Online algorithm for the Canadian Traveler Problem on outerplanar graphs

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Joint work with
Beaudou, Bergé, Chernyshev, Dailly, Gerard, Limouzy, Pastor

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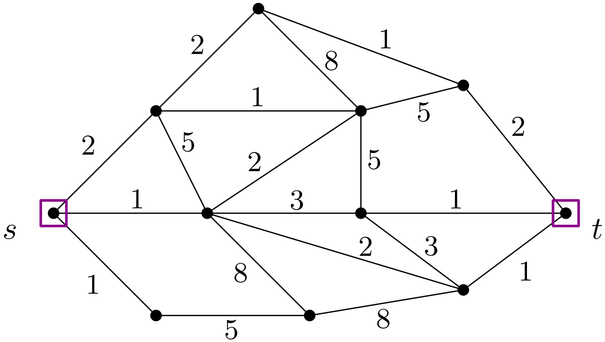
k-Canadian Traveler Problem



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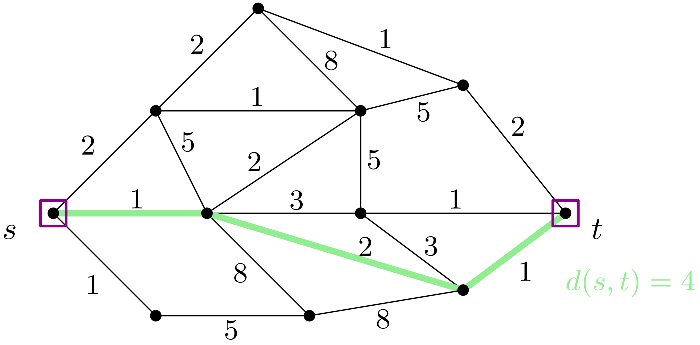

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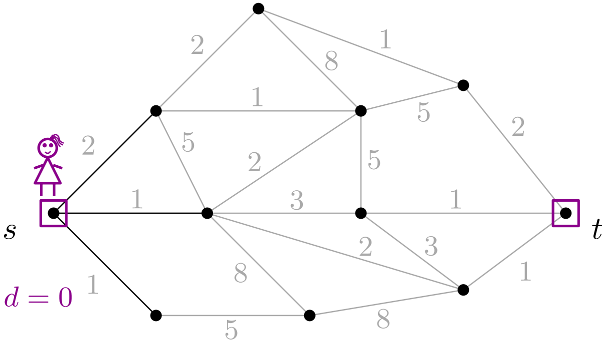

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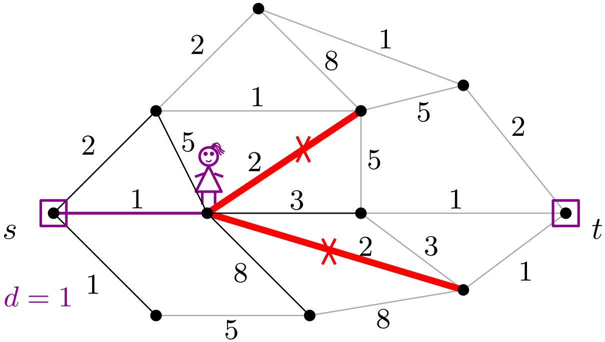

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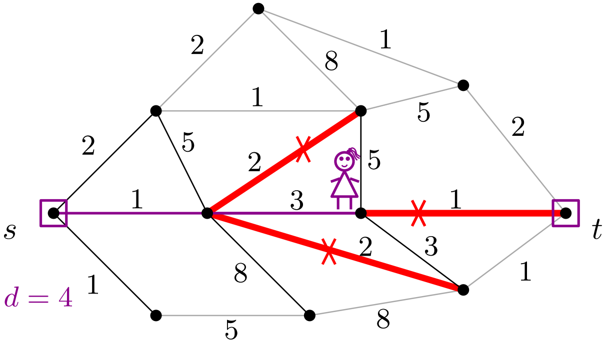

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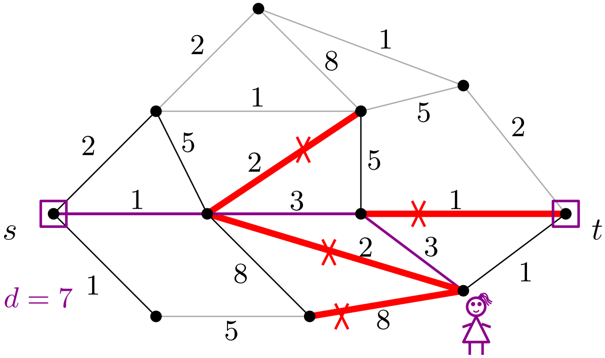

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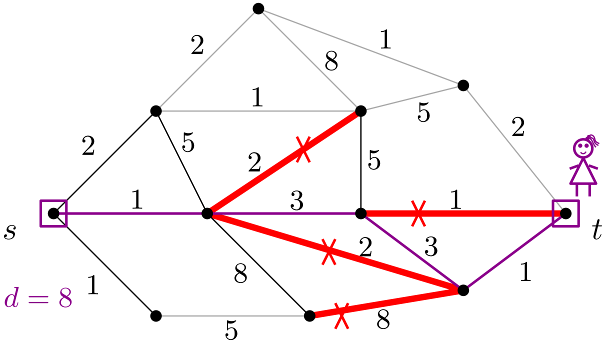

At most k blocked edges.

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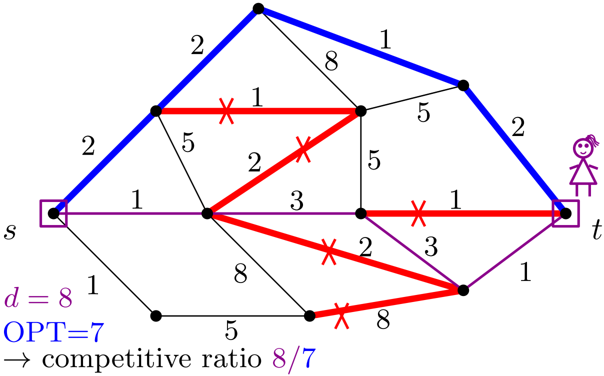

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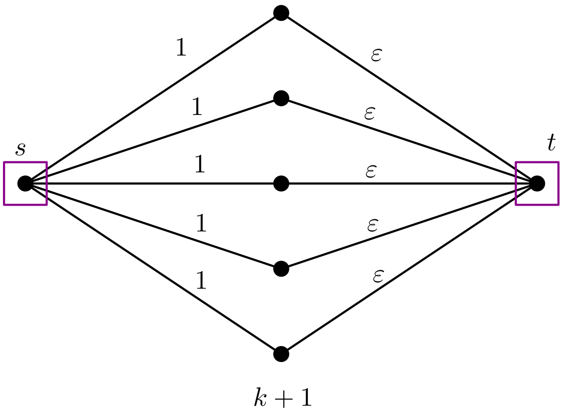

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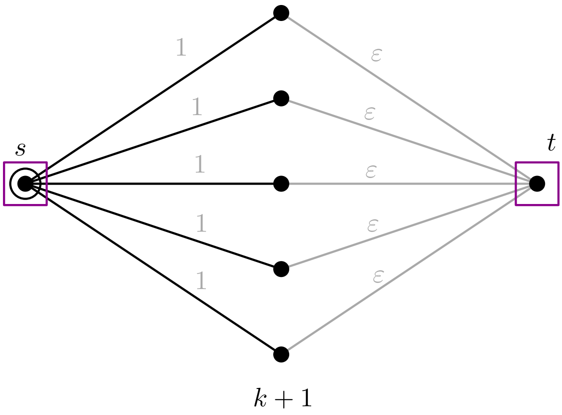

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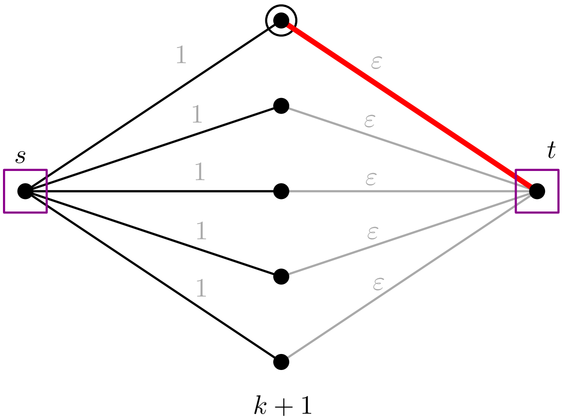

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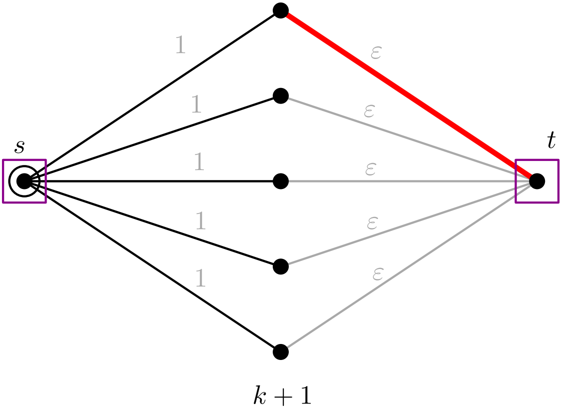

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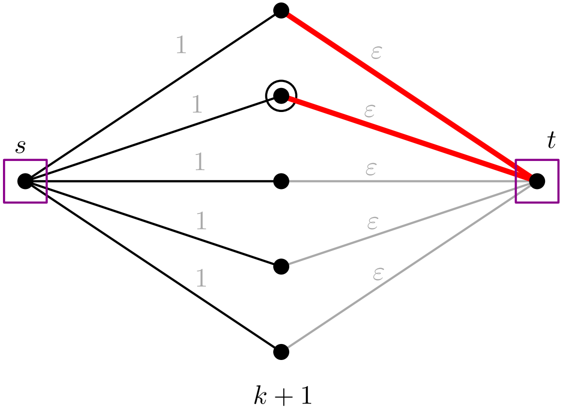

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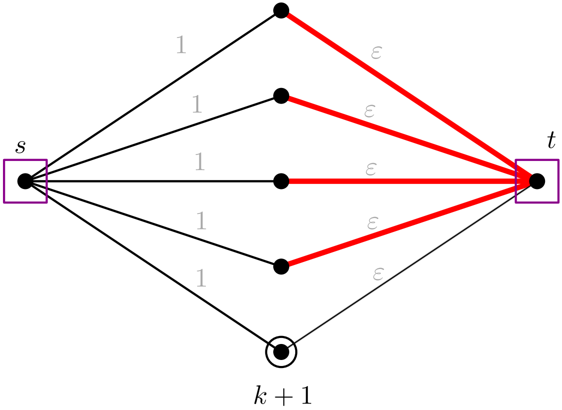

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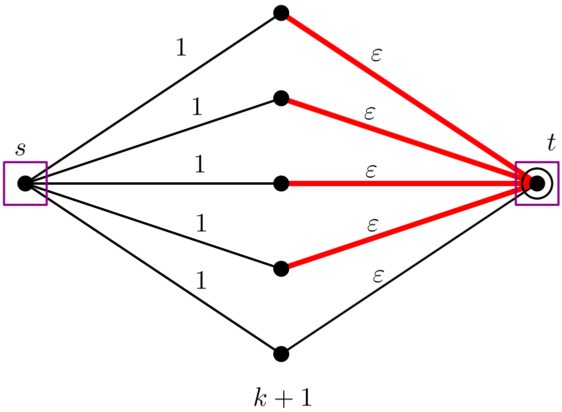

$\Rightarrow$ competitive ratio $\frac{2k+1+\varepsilon}{1+\varepsilon} = 2k + 1 + o(1)$

Lower bound $2k + 1$ even for planar graphs of treewidth 2.

Optimal strategies

Competitive ratio $2k+1$ in the general case :

- REPOSITION strategy :

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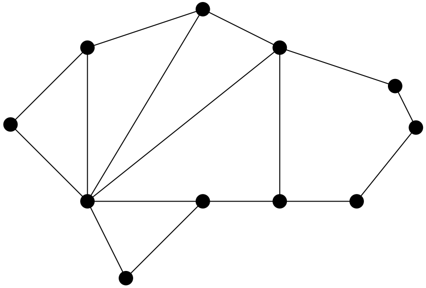
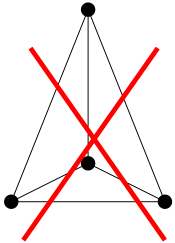
PSPACE-complete :
given a number r, an instance (G, w, s, t), decide if there is a strategy of competitive ratio $\leq r$.

Outerplanar graphs

Outerplanar : can be drawn in the plane with all vertices on the outer face.





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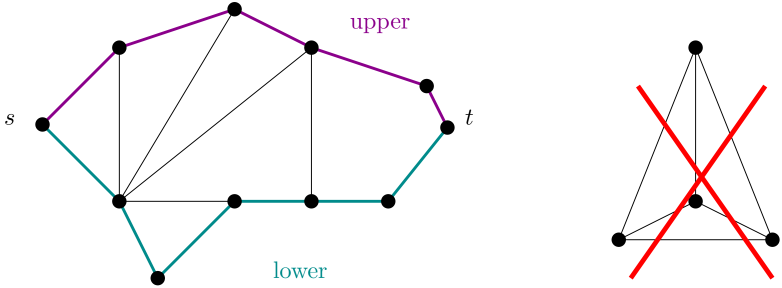

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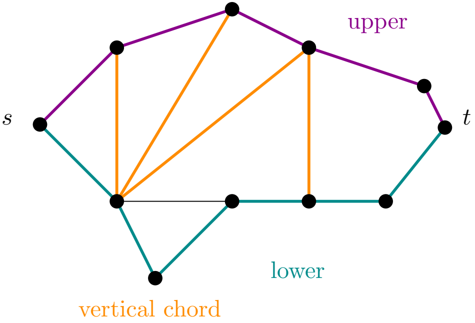

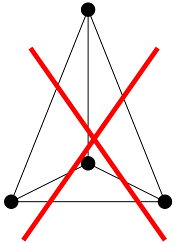

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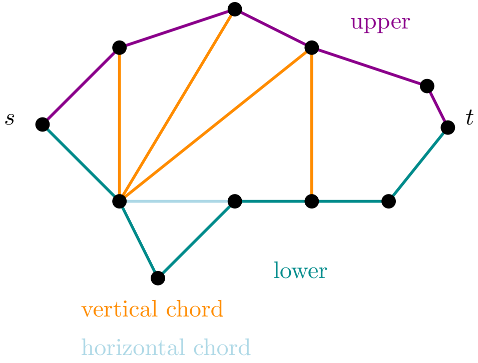

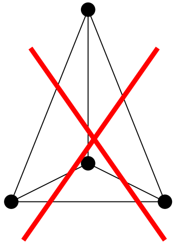

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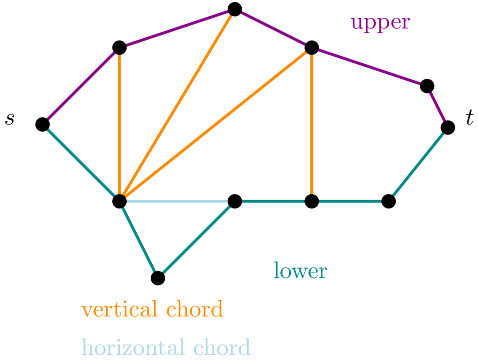

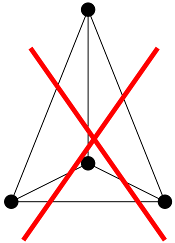

Unit-weighted : $w(e) = 1 \qquad \forall e \in E.$

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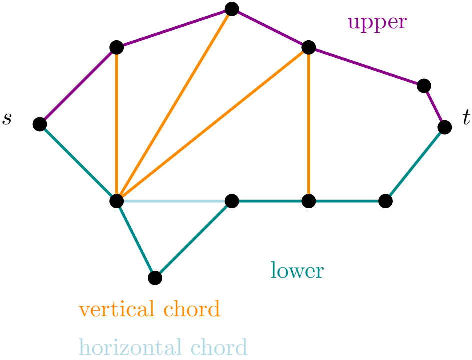
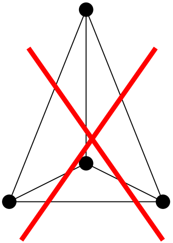
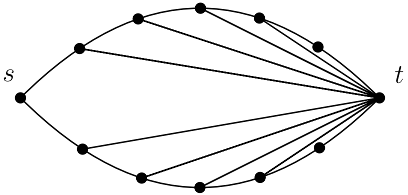




Unit-weighted : $w(e) = 1 \quad \forall e \in E$. $\rightarrow$ optimal ratio 9

A shell and a cow : Linear Search Problem



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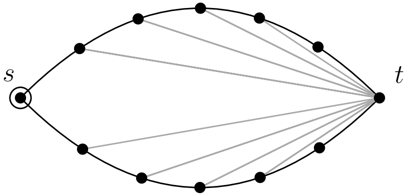

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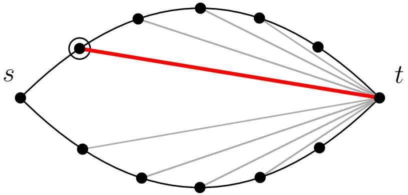

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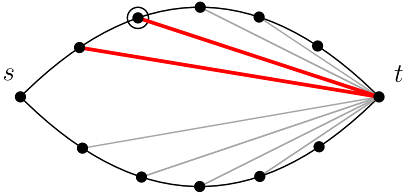

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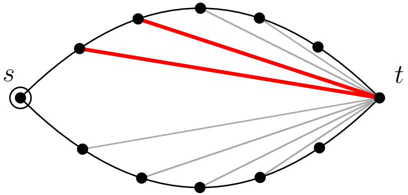

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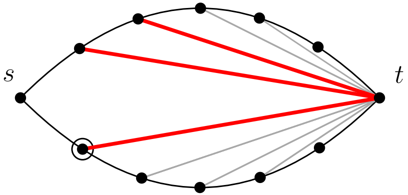

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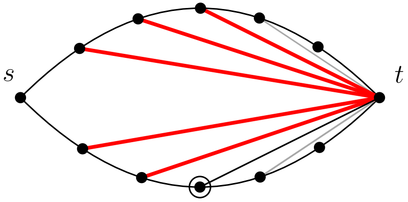

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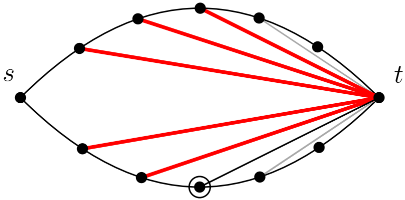

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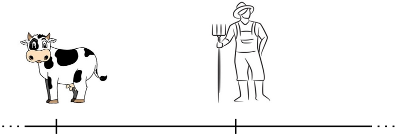

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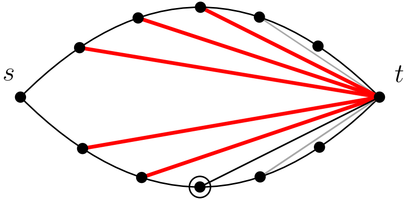

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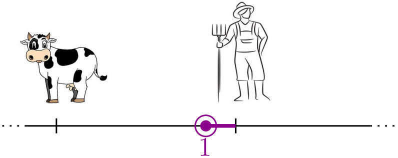

Observation

No competitive ratio < 9 on unit-weighted outerplanar graphs for the k-Canadian Traveler Problem.

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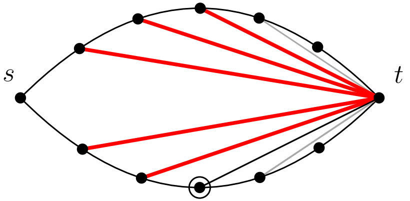

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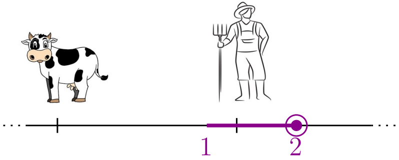

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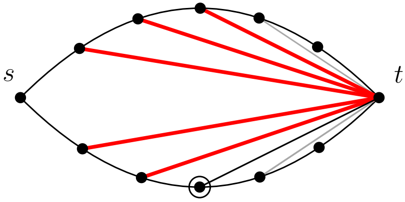

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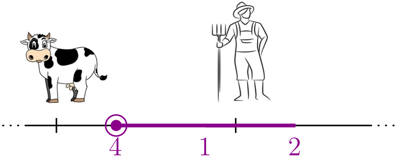

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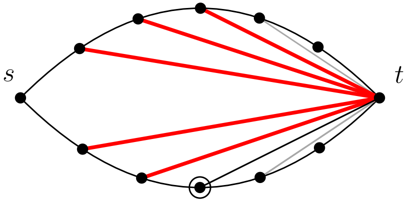

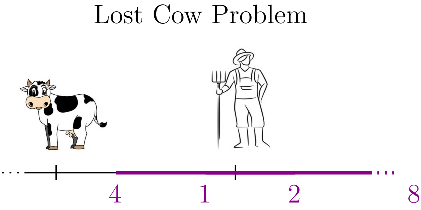

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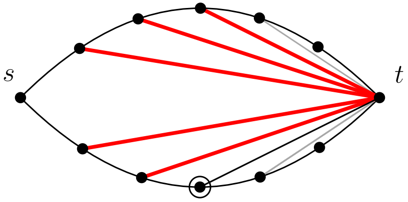

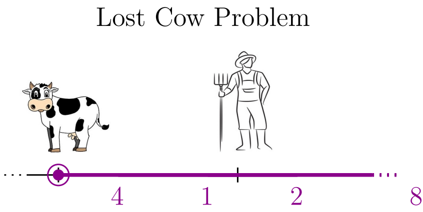

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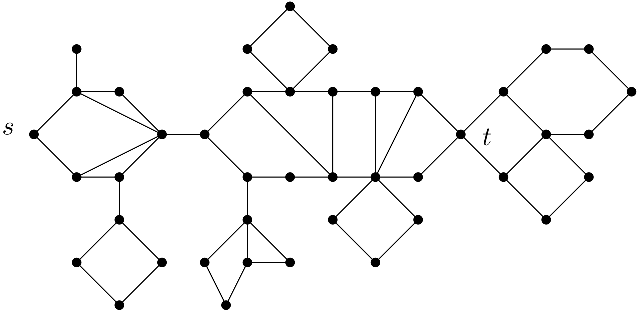
Competitive ratio 9

Theorem [BBCDGLLP24+]

There is a strategy with competitive ratio 9 for the k-Canadian Traveler Problem on all unit-weighted outerplanar graphs.



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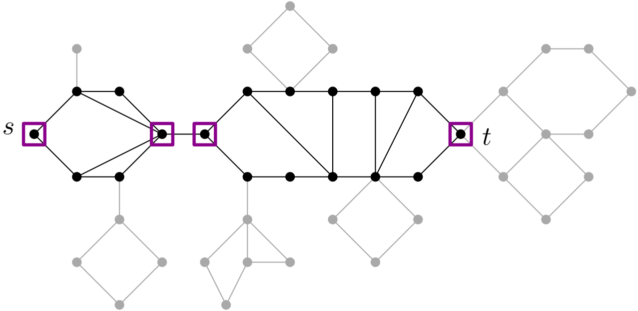
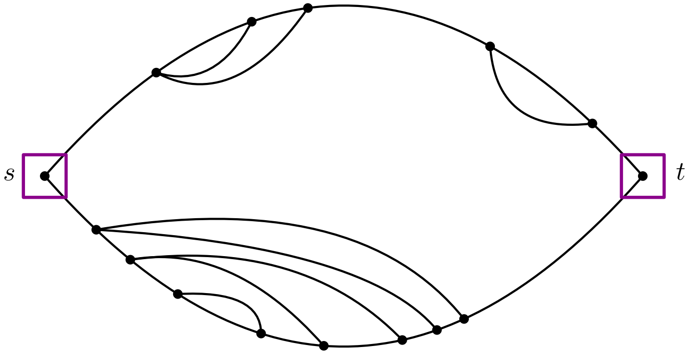


Horizontal chord



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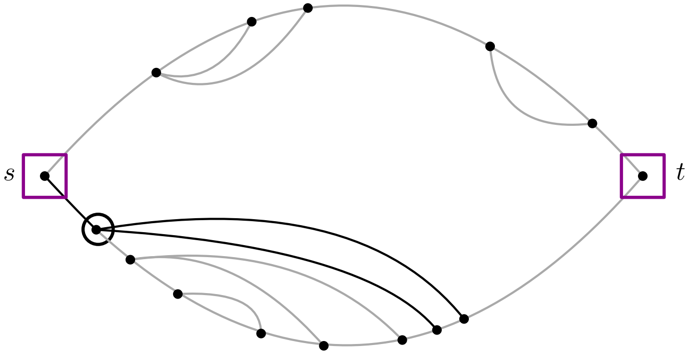
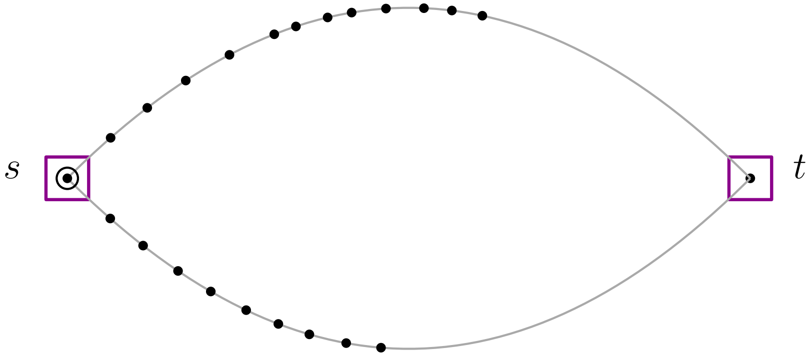


Exponential balancing & Vertical chord



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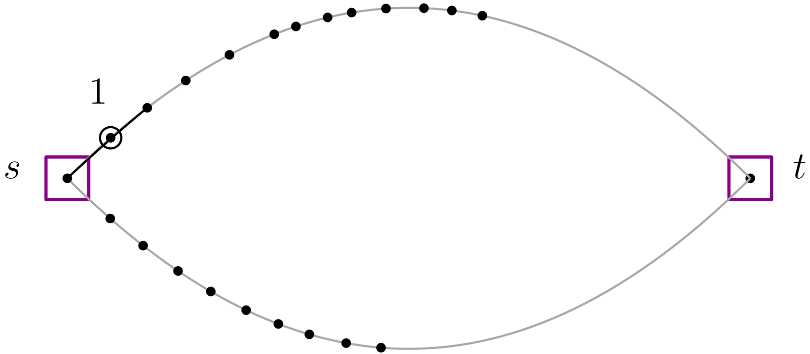

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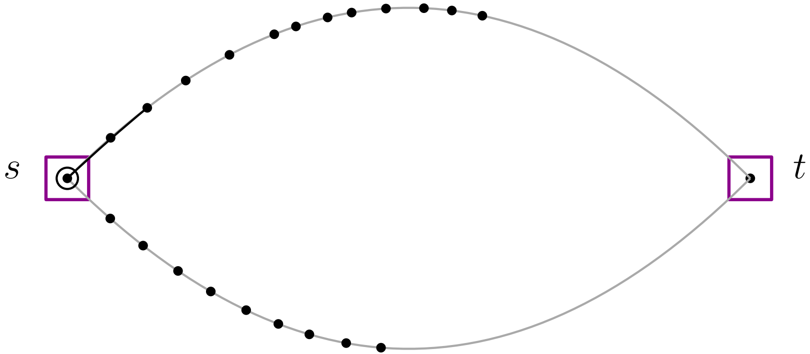

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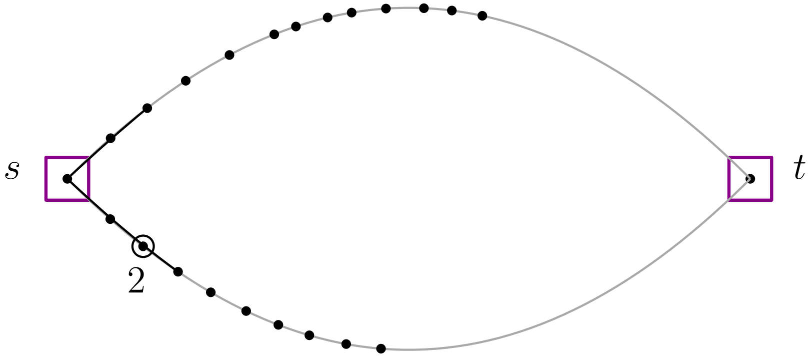

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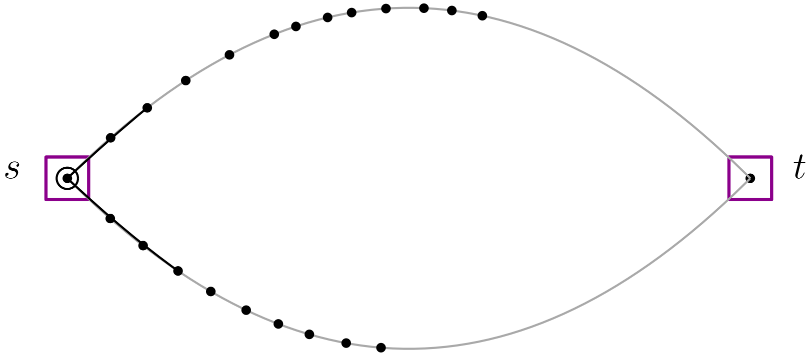

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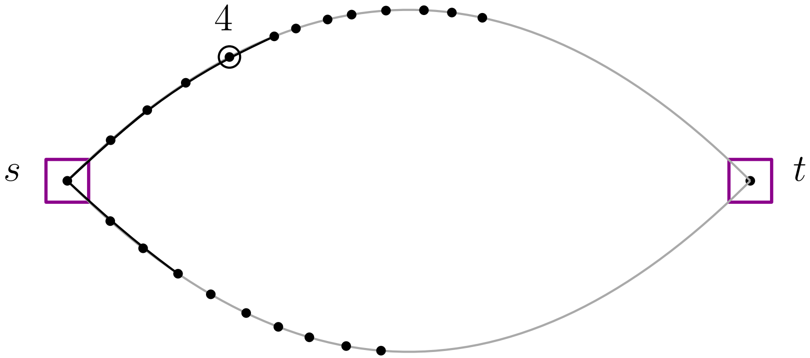

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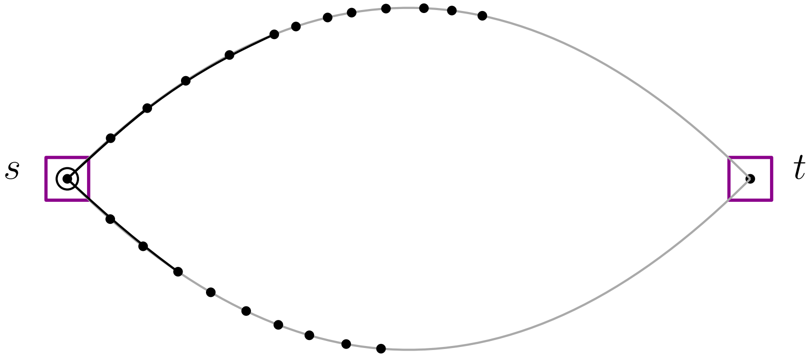

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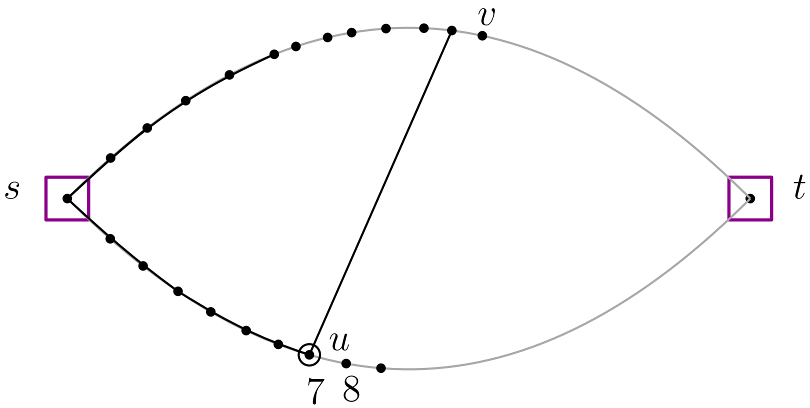

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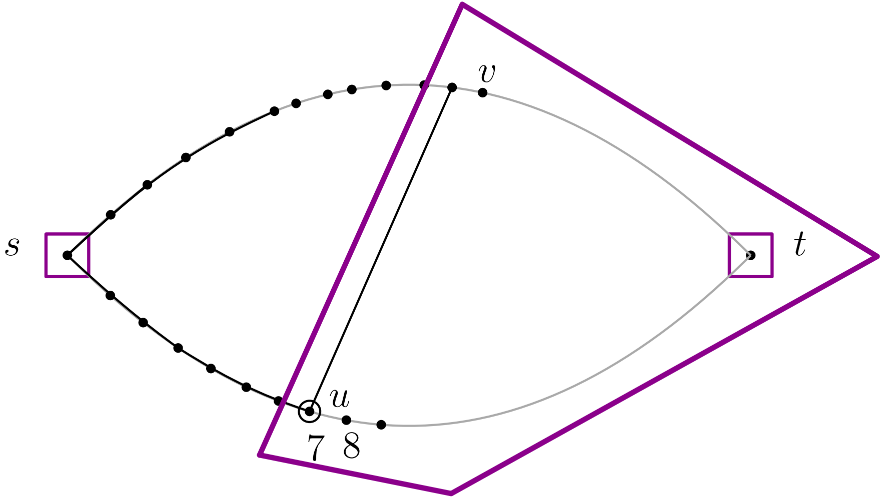

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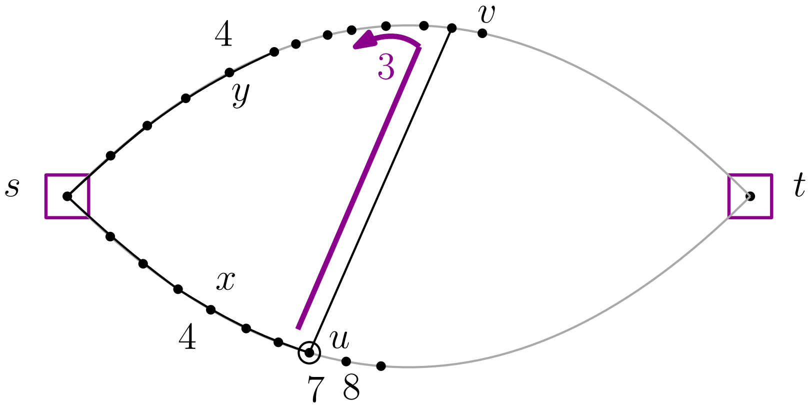

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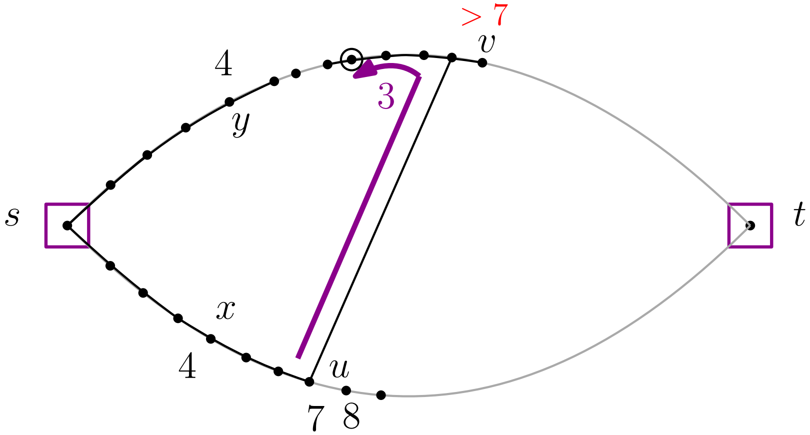

shortest sv-path go through $u \to v$

Exponential balancing & Vertical chord

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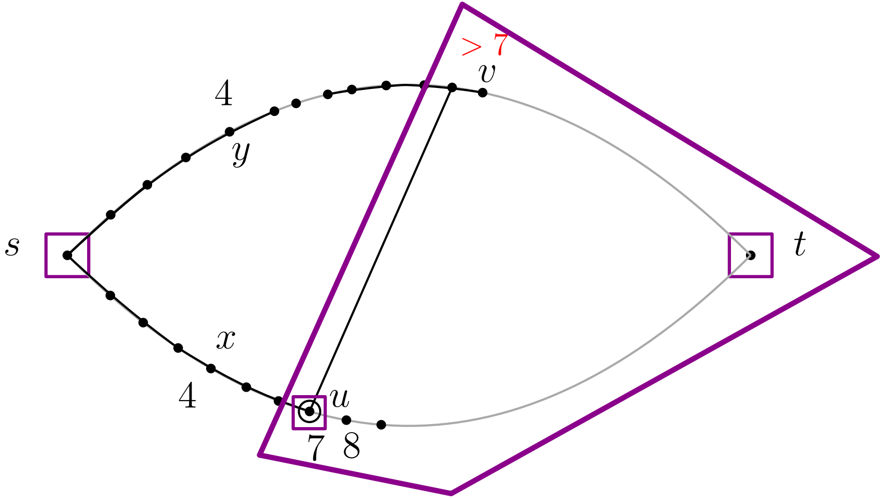

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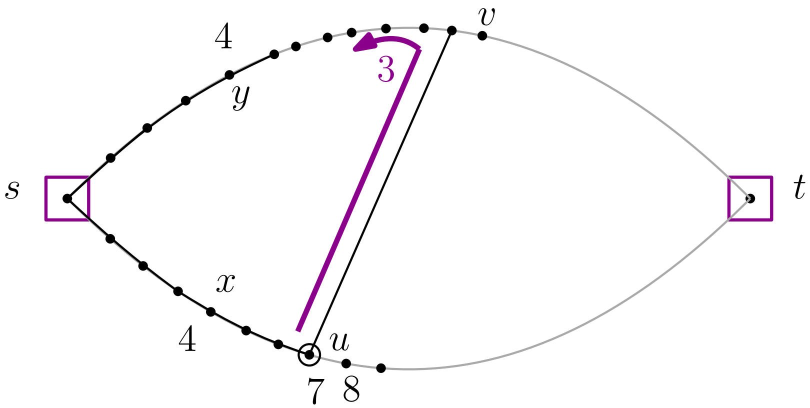

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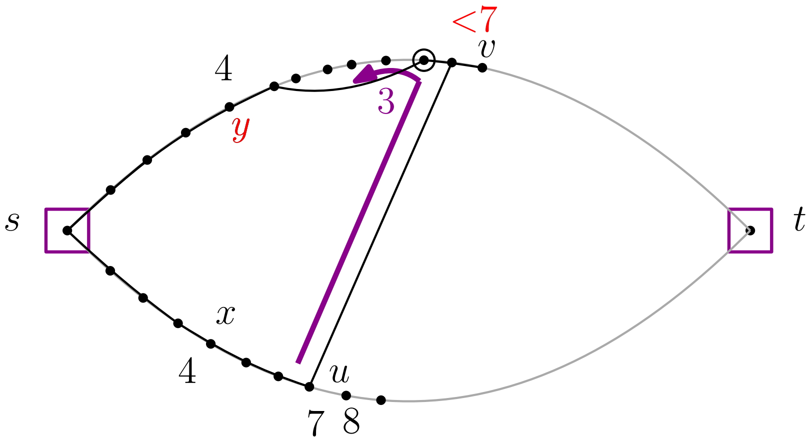

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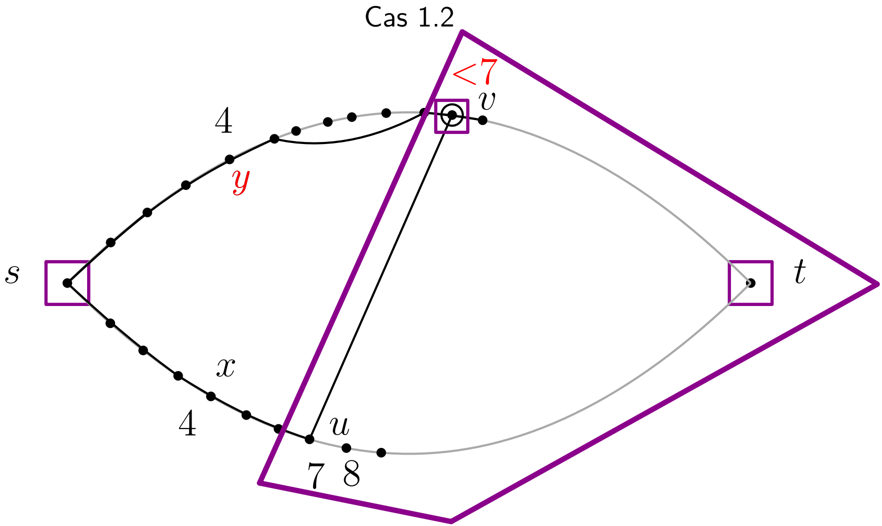

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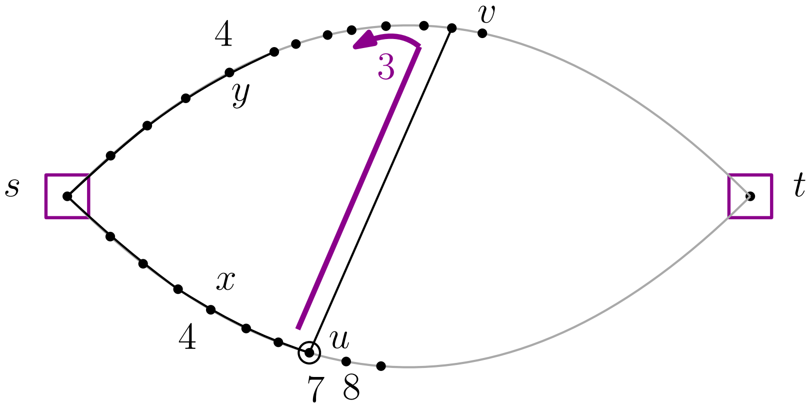
Exponential balancing & Vertical chord

Cas 1.3



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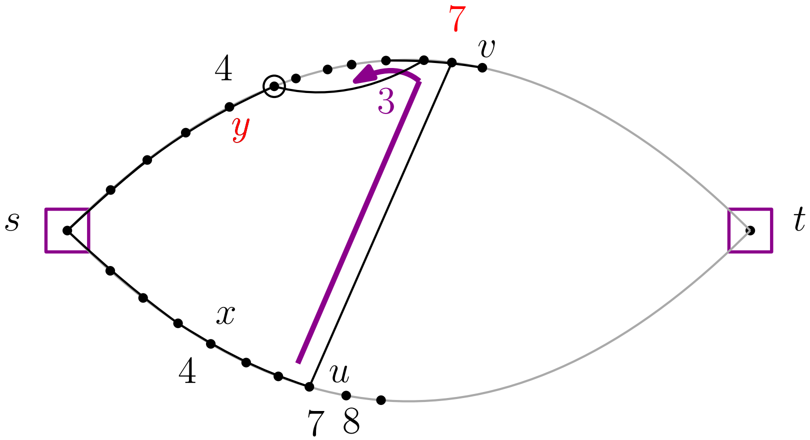

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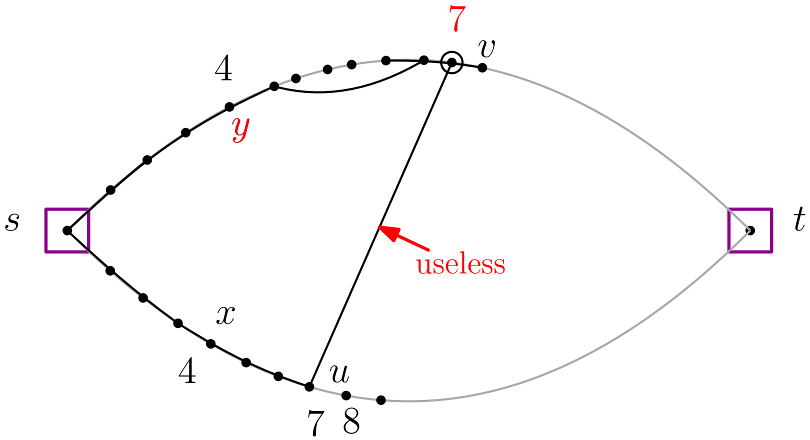

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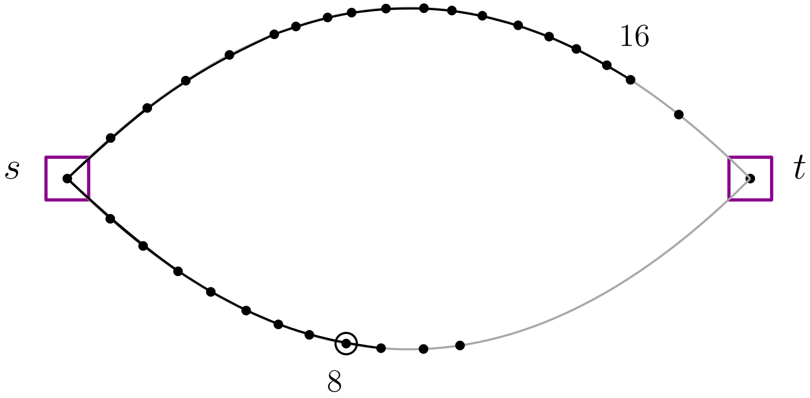
Cas 2



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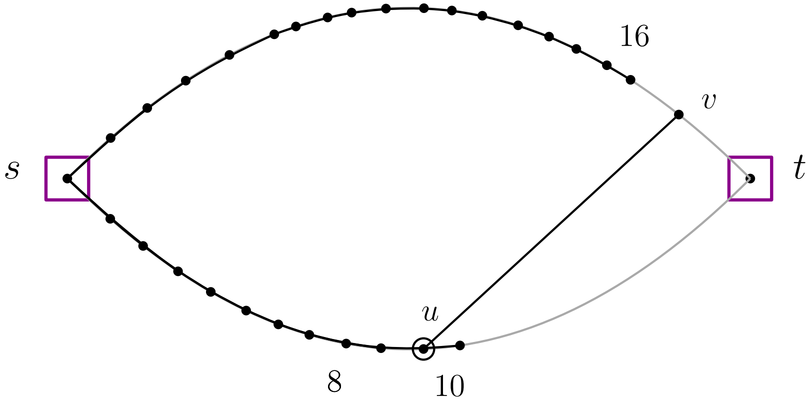

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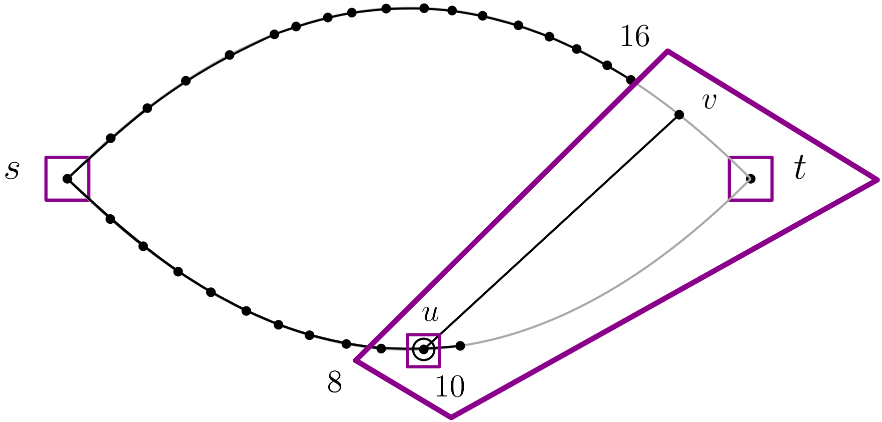
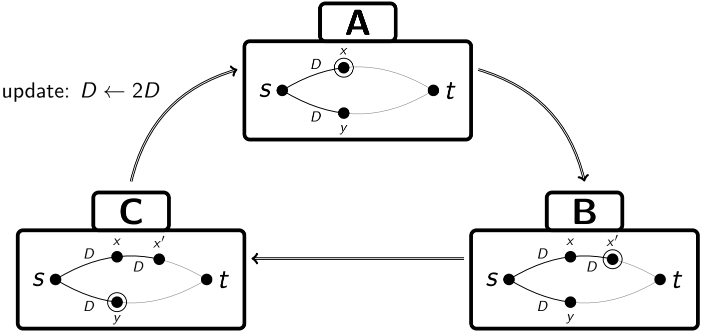


shortest sv-path go through $u \to v$

Proof with 3 steps



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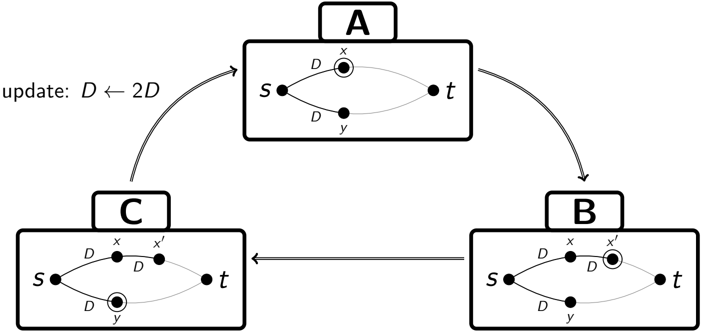

$\rightarrow$ Strategy with ratio 9.

Arbitrary weights

Theorem [BBCDGLLP24+]

There is no strategy with ratio better than $\Omega(\frac{\log k}{\log \log k})$ for the k-Canadian Traveler Problem on all outerplanar graphs.

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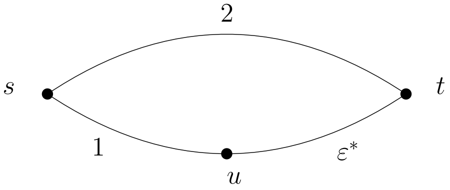
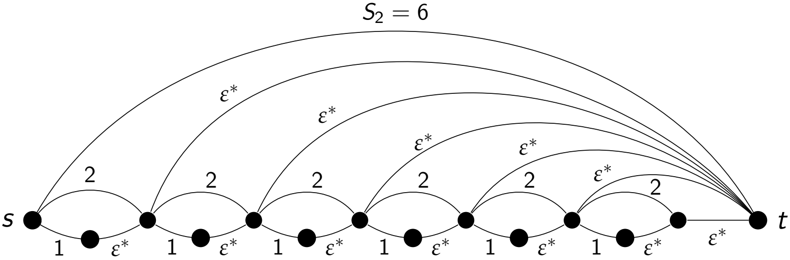
Sketch of proof Build H_i on which :

- we cannot achieve ratio better than $r_i = i + 1 - \varepsilon$.
- there are less than $((i+1)!)^2$ blocked edges
- the distance traversed by the traveler is at least $(i+1)!$



Build H_i from H_{i-1}



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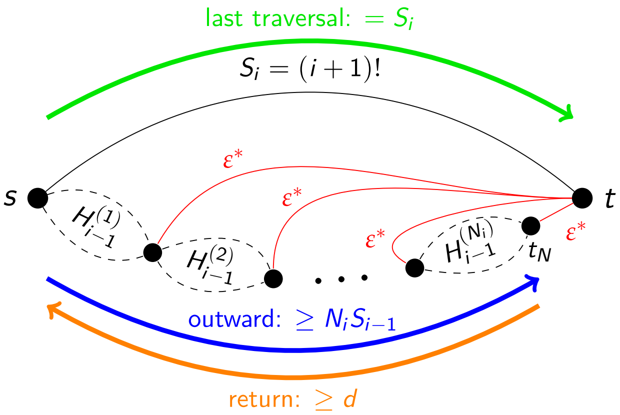

With $N_i = i(i+1)$, and $S_i = (i+1)!$
$\Rightarrow$ Cannot achieve ratio $i + 1 - \varepsilon$.

Open questions

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Thank you for your attention !