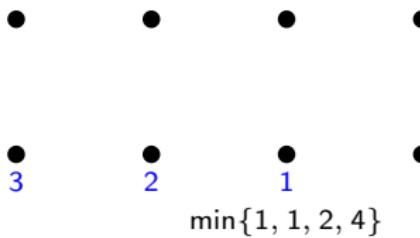
$$\min\{3, 9, 8, 7\}$$



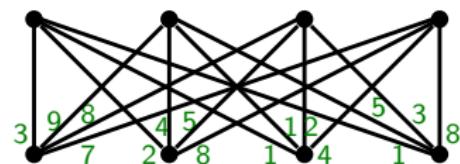
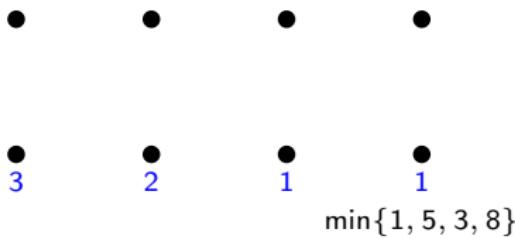
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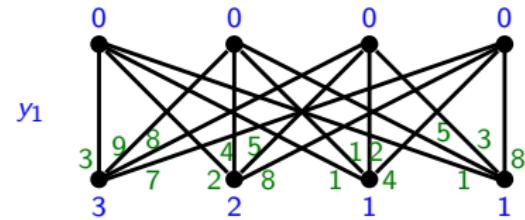
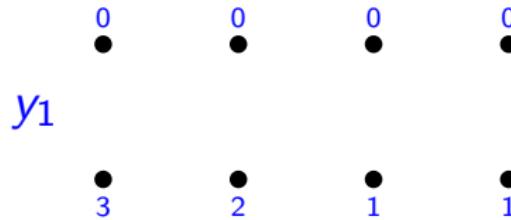
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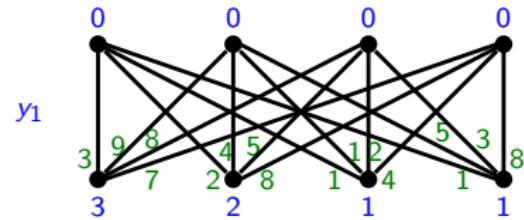
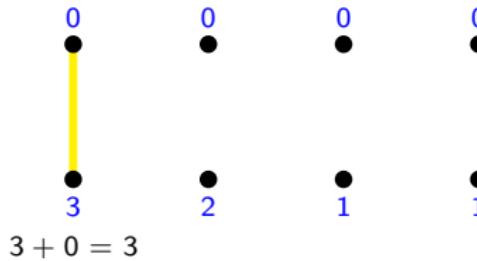
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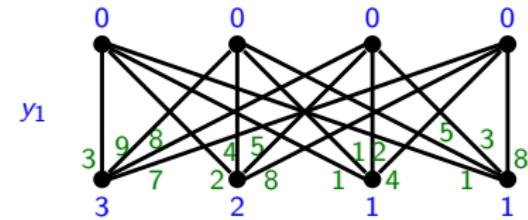
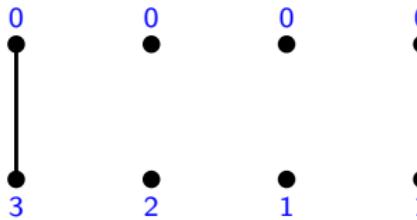
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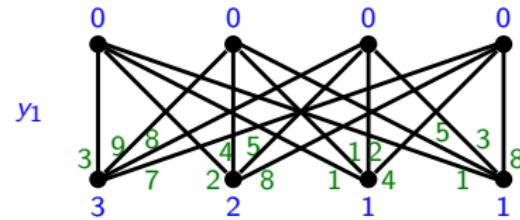
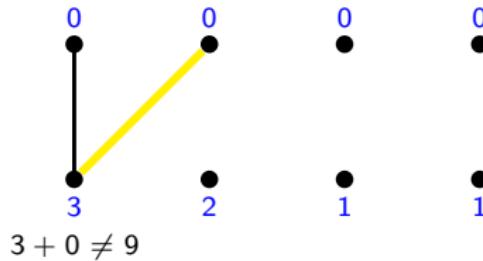
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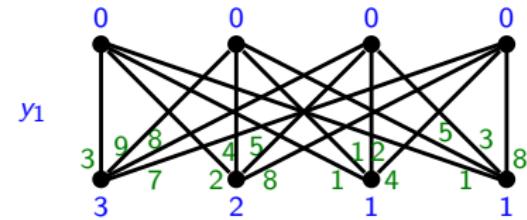
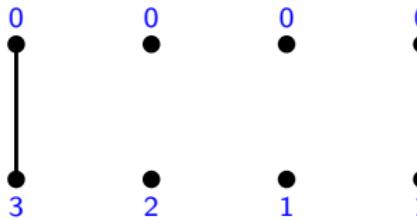
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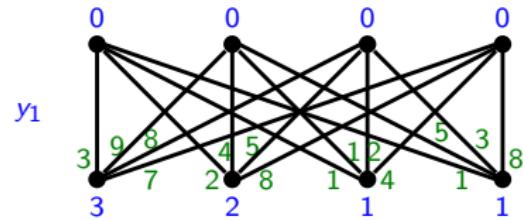
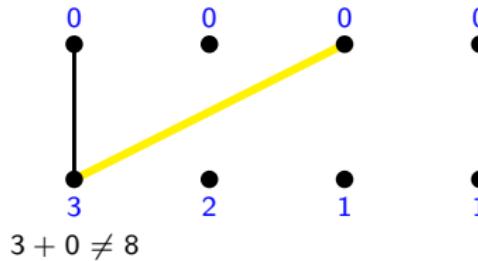
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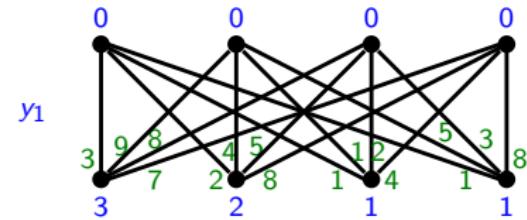
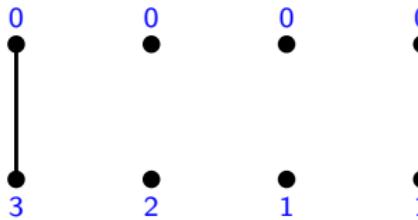
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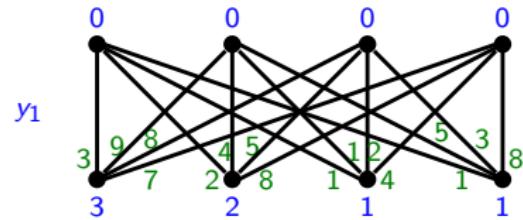
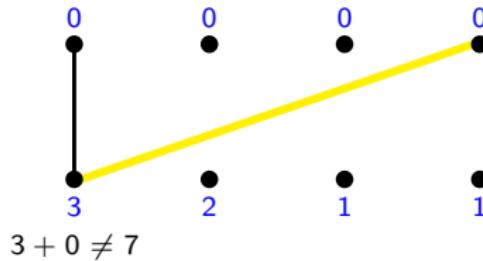
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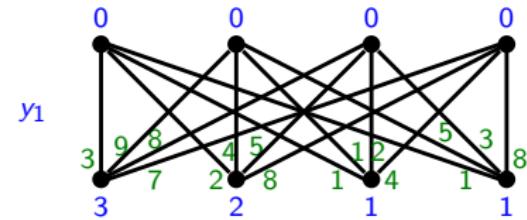
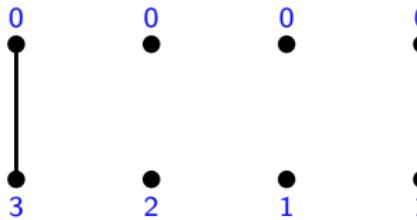
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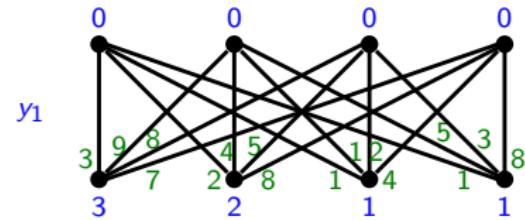
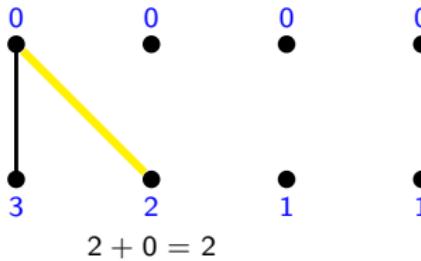
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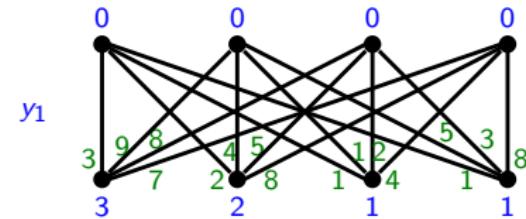
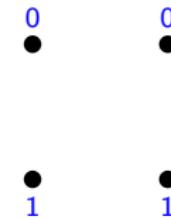
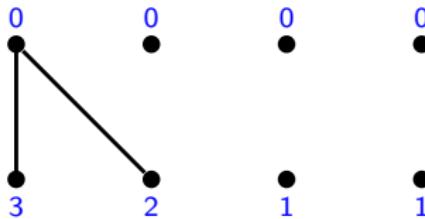
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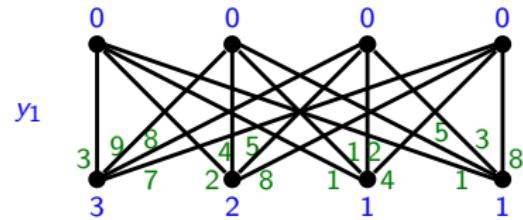
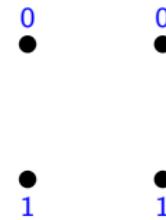
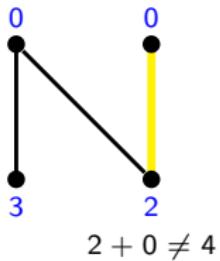
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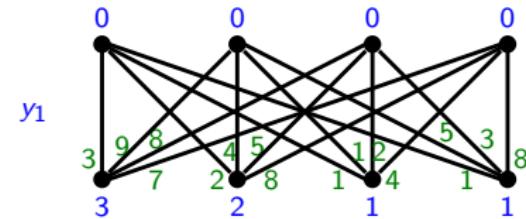
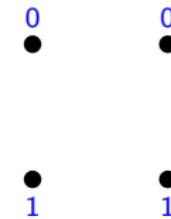
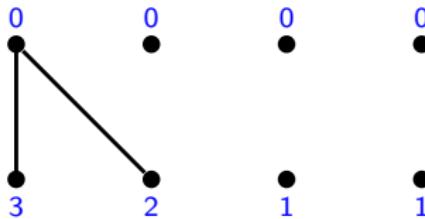
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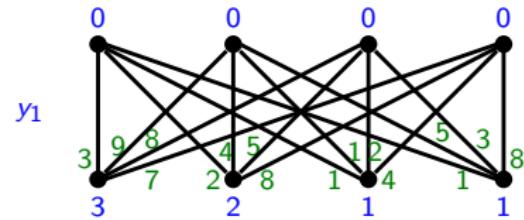
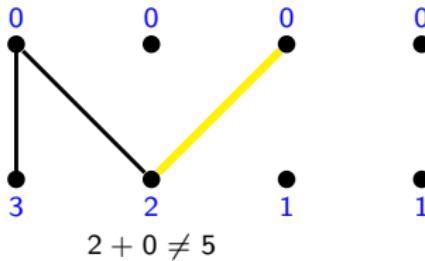
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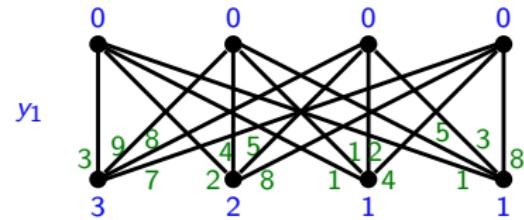
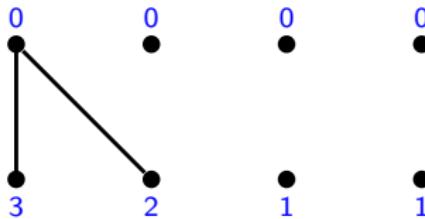
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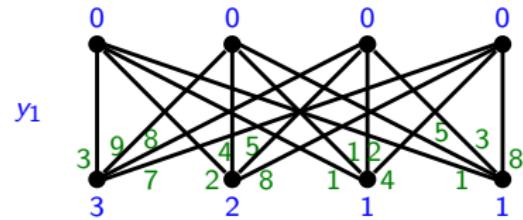
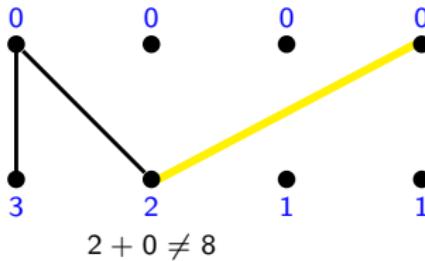
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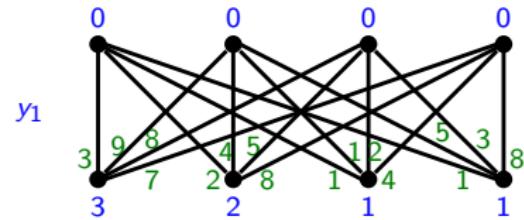
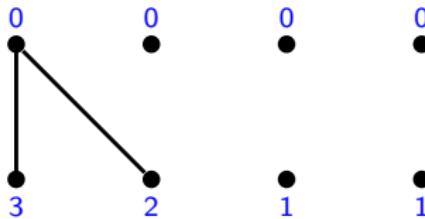
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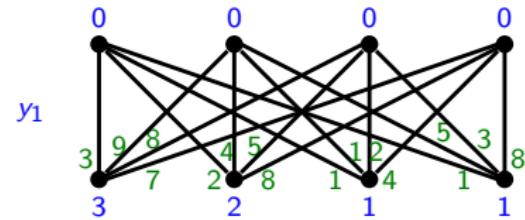
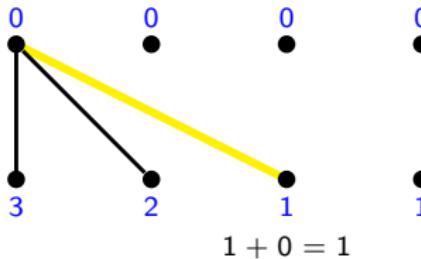
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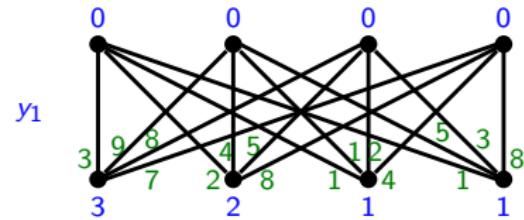
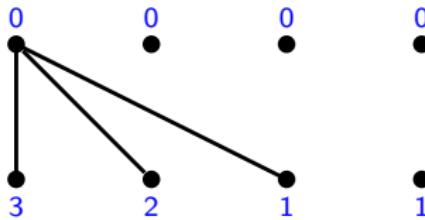
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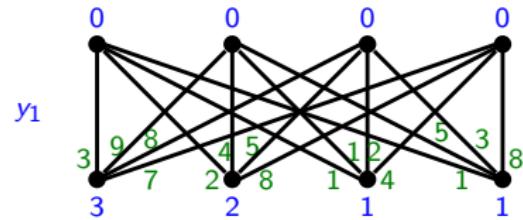
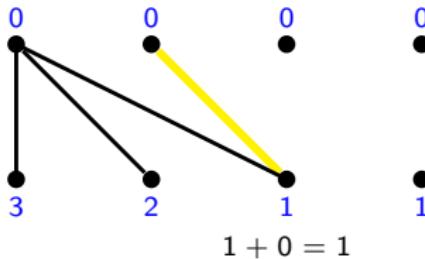
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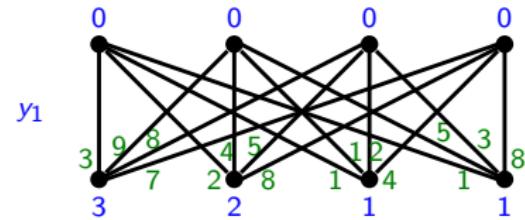
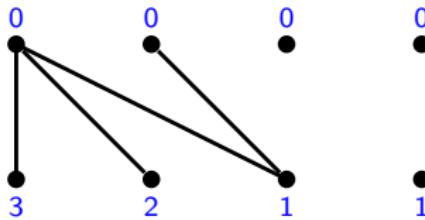
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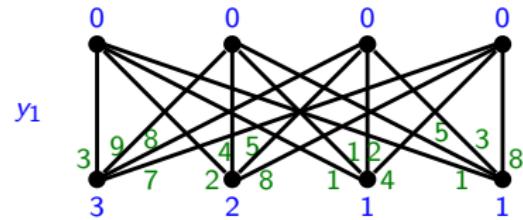
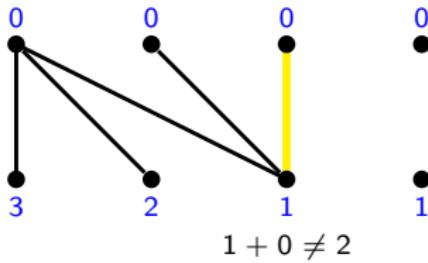
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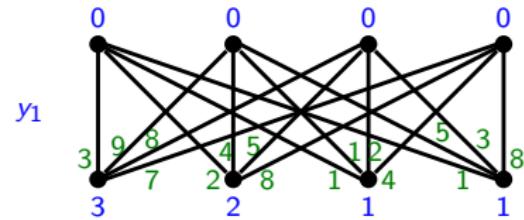
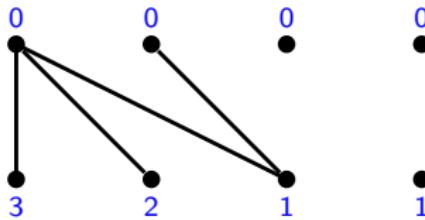
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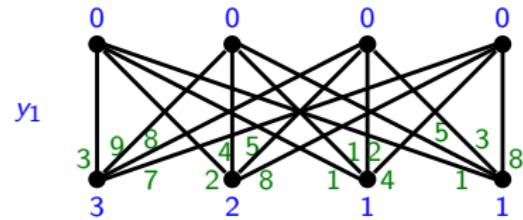
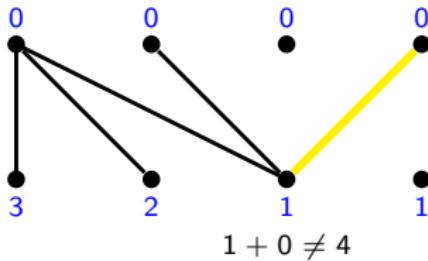
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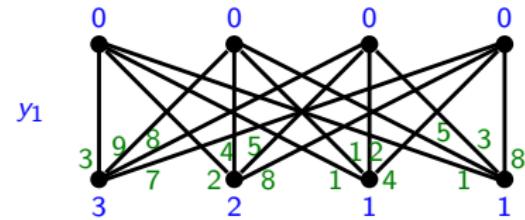
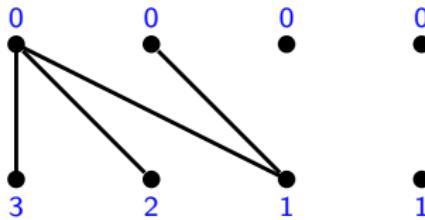
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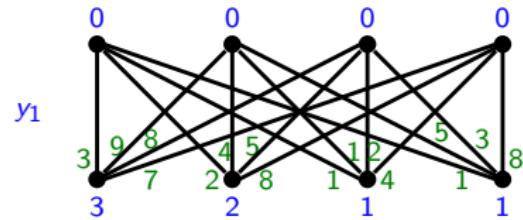
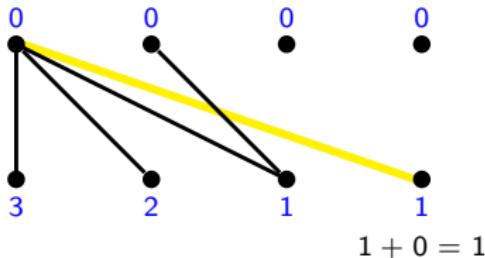
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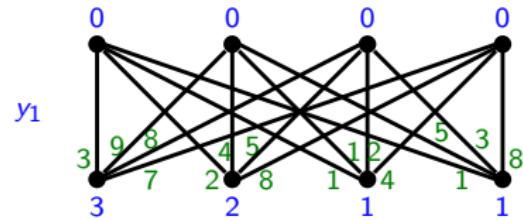
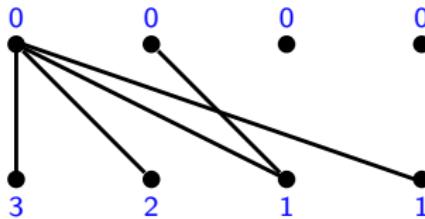
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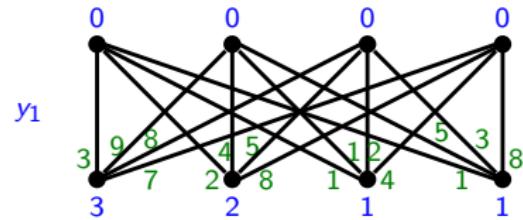
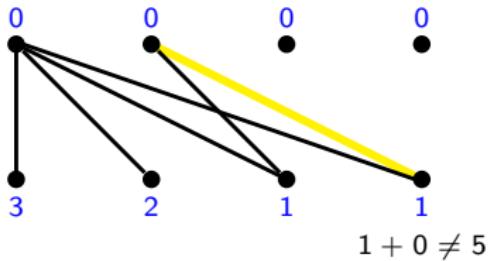
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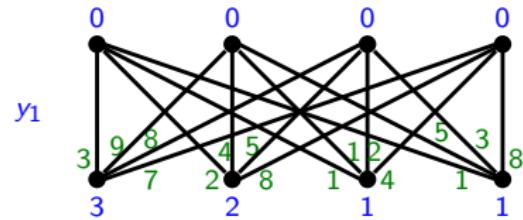
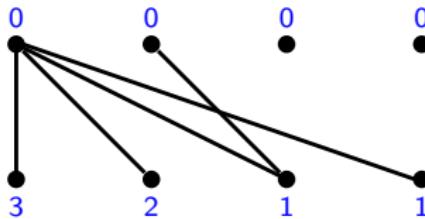
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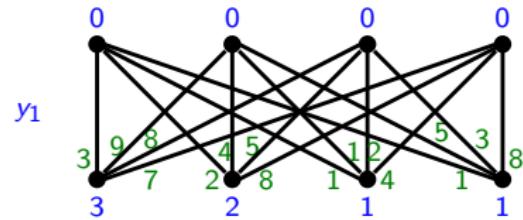
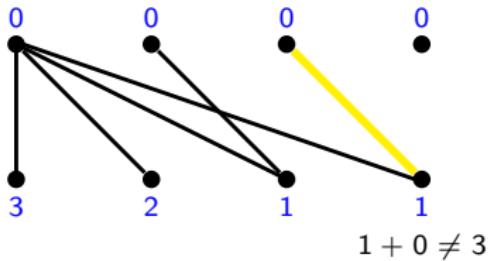
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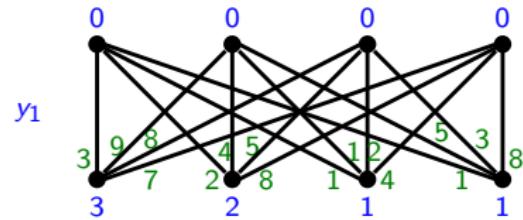
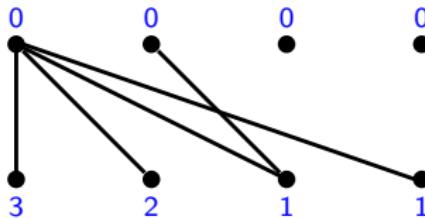
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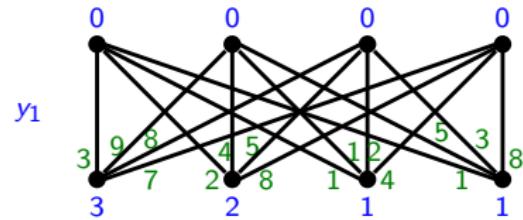
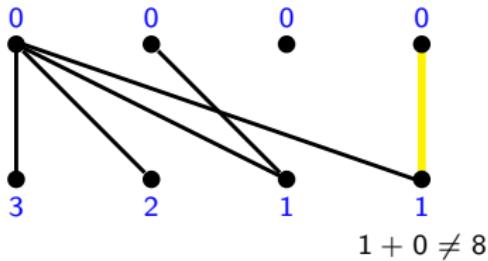
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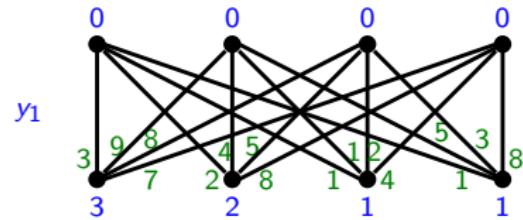
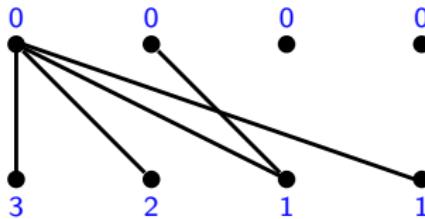
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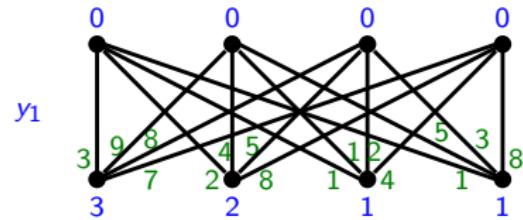
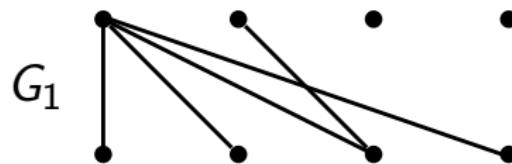
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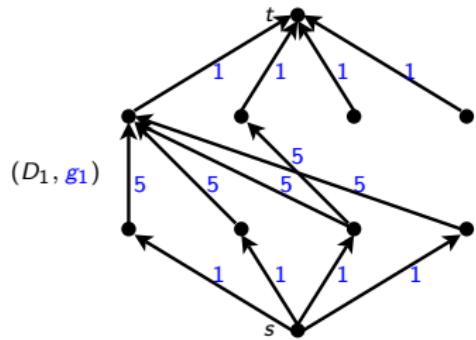
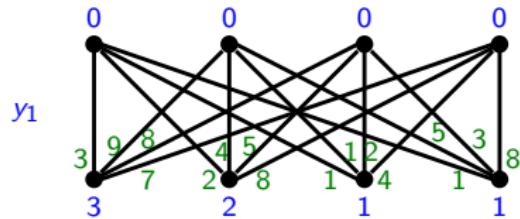
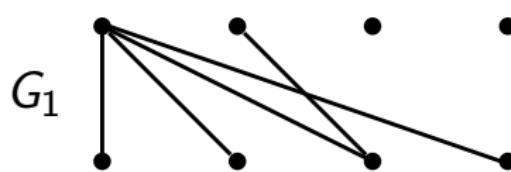
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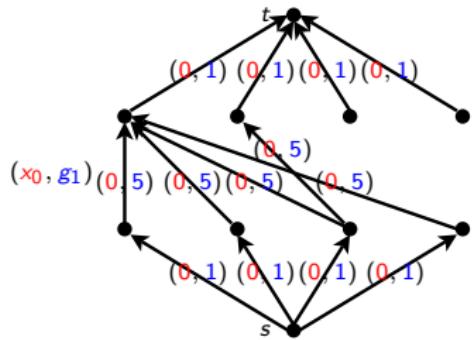
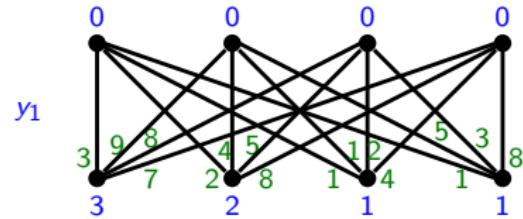
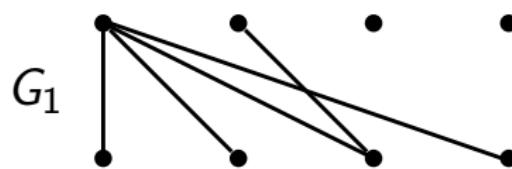
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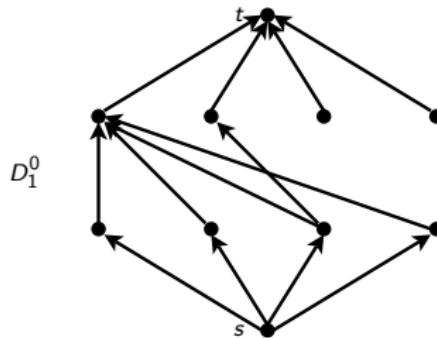
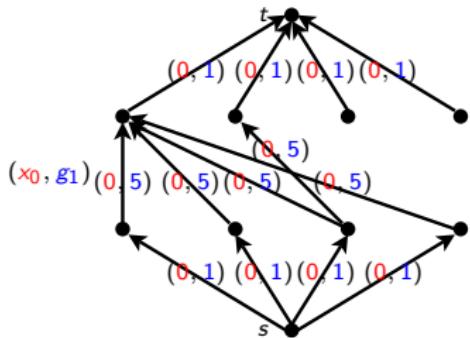
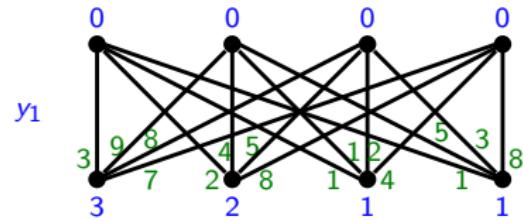
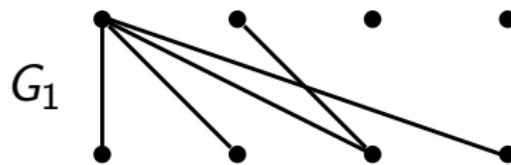
## Execution of the Hungarian method



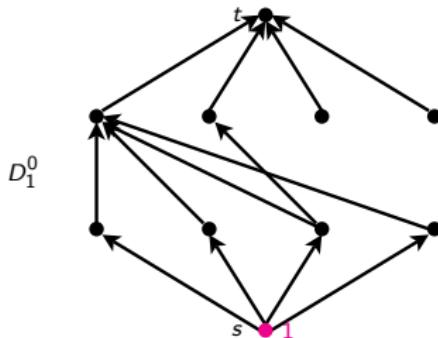
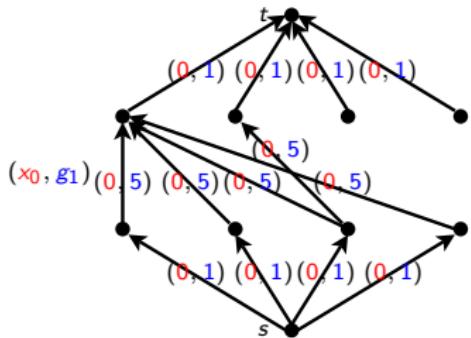
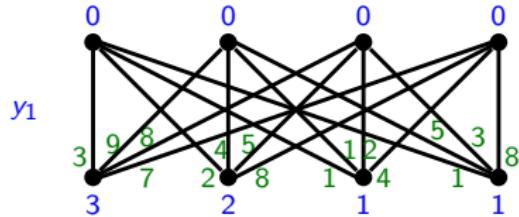
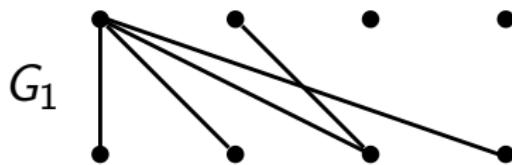
# Execution of the Hungarian method



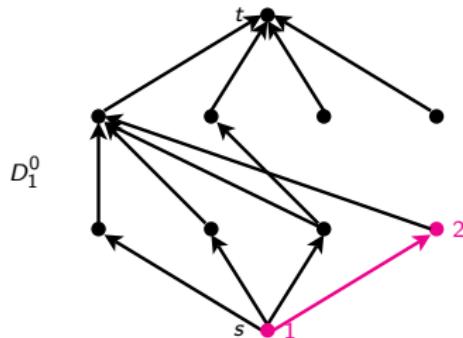
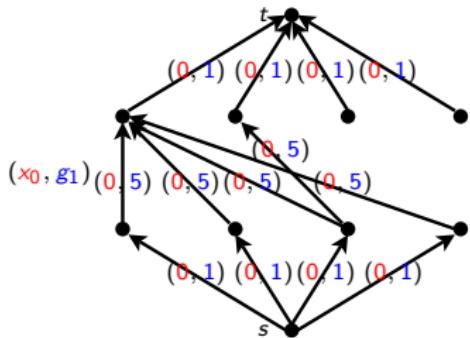
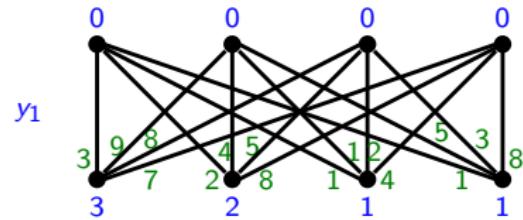
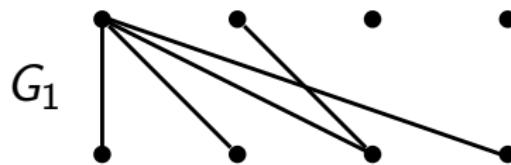
# Execution of the Hungarian method



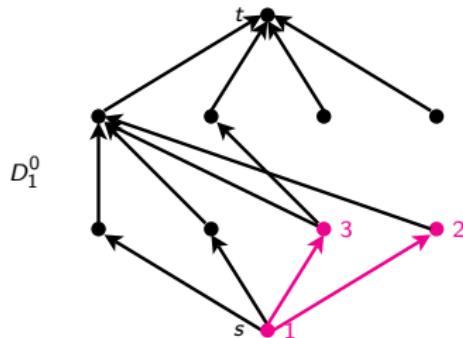
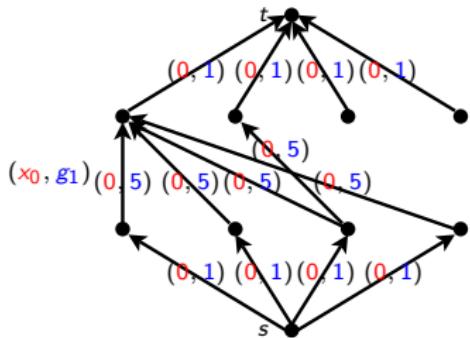
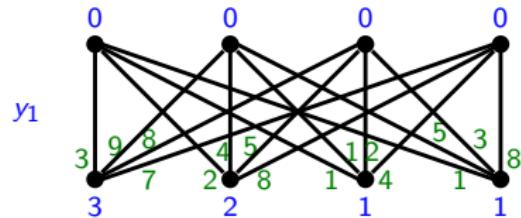
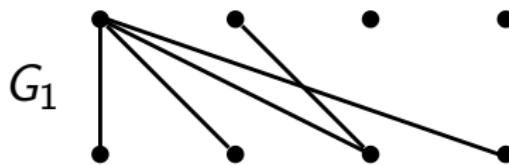
# Execution of the Hungarian method



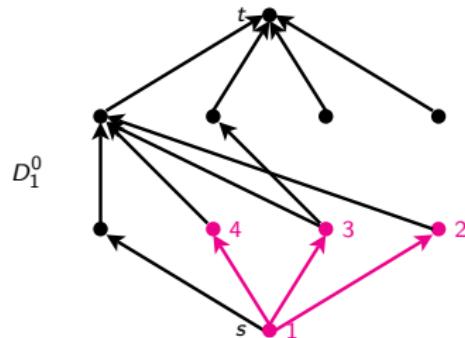
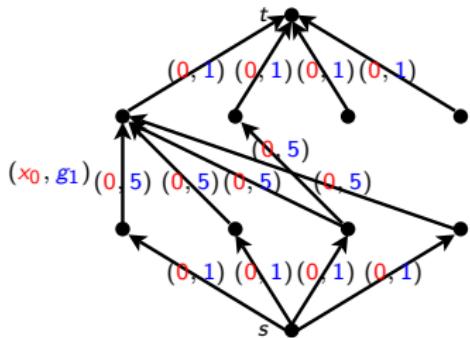
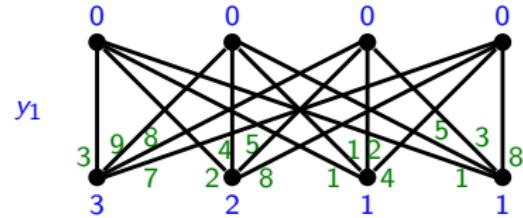
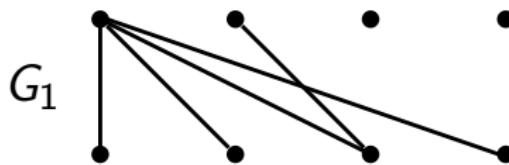
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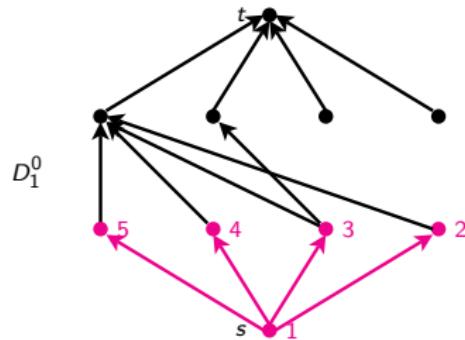
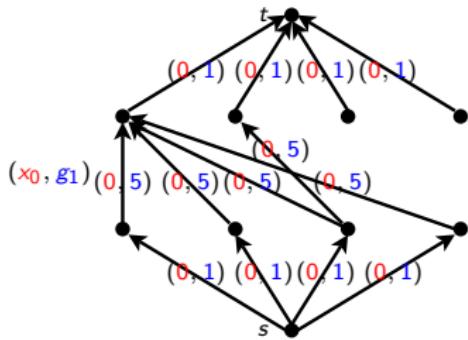
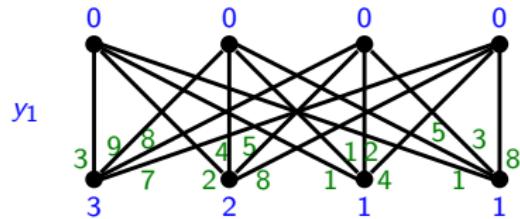
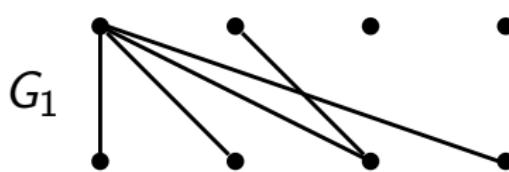
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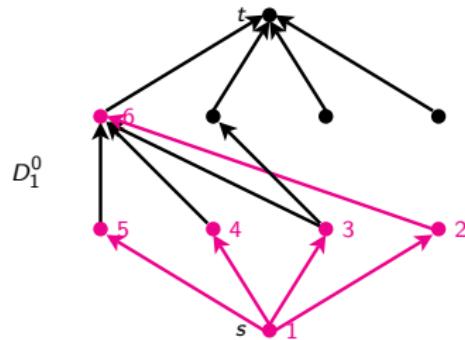
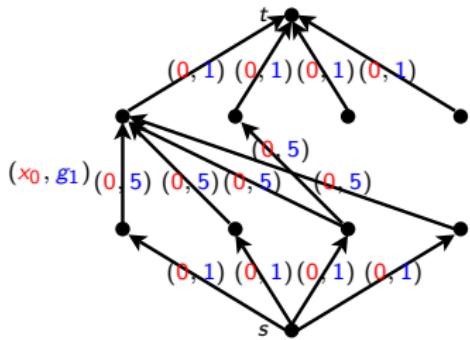
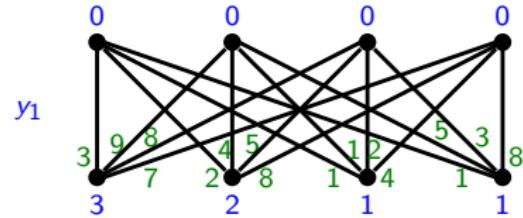
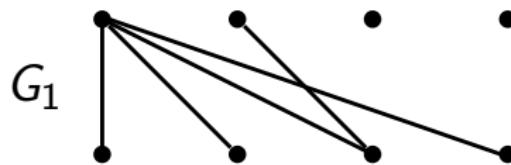
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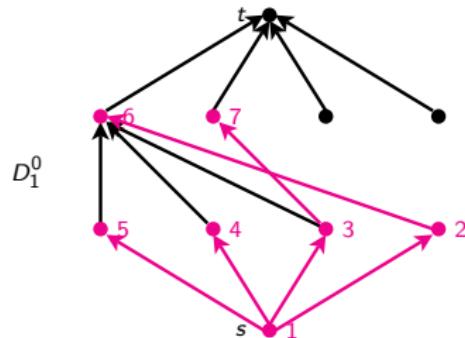
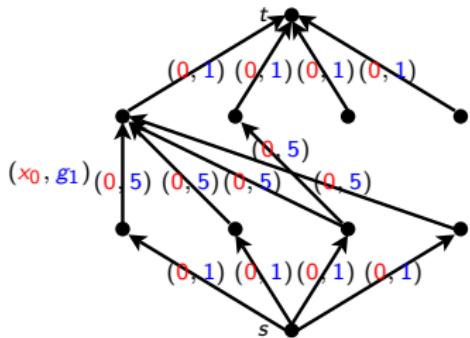
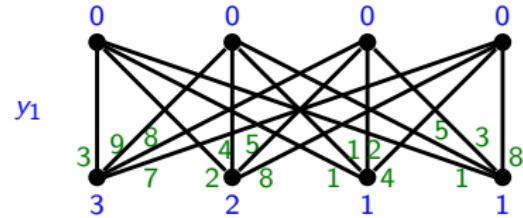
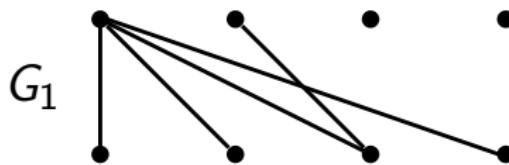
## Execution of the Hungarian method



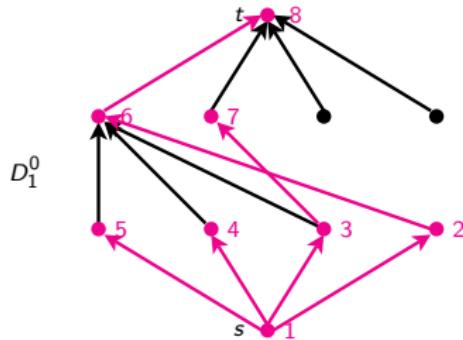
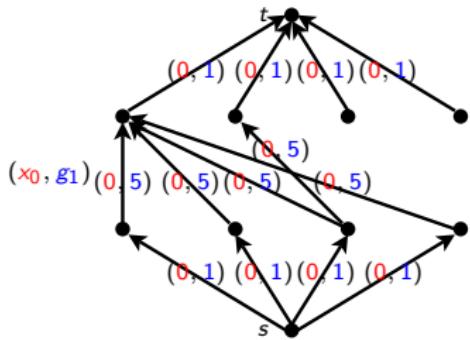
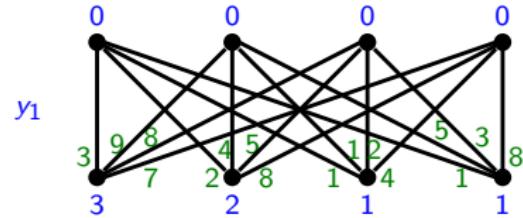
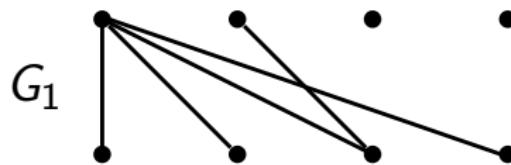
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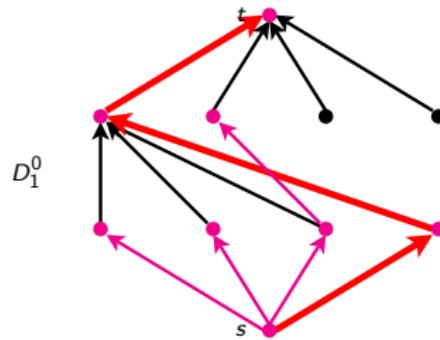
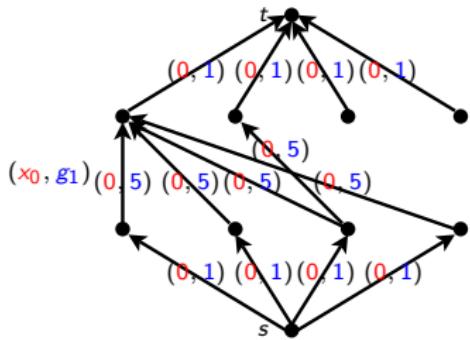
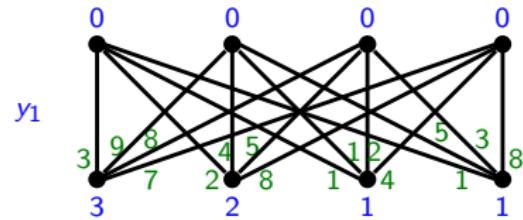
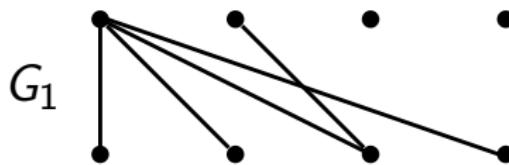
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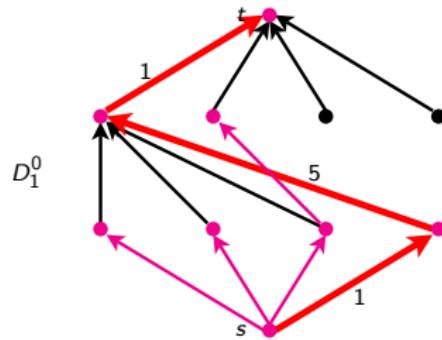
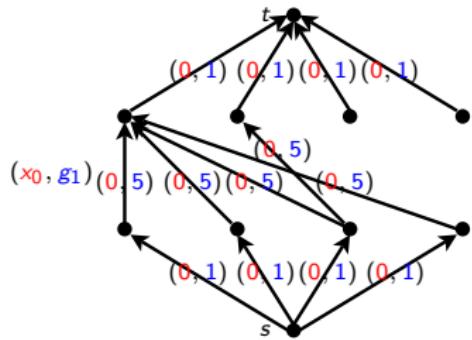
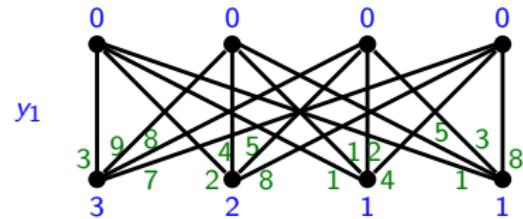
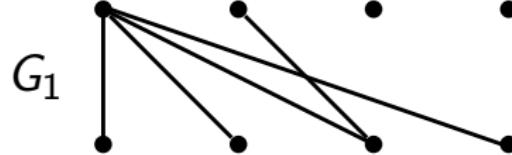
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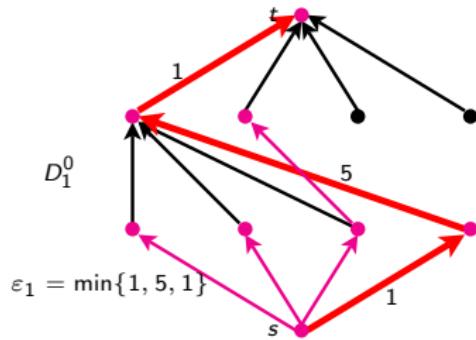
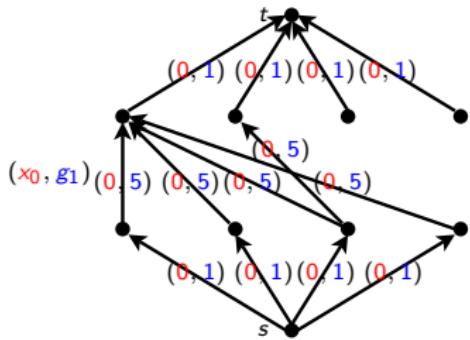
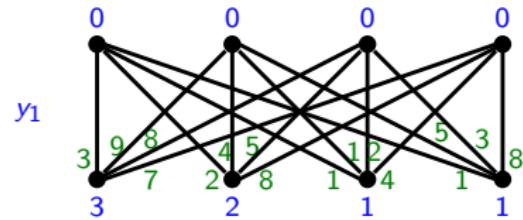
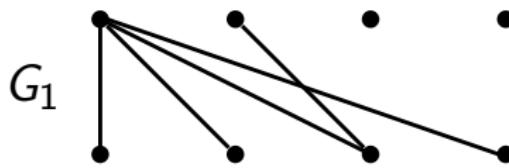
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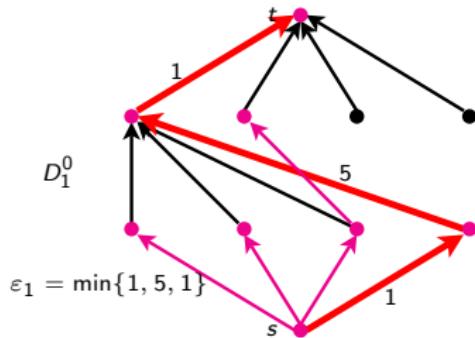
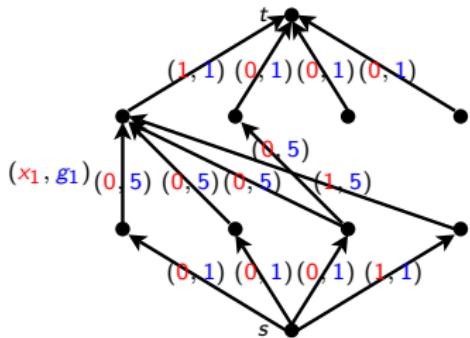
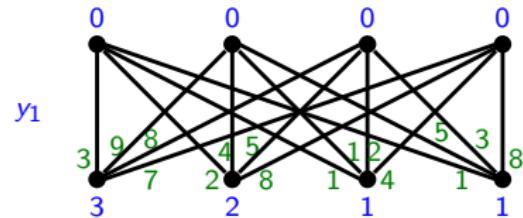
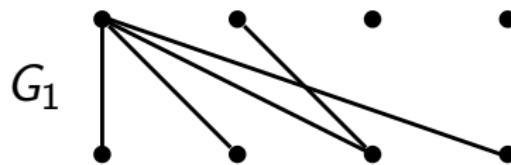
# Execution of the Hungarian method



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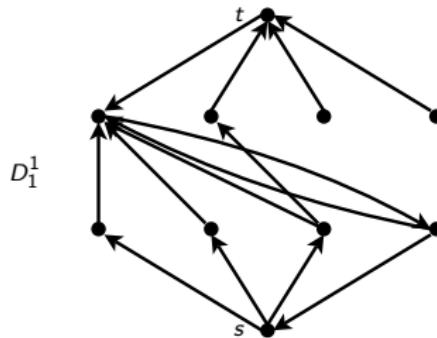
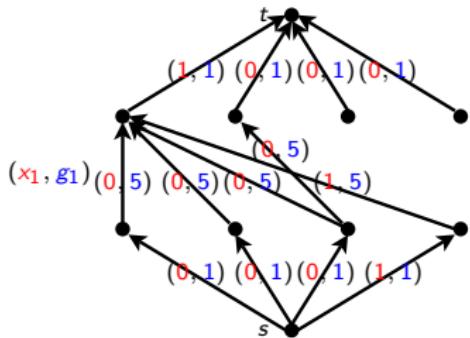
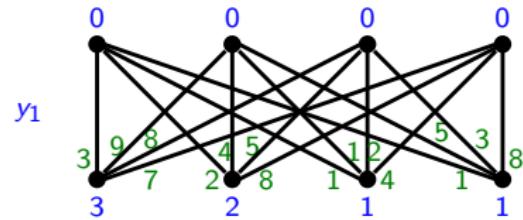
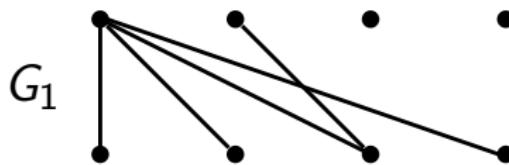


# Execution of the Hungarian method

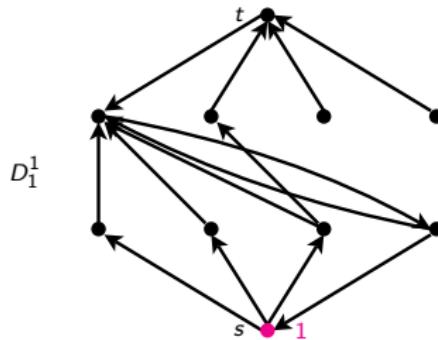
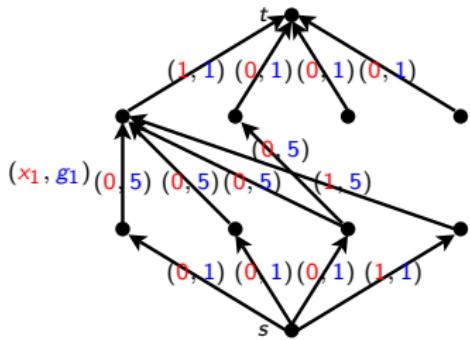
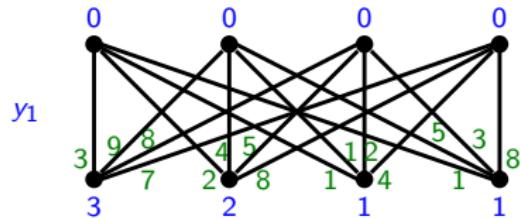
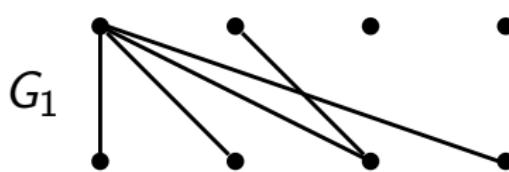


$$\varepsilon_1 = \min\{1, 5, 1\}$$

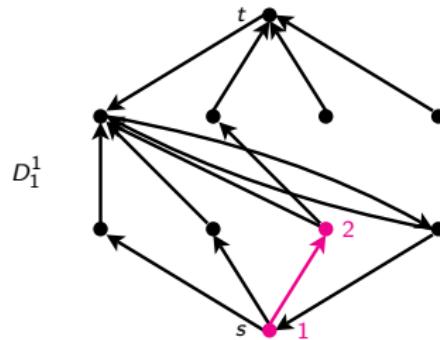
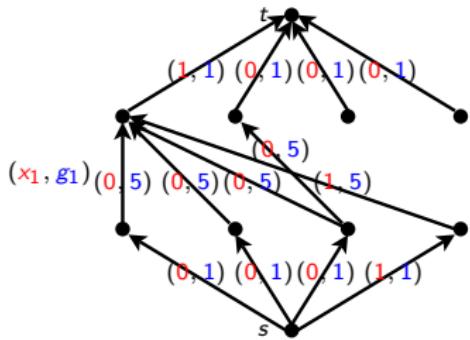
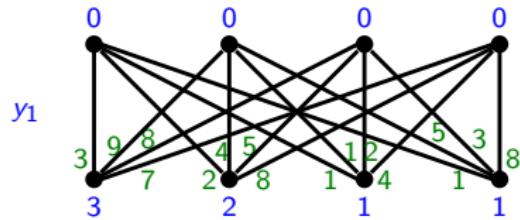
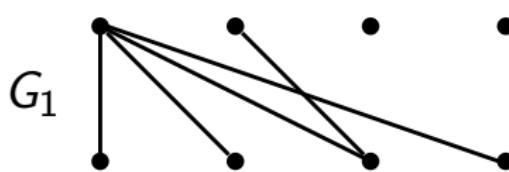
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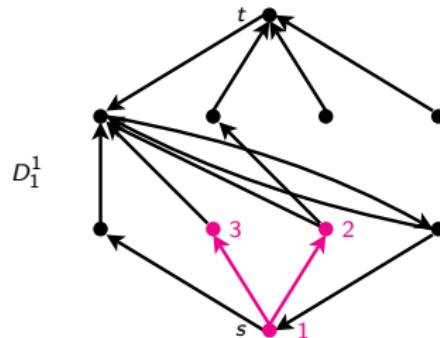
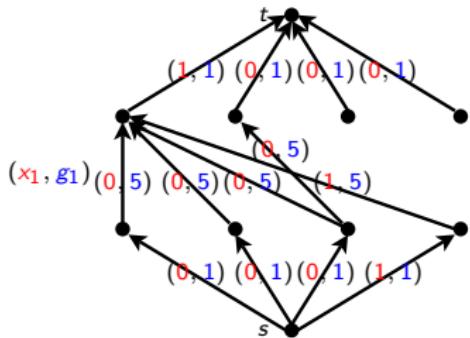
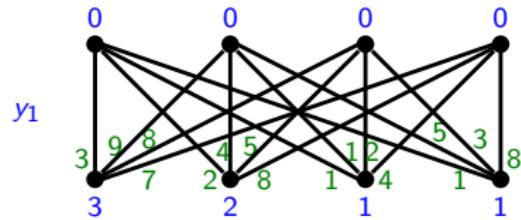
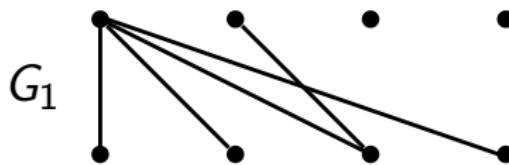
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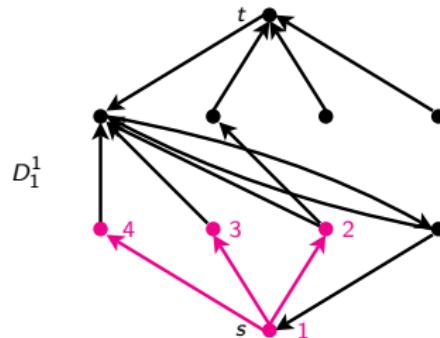
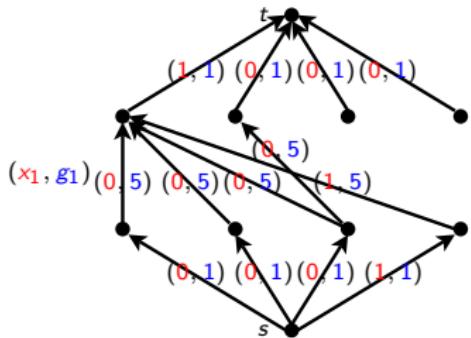
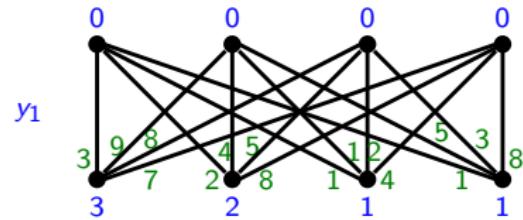
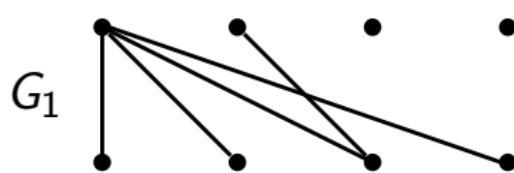
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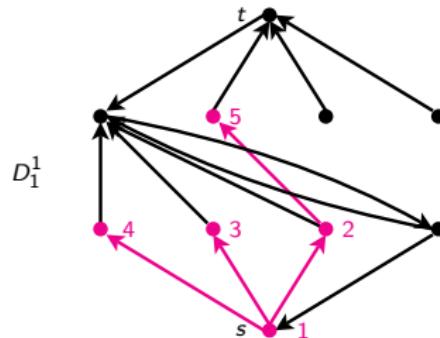
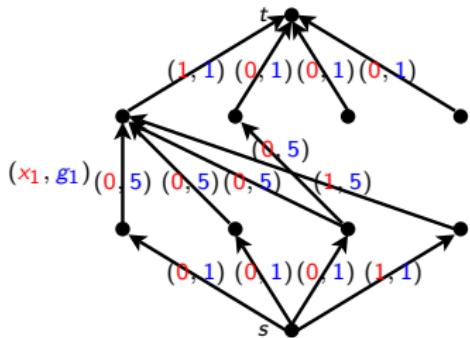
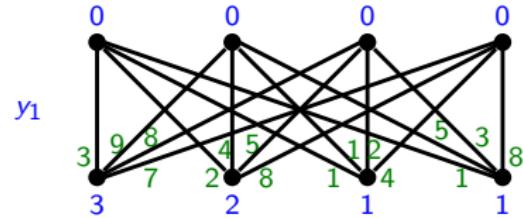
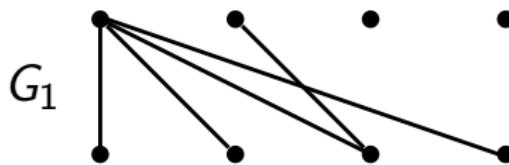
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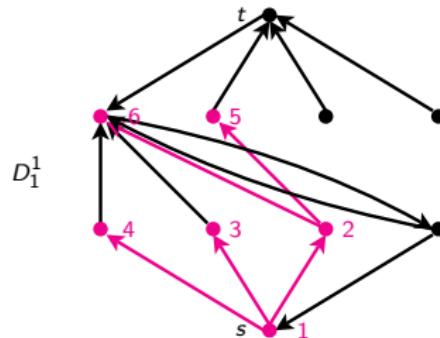
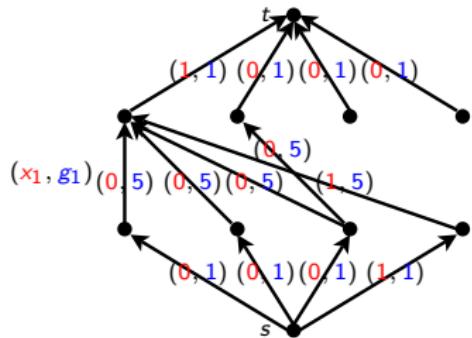
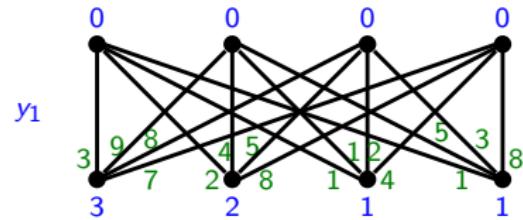
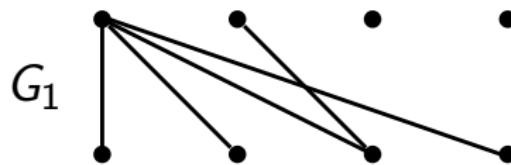
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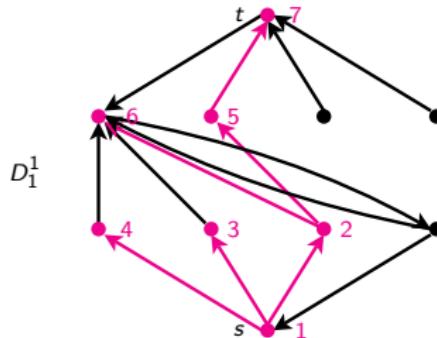
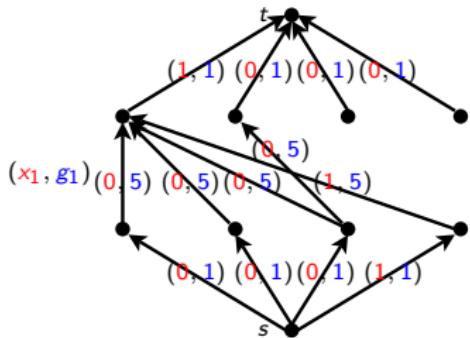
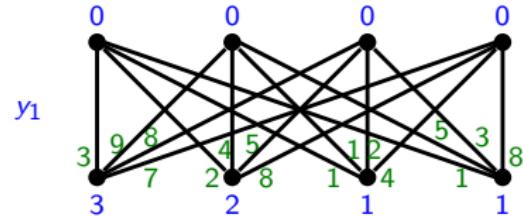
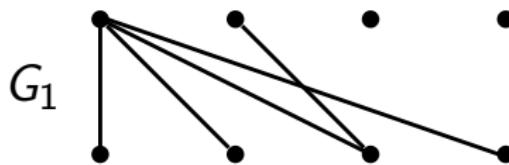
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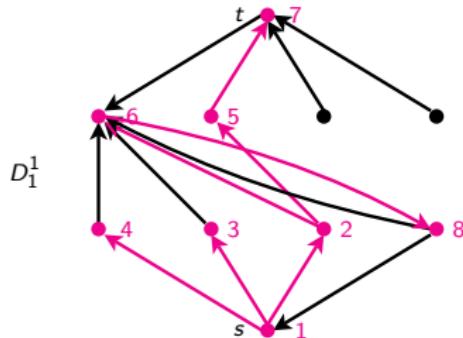
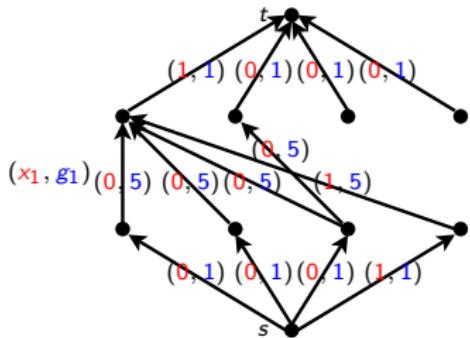
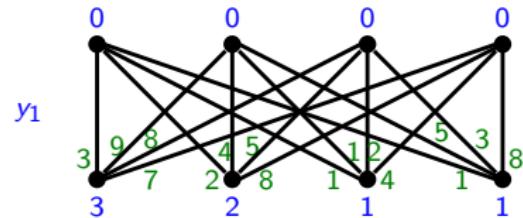
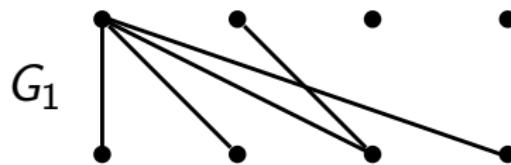
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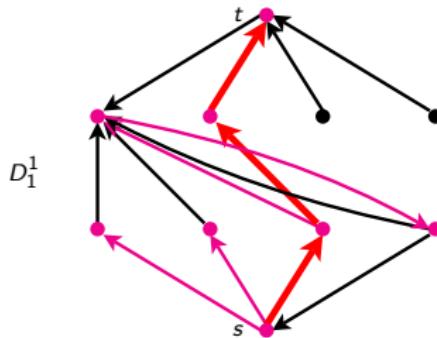
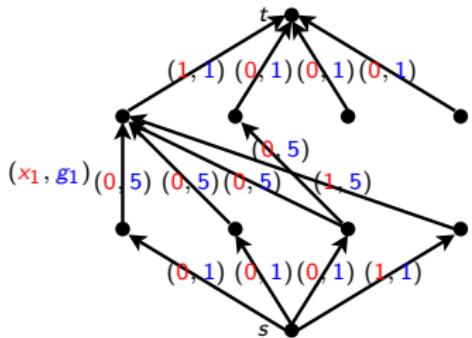
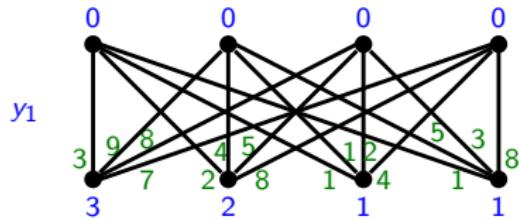
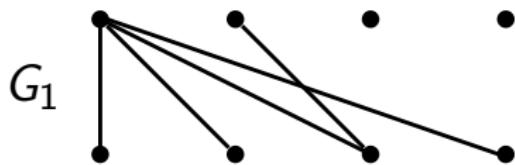
# Execution of the Hungarian method



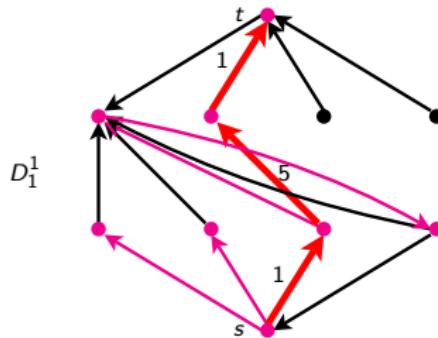
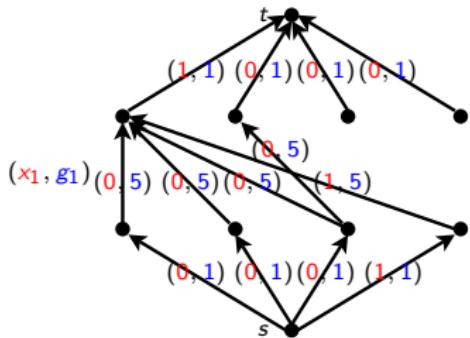
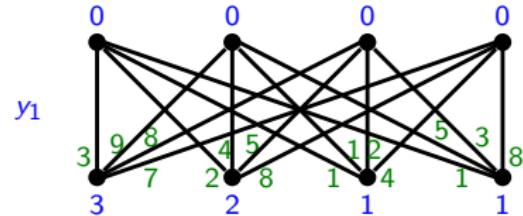
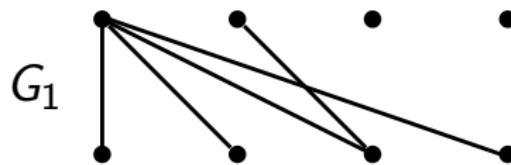
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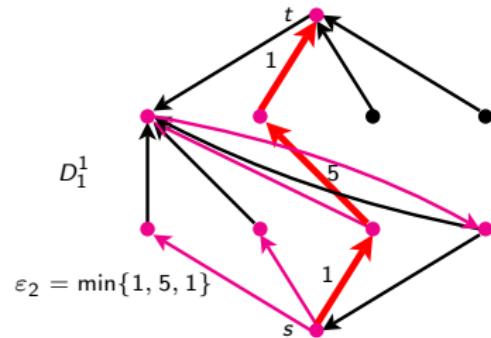
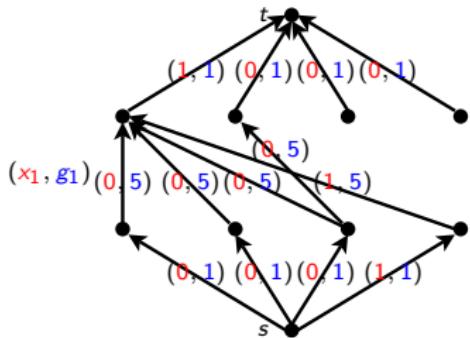
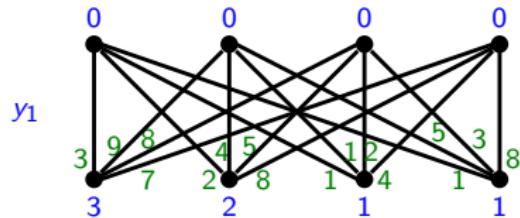
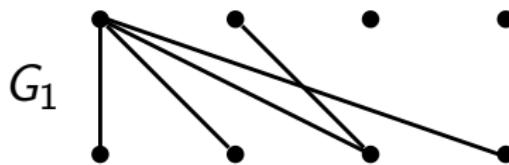
# Execution of the Hungarian method



# Execution of the Hungarian method

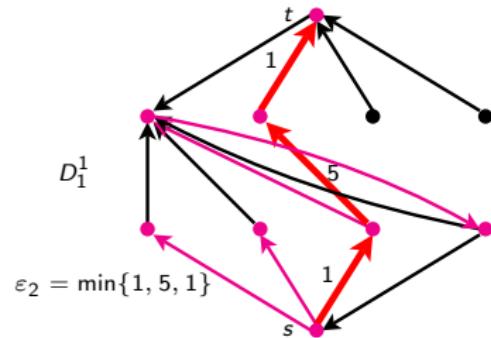
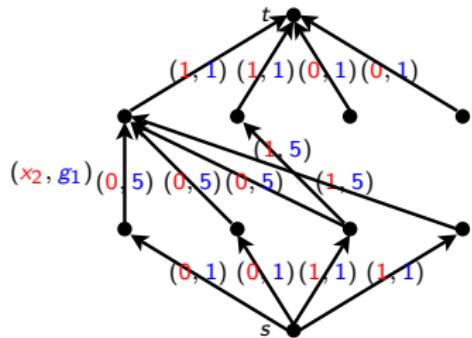
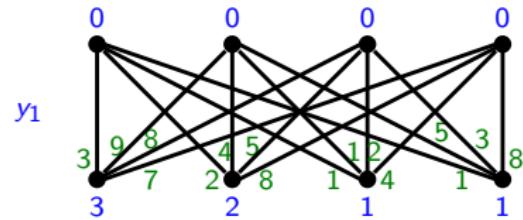
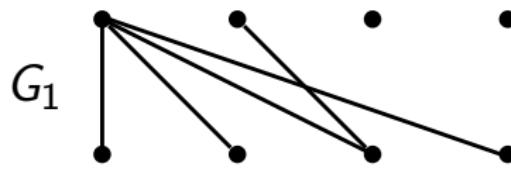


# Execution of the Hungarian method

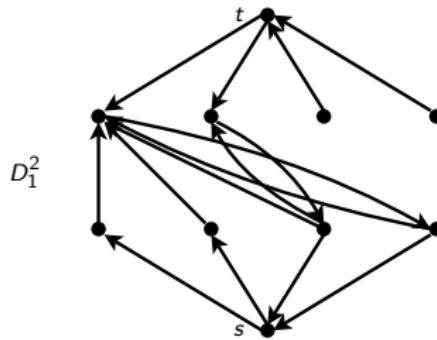
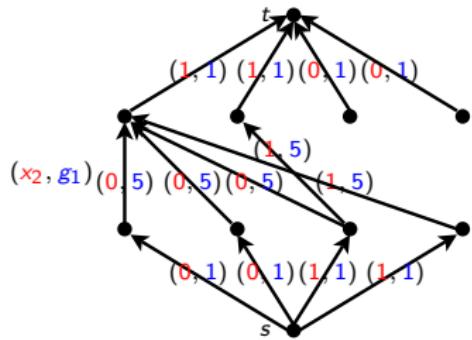
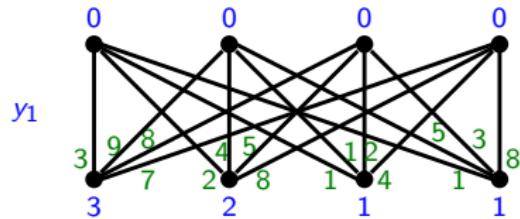
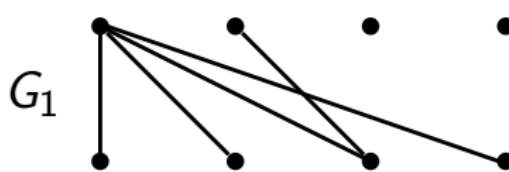


$$\varepsilon_2 = \min\{1, 5, 1\}$$

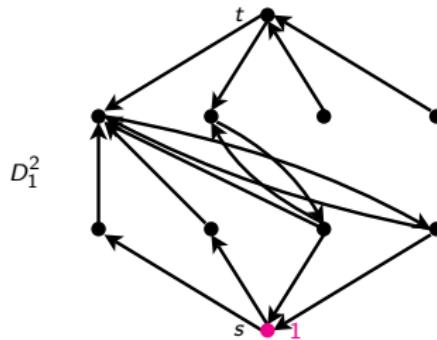
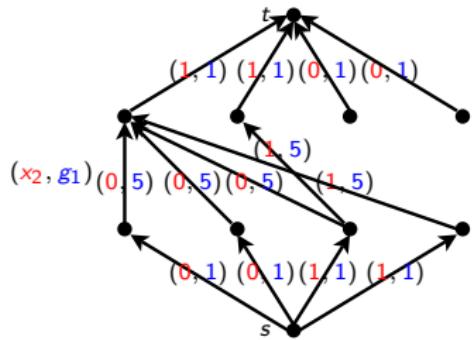
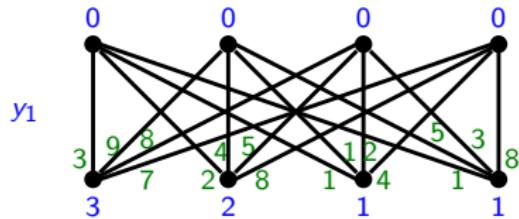
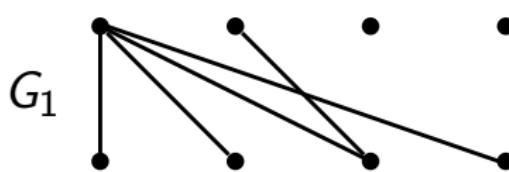
# Execution of the Hungarian method



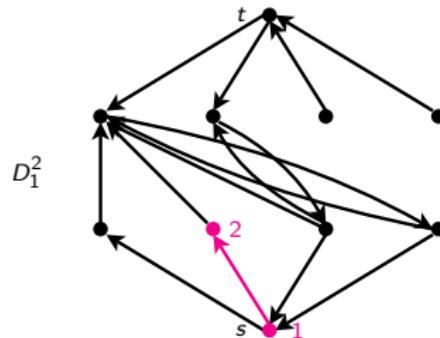
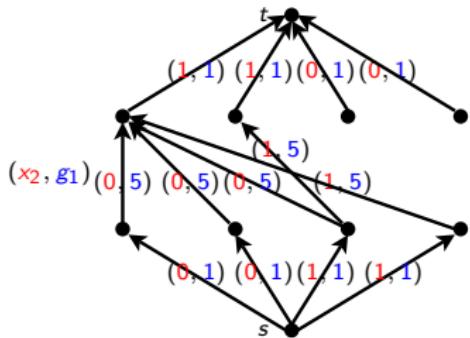
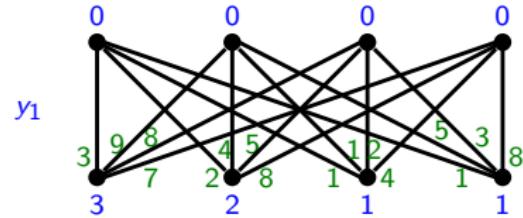
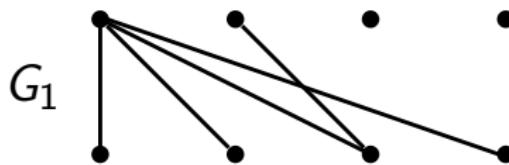
## Execution of the Hungarian method



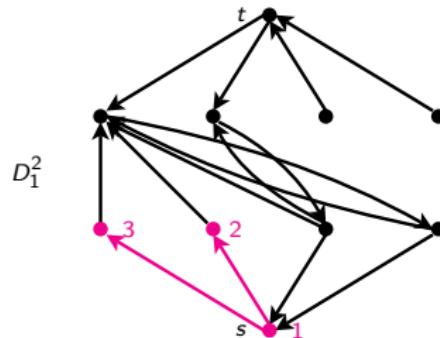
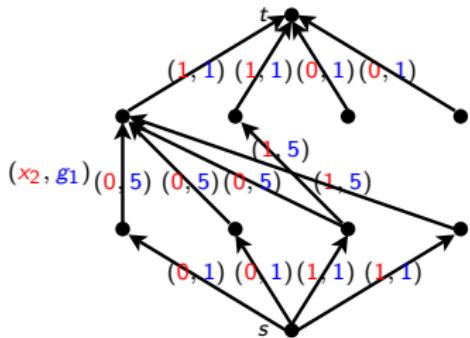
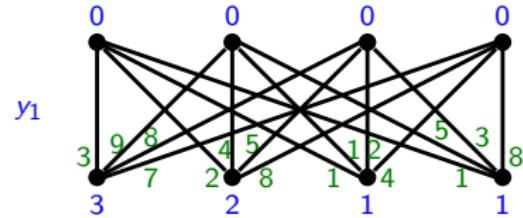
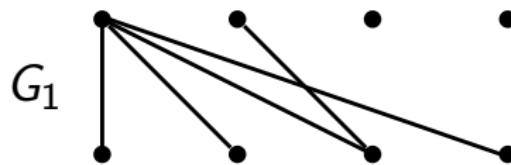
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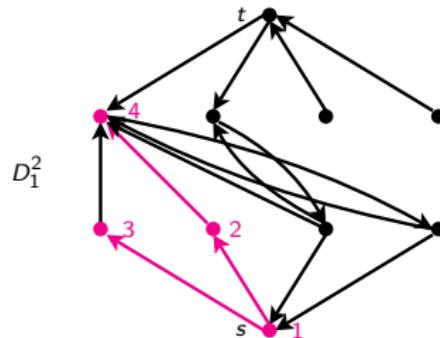
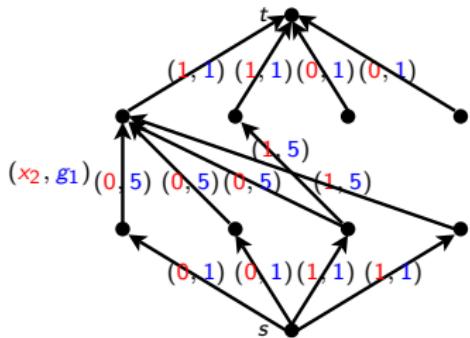
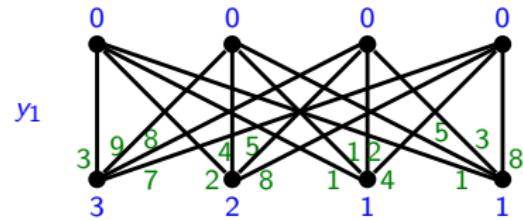
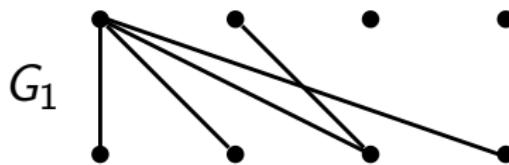
# Execution of the Hungarian method



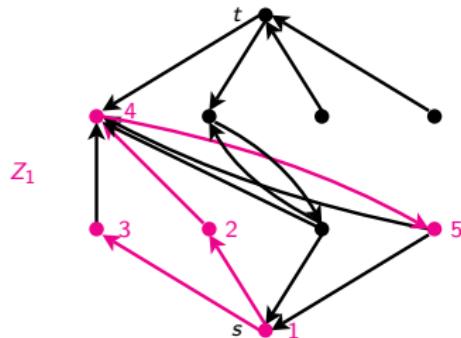
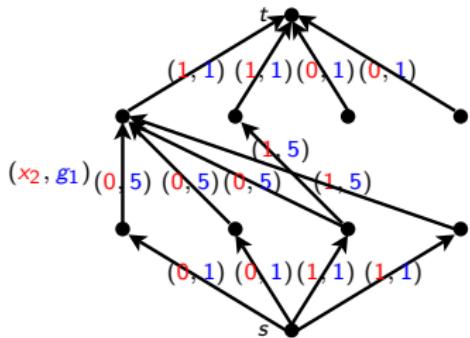
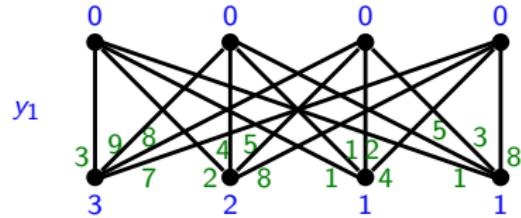
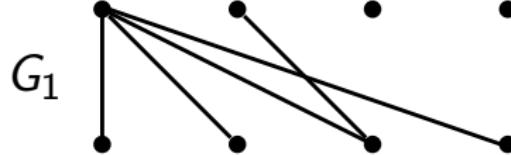
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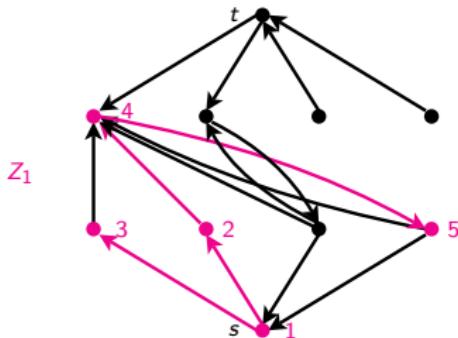
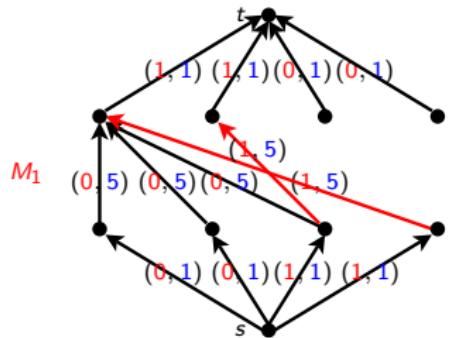
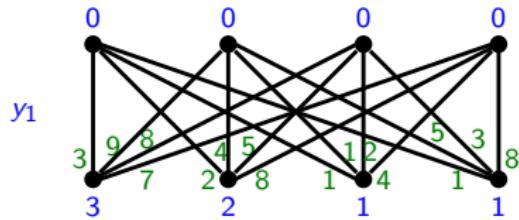
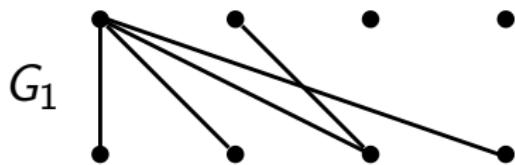
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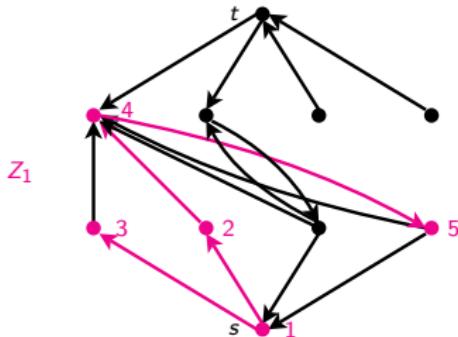
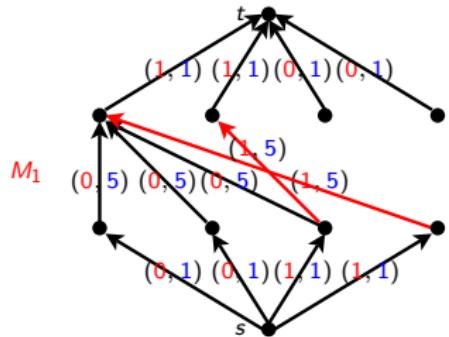
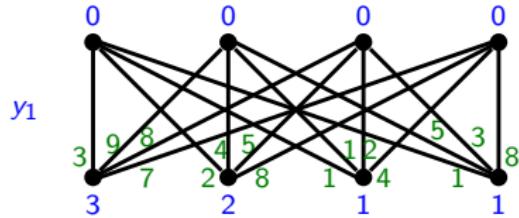
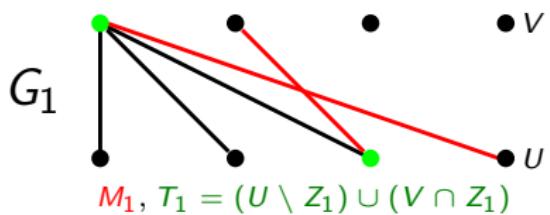
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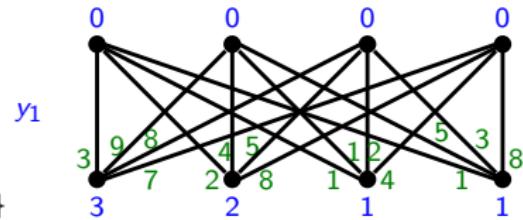
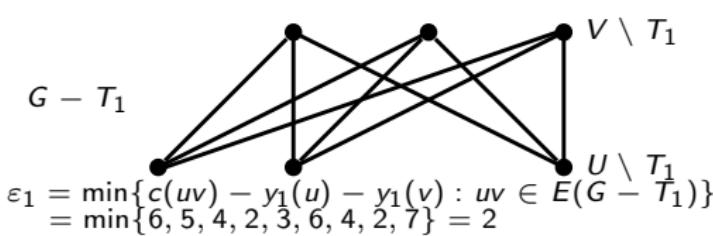
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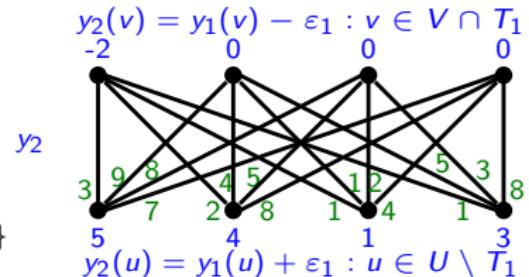
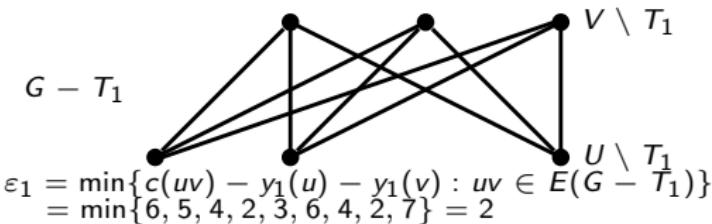
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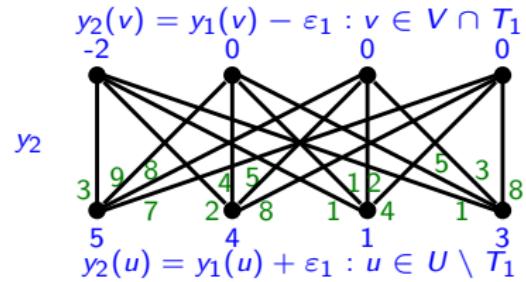
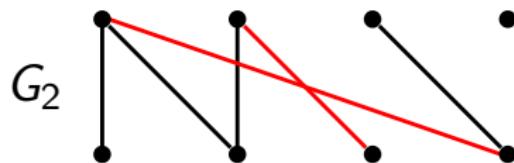
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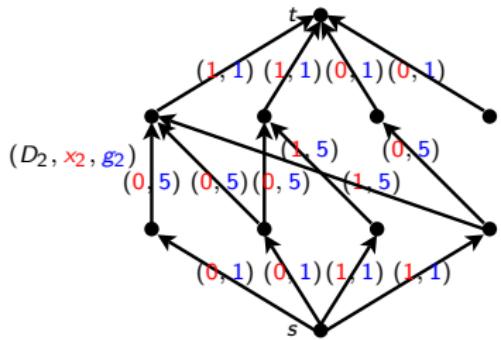
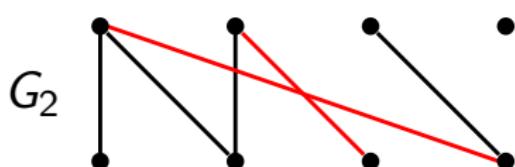
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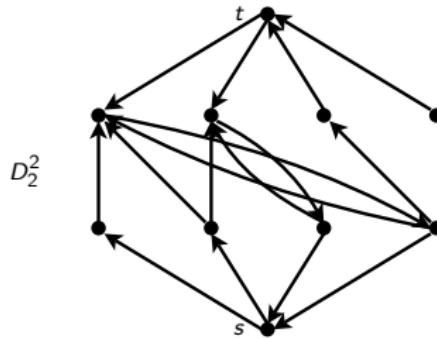
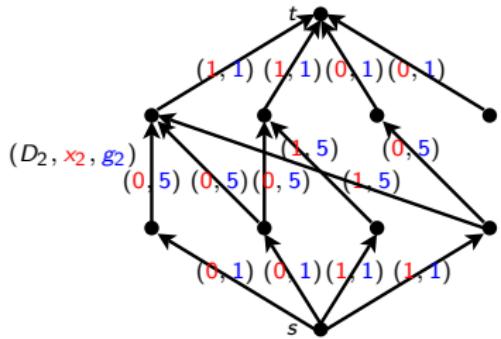
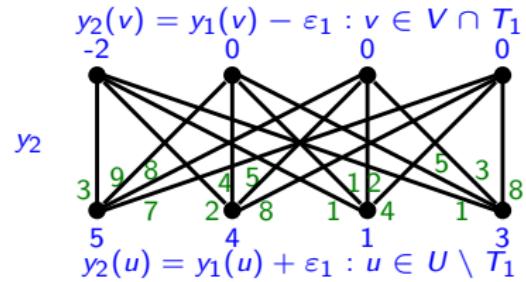
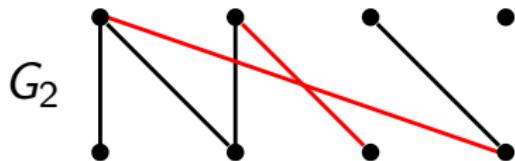
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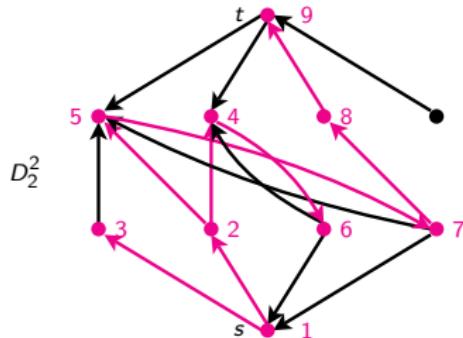
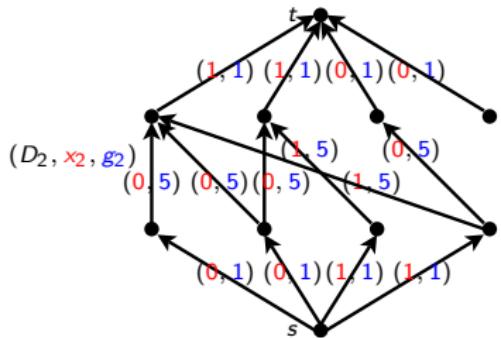
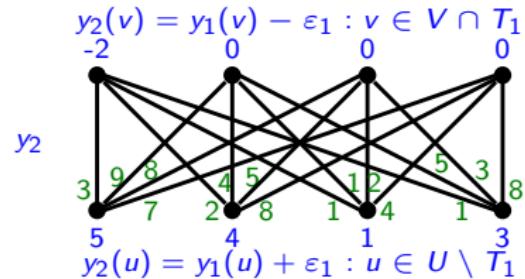
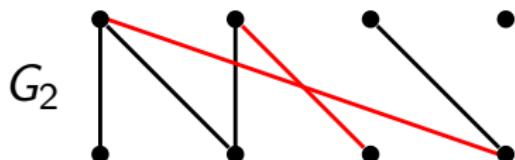
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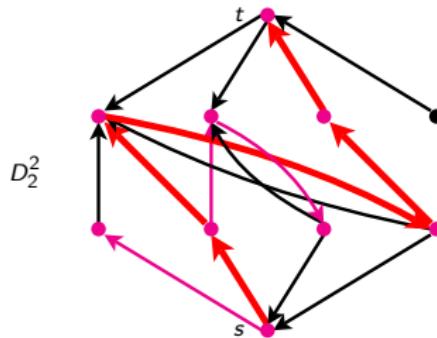
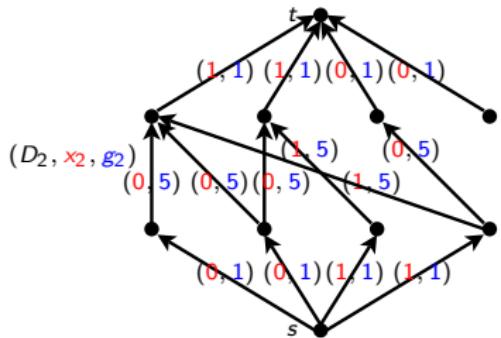
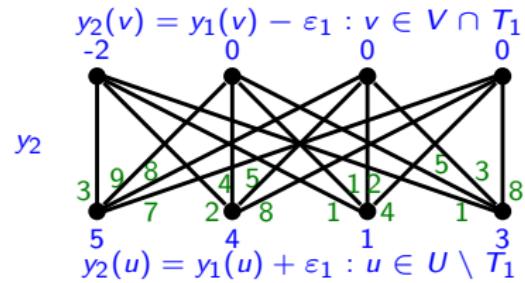
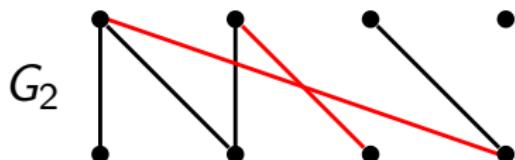
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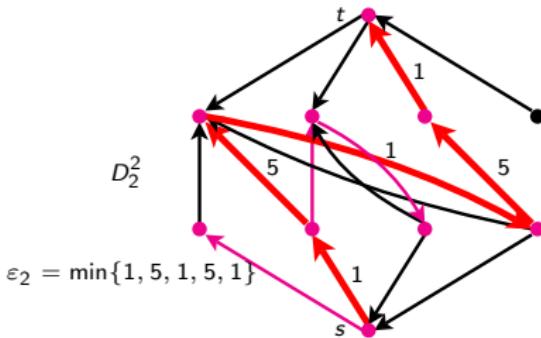
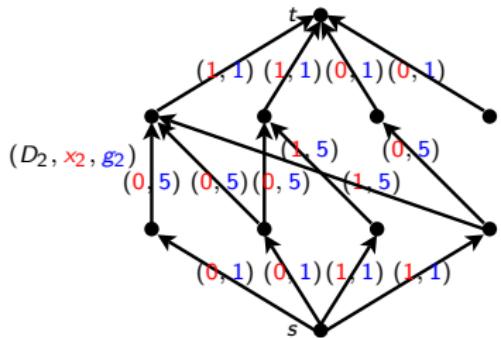
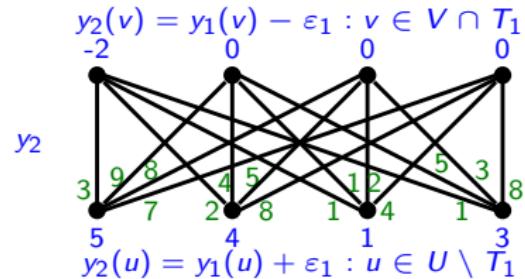
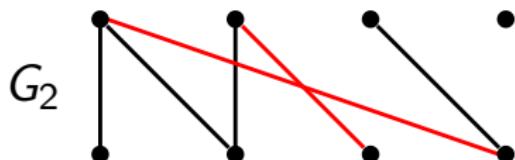
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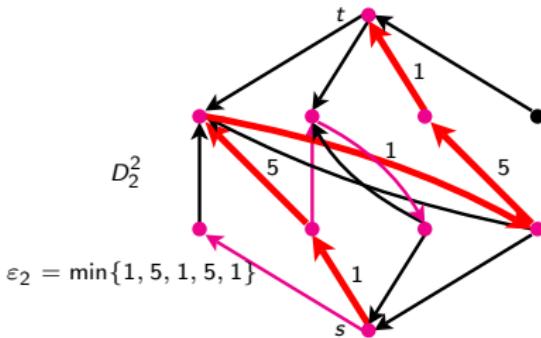
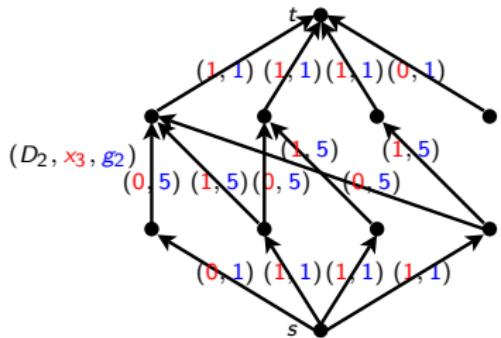
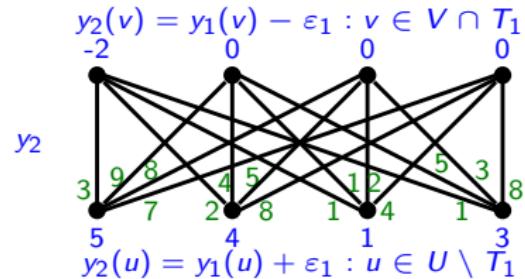
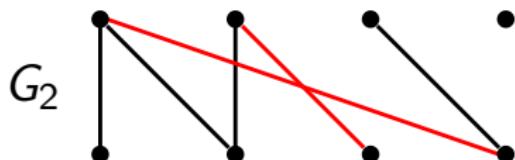
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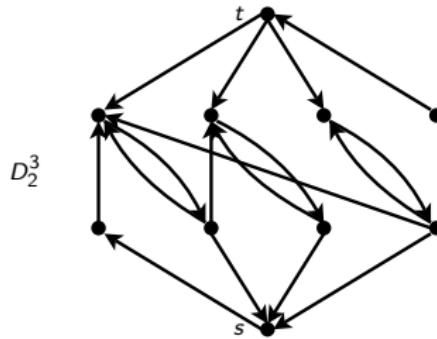
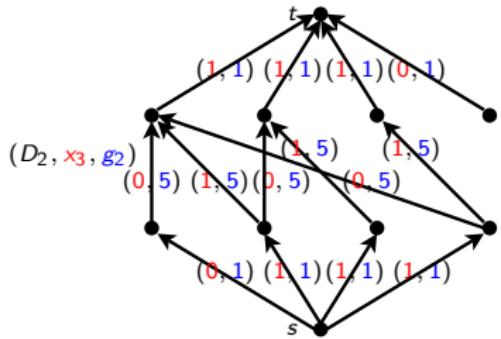
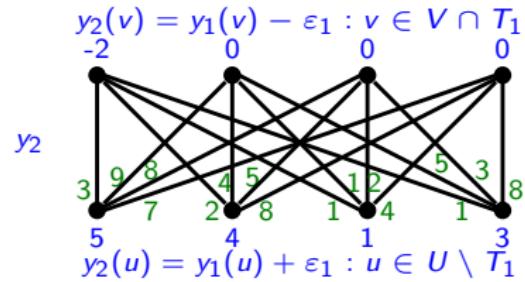
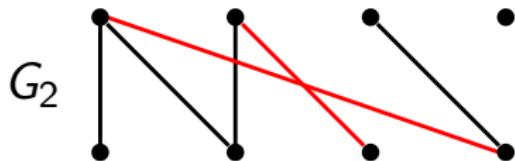
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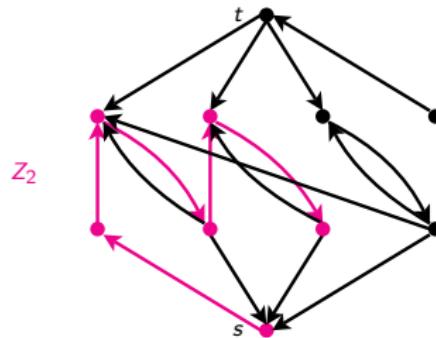
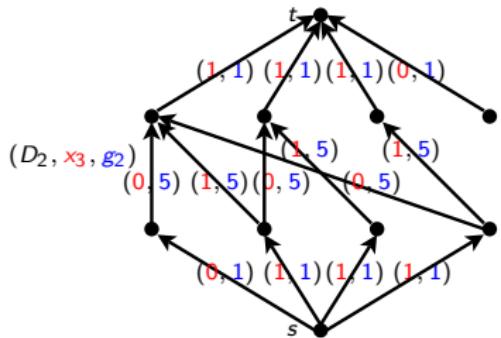
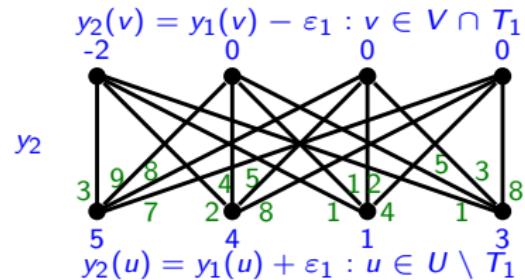
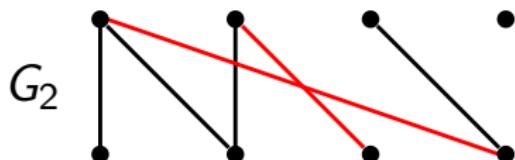
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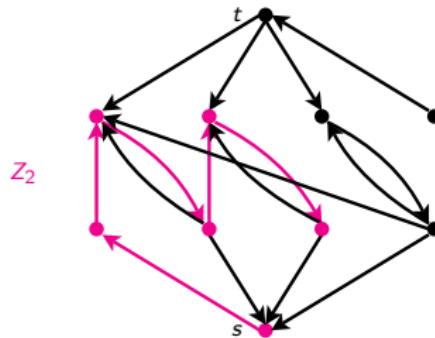
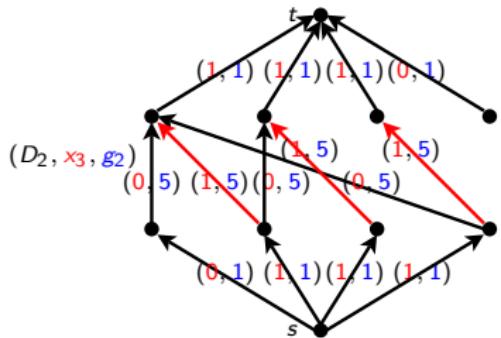
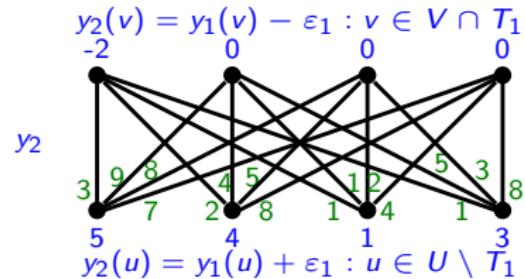
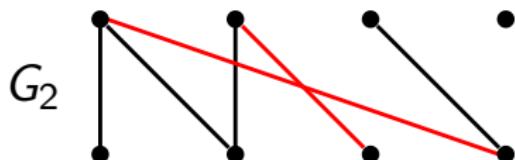
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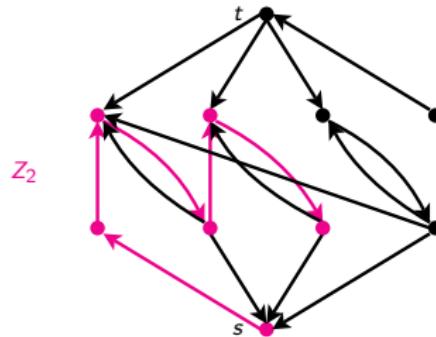
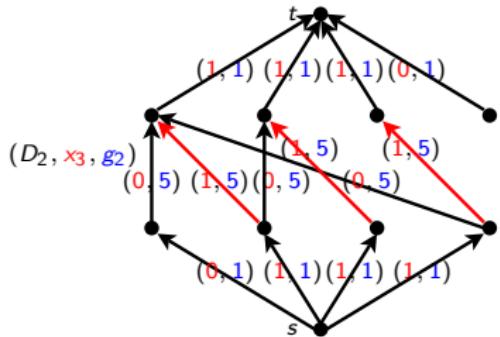
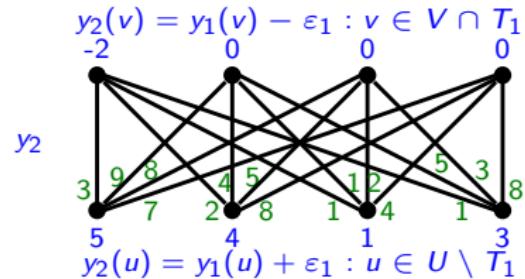
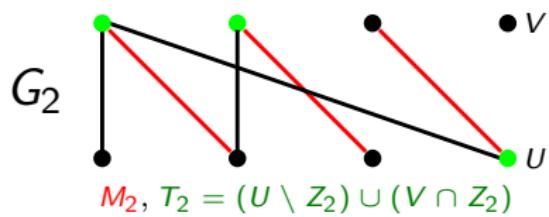
# Execution of the Hungarian method



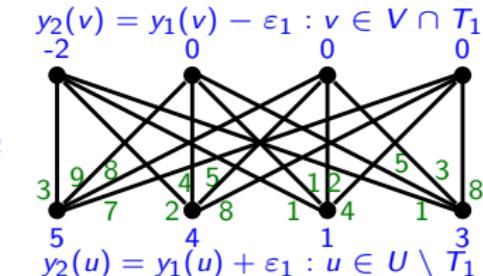
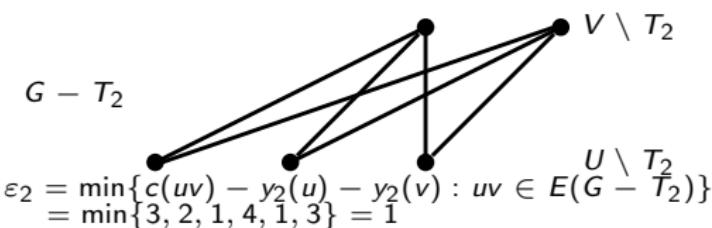
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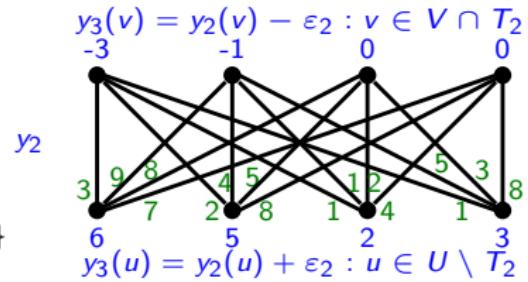
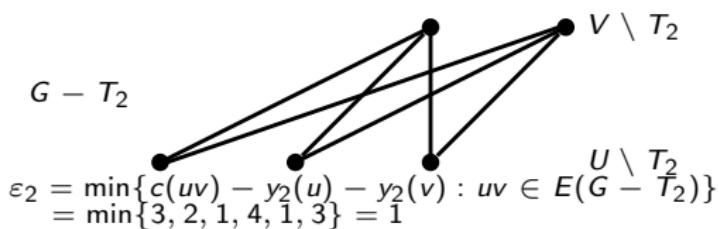
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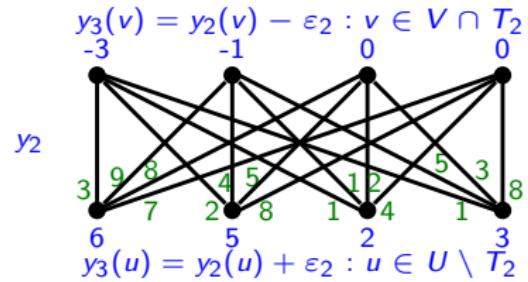
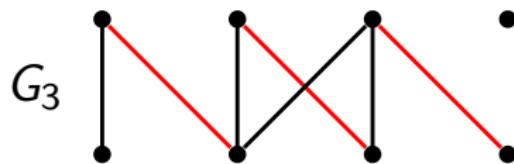
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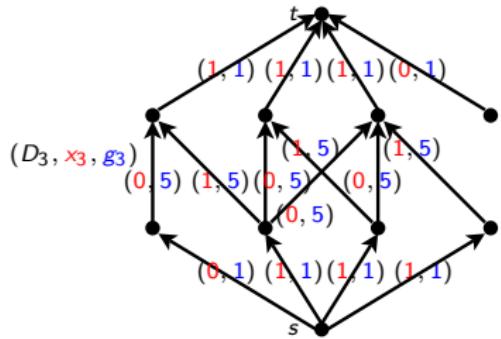
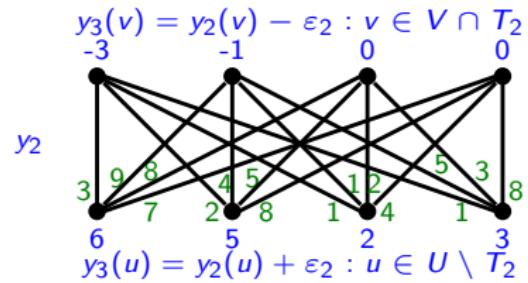
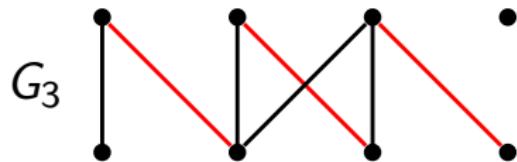
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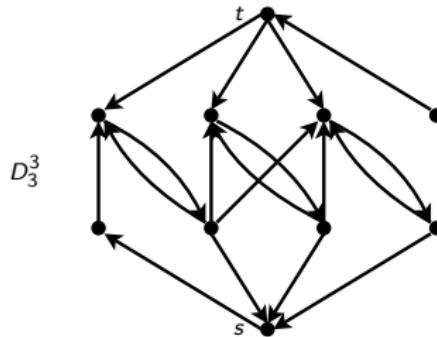
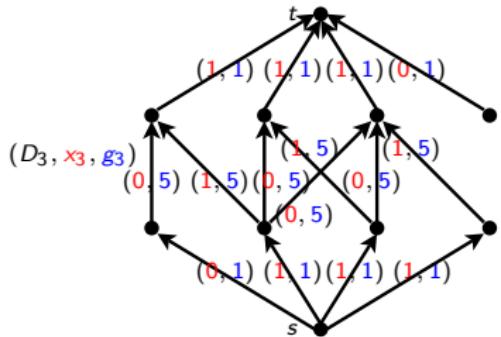
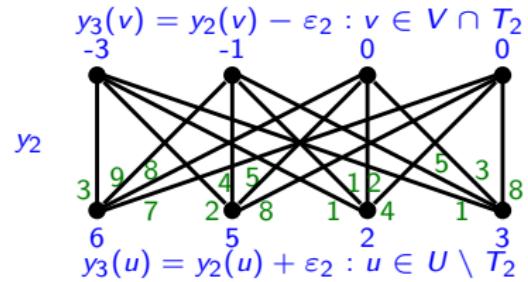
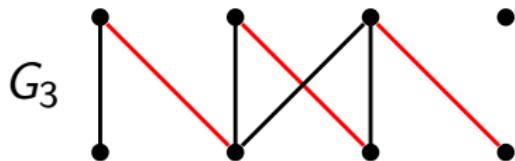
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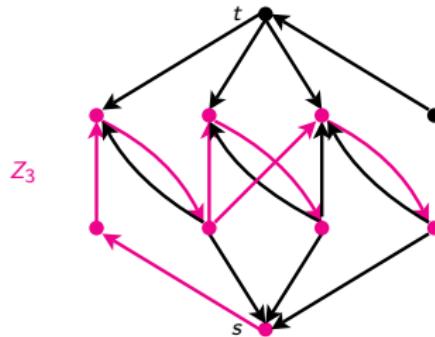
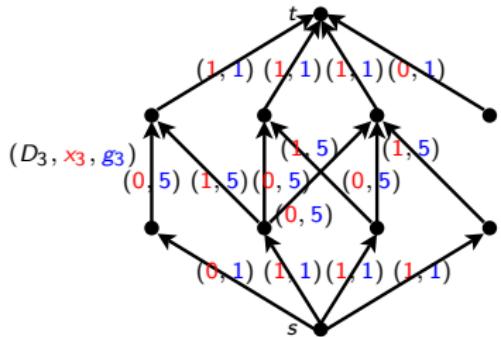
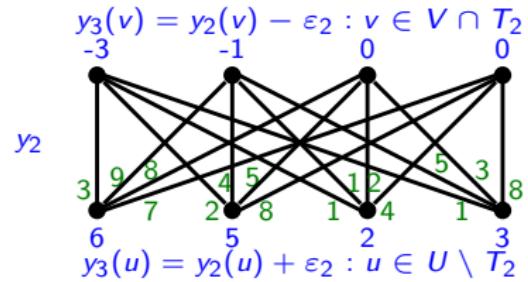
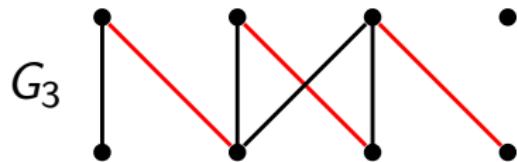
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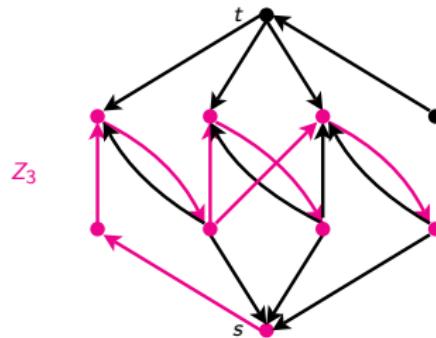
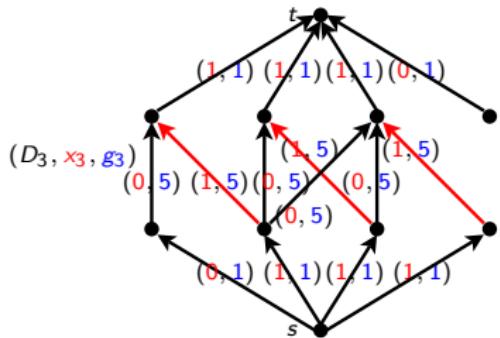
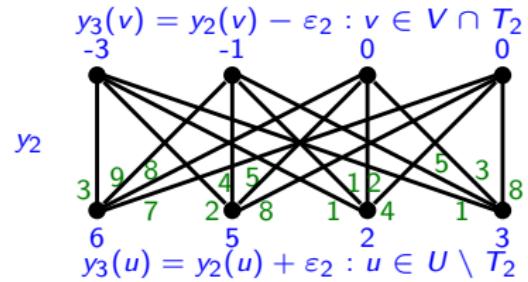
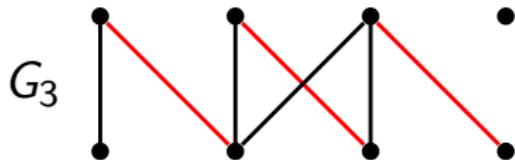
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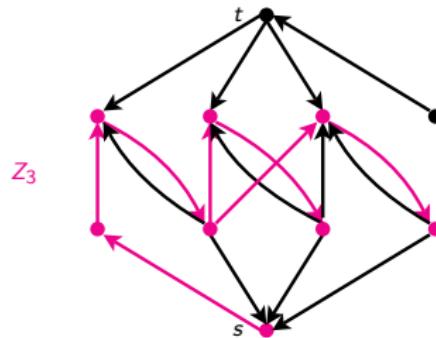
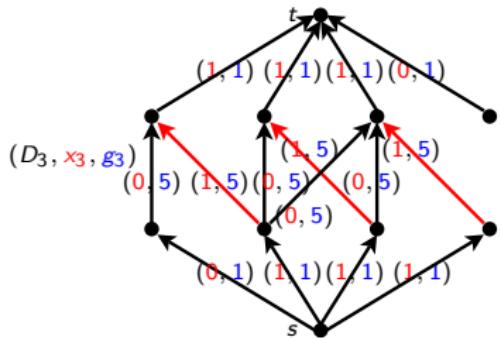
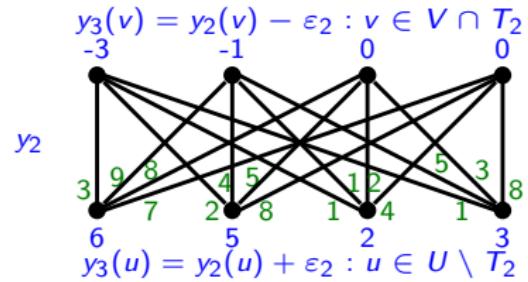
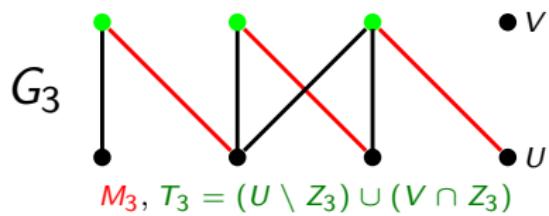
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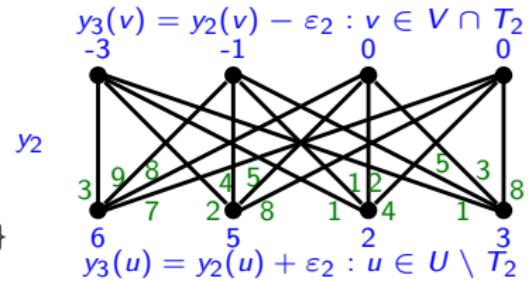
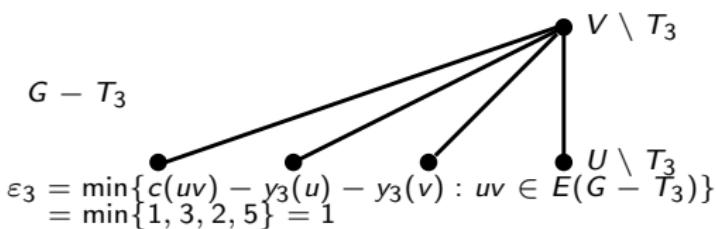
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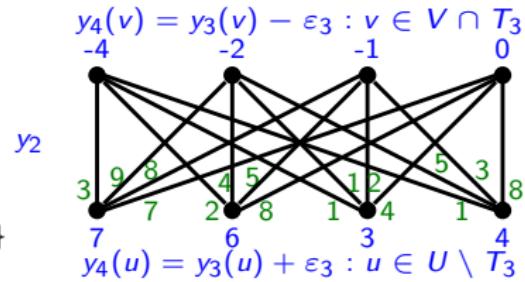
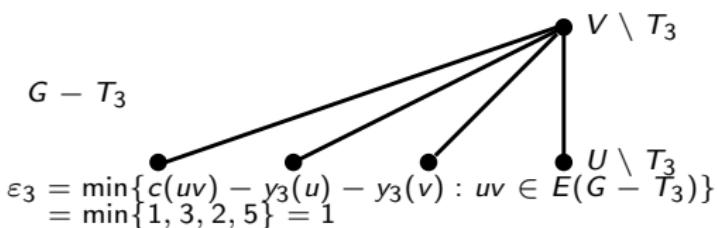
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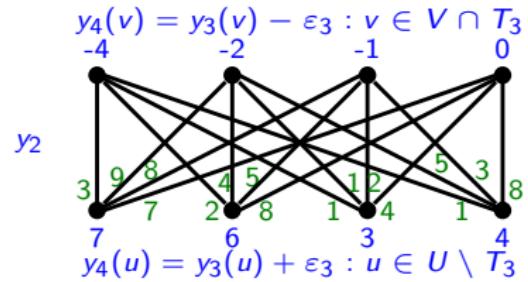
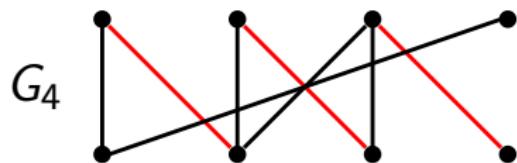
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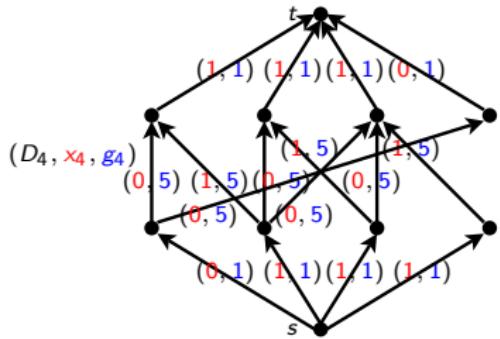
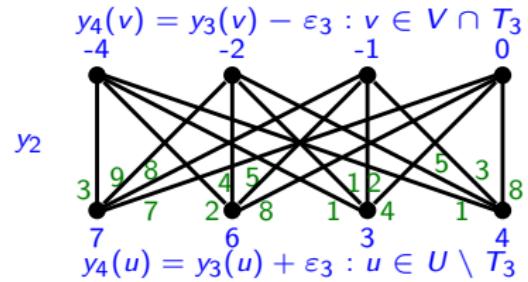
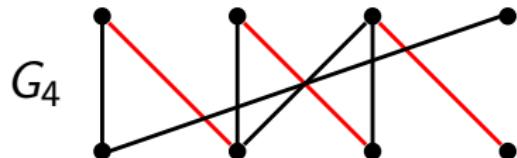
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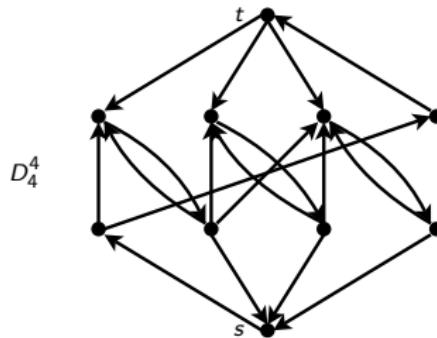
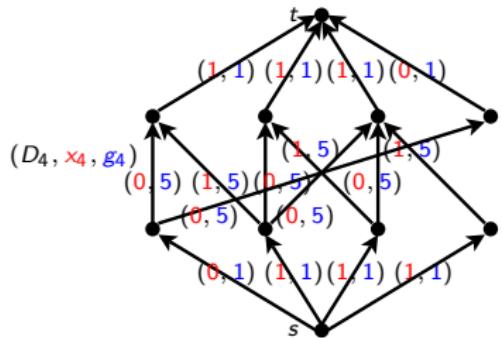
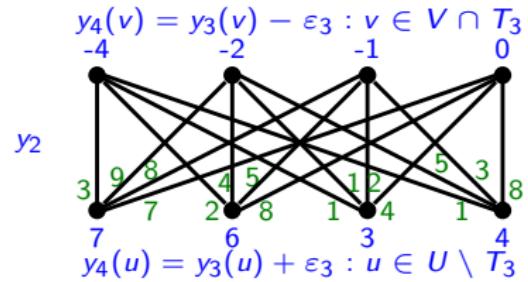
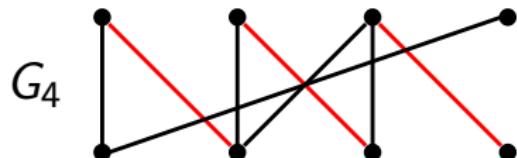
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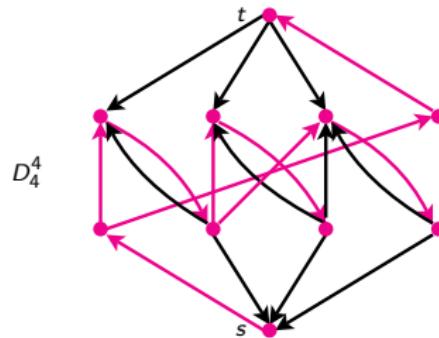
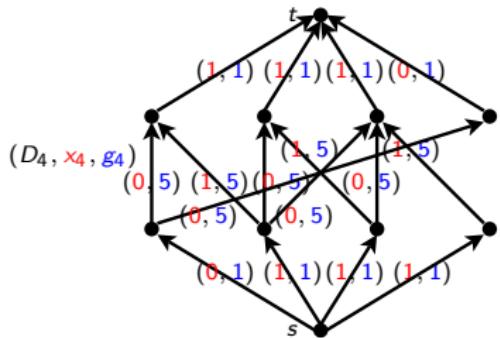
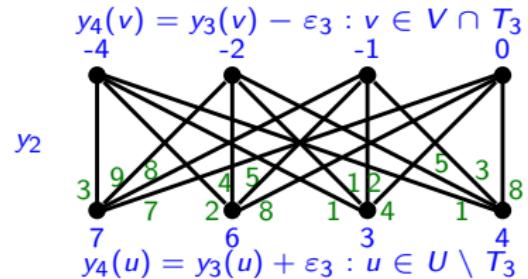
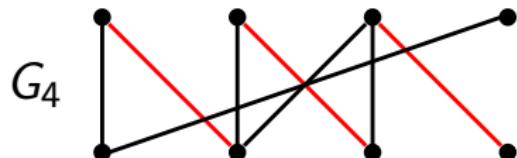
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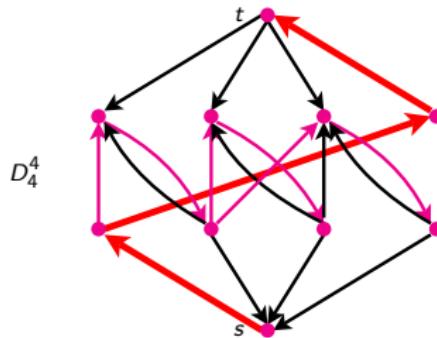
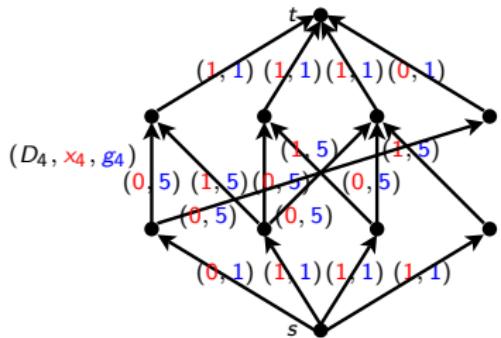
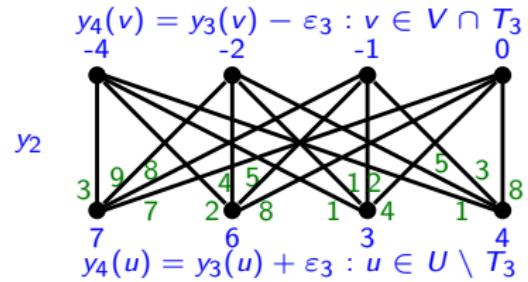
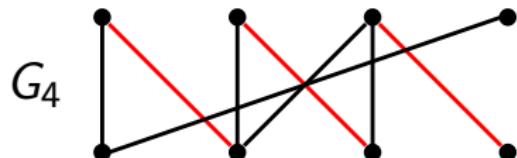
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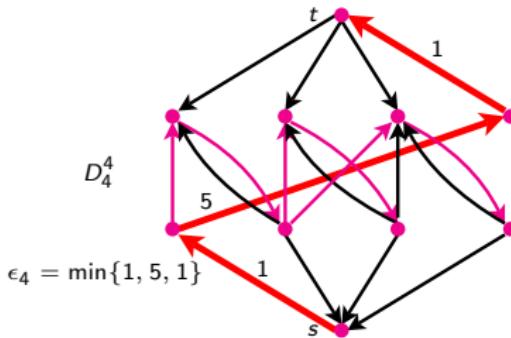
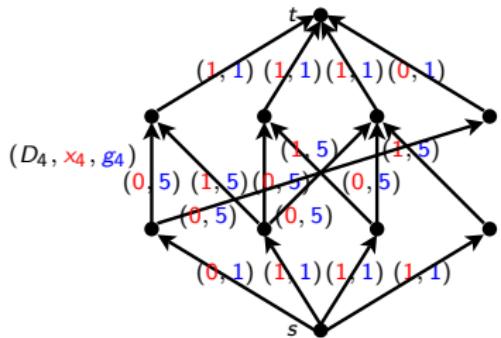
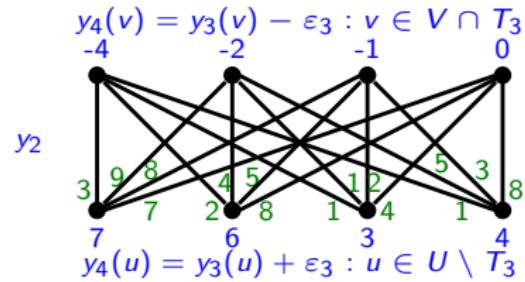
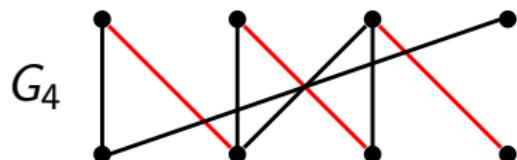
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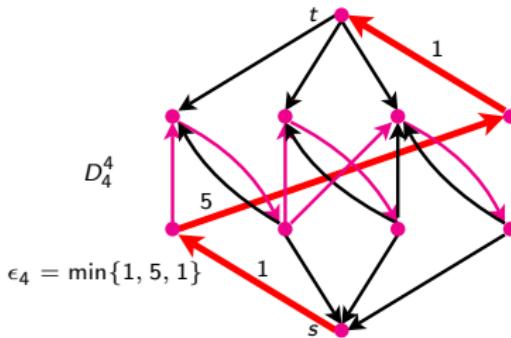
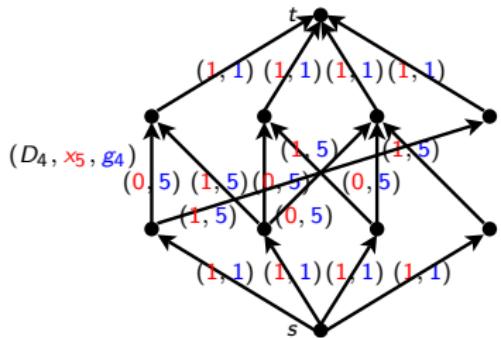
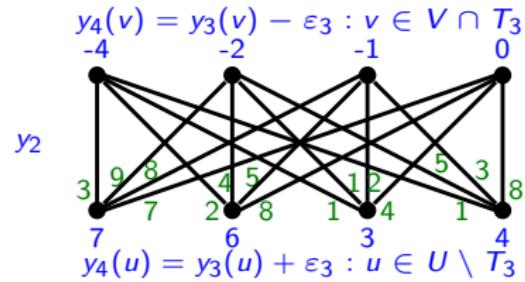
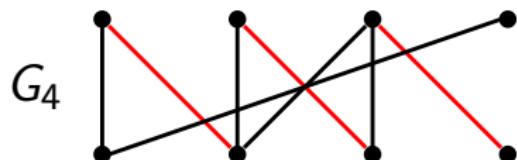
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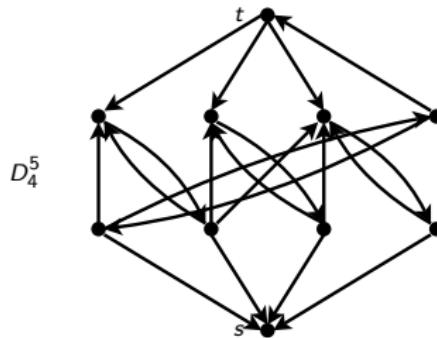
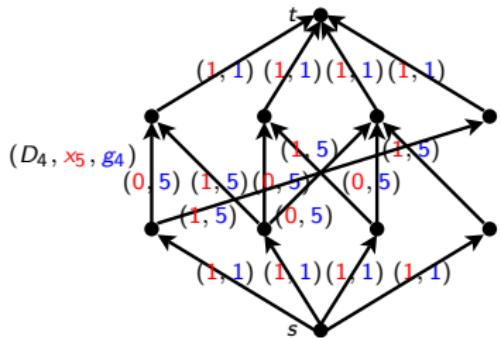
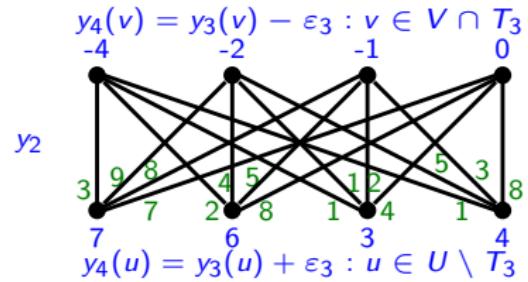
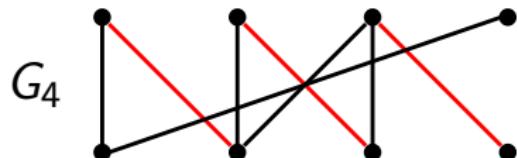
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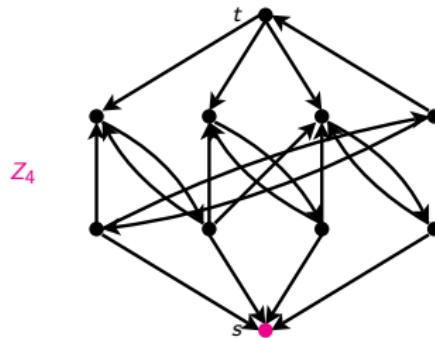
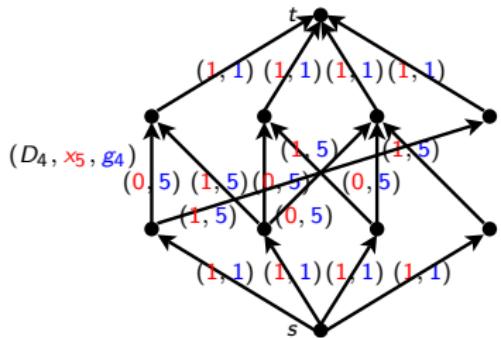
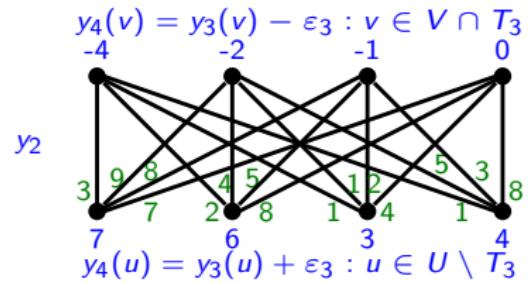
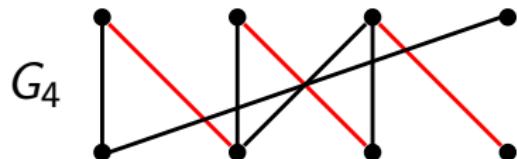
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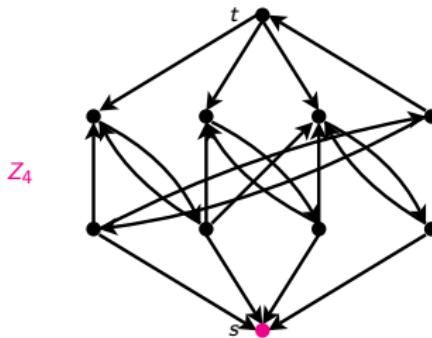
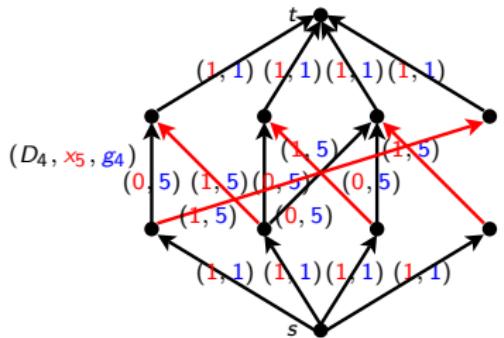
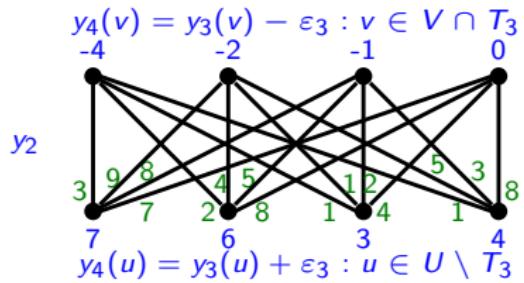
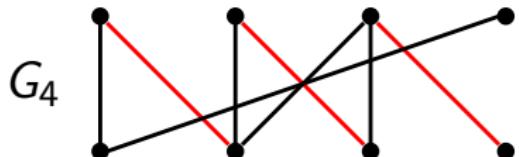
# Execution of the Hungarian method



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