1. Let $p : \text{Flower}_5 \to B_2$ be the covering represented in the following Figure. 

Call $a$ the lower loop edge of $B_2$. On what condition related to $a$ does a loop of $B_2$ lift to a loop in $\text{Flower}_5$? Deduce that for any vertex $v$ of $\text{Flower}_5$, we have $p_*\pi_1(\text{Flower}_5, v) \triangleleft \pi_1(B_2)$, i.e. $\pi_1(\text{Flower}_5, v)$ is normal in $\pi_1(B_2)$. What is the quotient group? Justify your answer in two different ways.

2. Let $f$ be a morphism from the covering $p : H \to G$ to the covering $q : K \to G$. Consider a vertex $v$ in $H$ and a path $\alpha$ in $G$ with initial vertex $p(v)$. Show the identity

$$f(v)\cdot \alpha = f(v, \alpha)$$

3. Show that a covering morphism with $H$ connected, must be the identity.

4. Consider the Petersen graph $P = P(5, 2)$ (see Exercise sheet #2). Let $r$ be the automorphism of $P$ acting on $C_5(5)$ by a one-shift (vertex $i$ is sent to vertex $i + 1$). What is the quotient graph $P/\langle r \rangle$? Can you obtain the same quotient graph as a quotient of the graph (1-skeleton) of the 3 dimensional cube?

5. Give a characterisation of the embedding of $\pi_1(P)$ induced by the quotient in the previous exercise.

6. Prove that there exists a free action of $\mathbb{Z}/n\mathbb{Z}$ on the complete graph $K_n$ if and only if $n$ is odd. What is the quotient graph for $n = 5$? Give a characterisation of the corresponding embedding of $\pi_1(K_5)$ in the fundamental group of the quotient.

7. Give an example of a graph morphism between connected graphs whose edge or vertex fibers all have the same size though the morphism is not a covering.