1. Propose an algorithm to compute a basis of the fundamental group of a finite map. Analyse its complexity.

2. Suppose that the edges of a finite map $M$ are positively weighted. Propose an algorithm to compute a minimal basis of $M$. Analyse its complexity.

3. With the assumptions of the previous exercise, propose an algorithm to compute a shortest non-contractible circuit in $M$. Analyse its complexity. (When all the edges have unit weight, the length of this shortest circuit is called the *edge-width* of $M$ in graph theory. It is a combinatorial version of the systole in Riemannian geometry.)

4. Let $M$ be a map and let $b : F(M) \to \{0, 1\}$, be a *boundary indicator* defined over the faces of $M$. A face of $M$ is said *perforated* if its boundary indicator is 1, and *plain* otherwise. We realize the pair $(M, b)$ as a topological surface with boundary as follows. Thicken the graph $G(M)$ as for the topological realization of the map $M$ and close each plain facial circuit of this thickening with a disk. Equivalently, one can realize $(M, b)$ by gluing facial polygons and remove a small open disk in each polygon corresponding to a perforated face.

We consider the homotopy relation for $(M, b)$ generated by addition/removal of spurs and replacement of complementary paths of facial circuits of plain faces only.

Generalize the construction of a quad system for such pairs $(M, b)$, that allows to transform a path of $M$ into a homotopic path of the quad system in linear time.