Online algorithm for the Canadian Traveler Problem on outerplanar graphs

Aurélie Lagoutte
G-SCOP, Grenoble INP / Université Grenoble Alpes

Joint work with
Beaudou, Bergé, Chernyshev, Dailly, Gerard, Limouzy, Pastor

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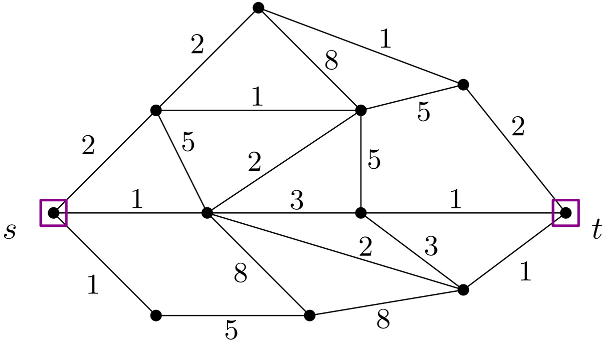
k-Canadian Traveler Problem



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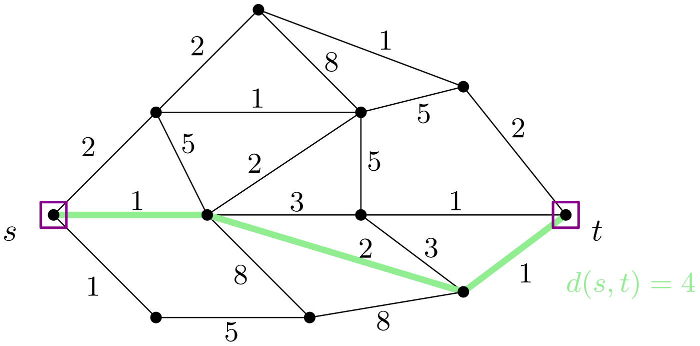

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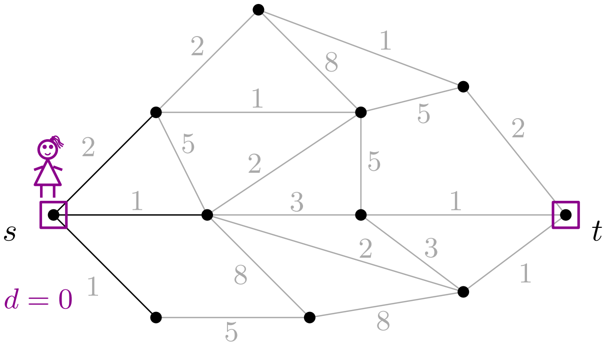

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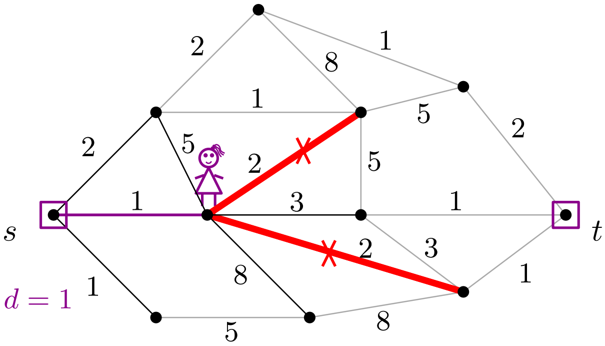

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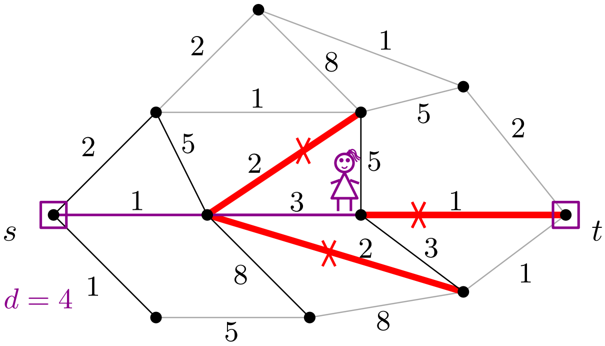

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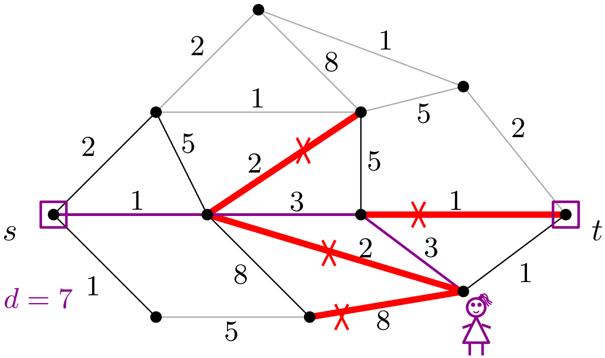

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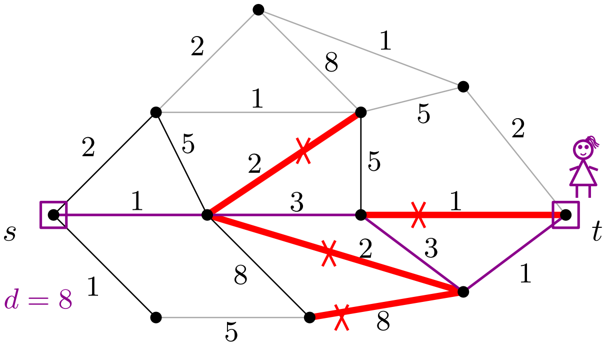

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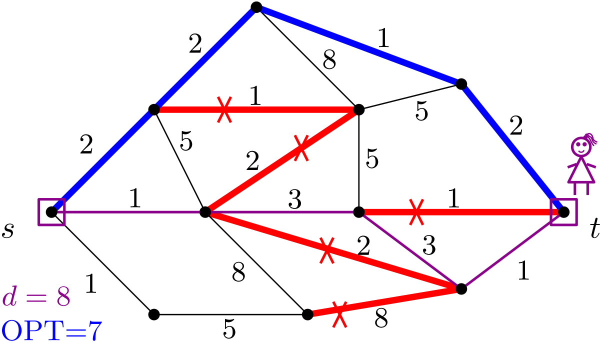
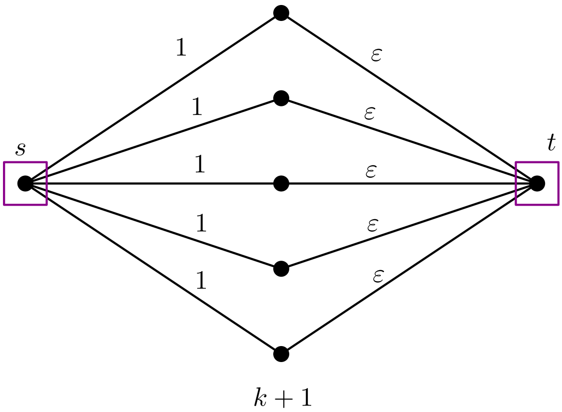


$d = 8$

OPT=7

$\rightarrow$ competitive ratio $8/7$

What can we hope in the general case ?



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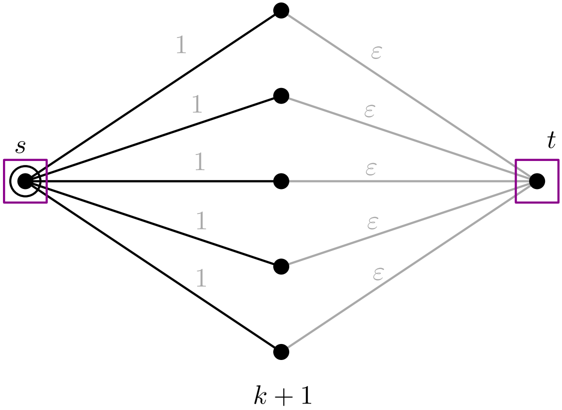

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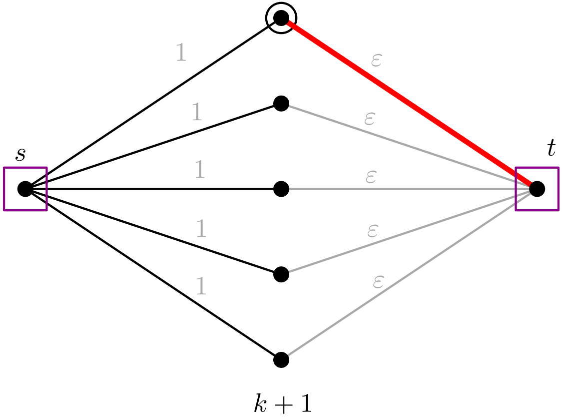

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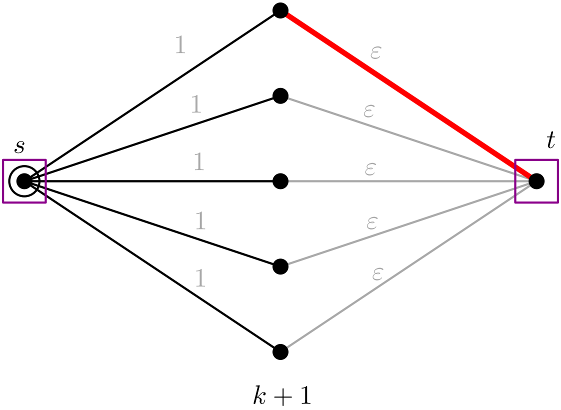

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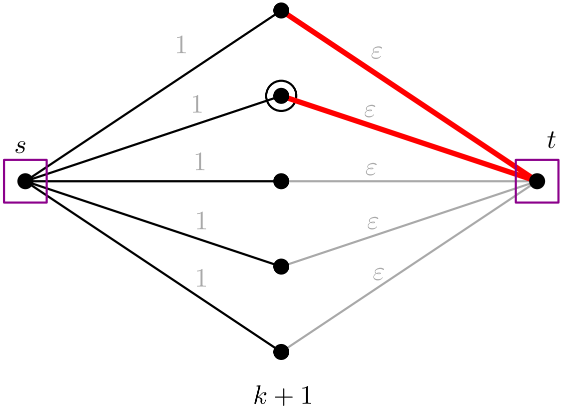

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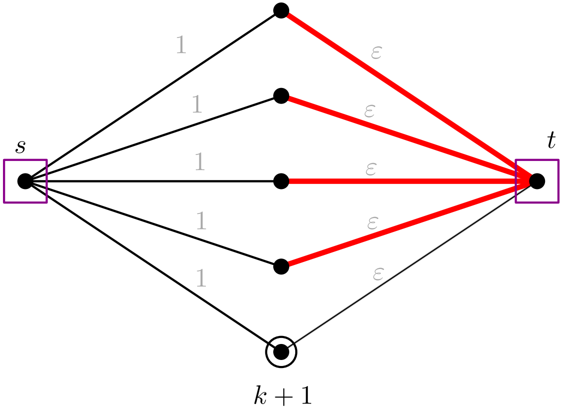

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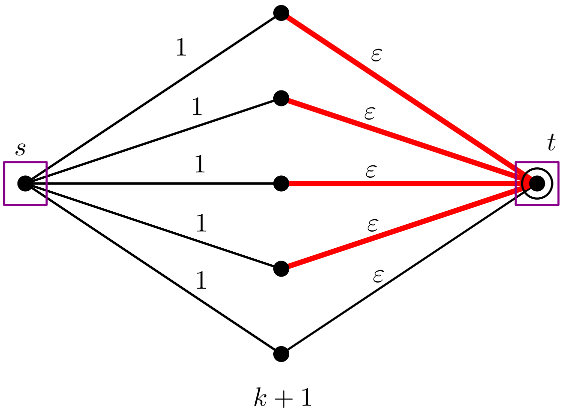

$\Rightarrow$ competitive ratio $\frac{2k+1+\varepsilon}{1+\varepsilon} = 2k + 1 + o(1)$

Lower bound $2k + 1$ even for planar graphs of treewidth 2.

Optimal strategies

Competitive ratio $2k+1$ in the general case :

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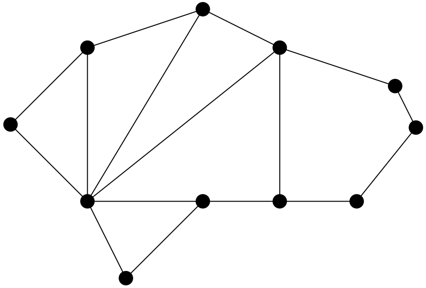
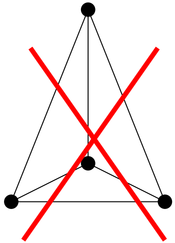
PSPACE-complete :
given a number r, an instance (G, w, s, t), decide if there is a strategy of competitive ratio $\leq r$.

Outerplanar graphs

Outerplanar : can be drawn in the plane with all vertices on the outer face.





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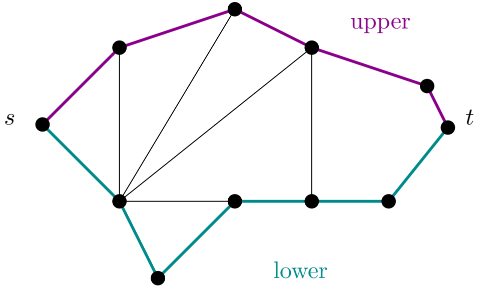

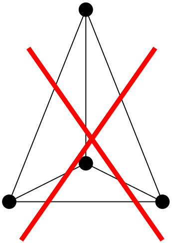

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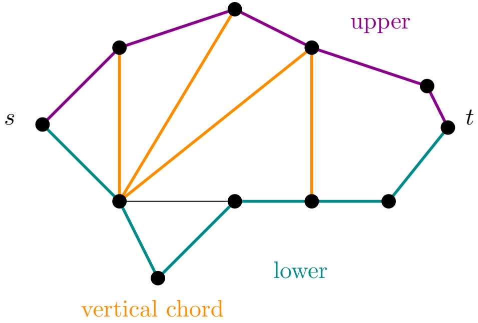

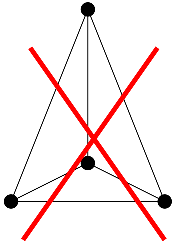

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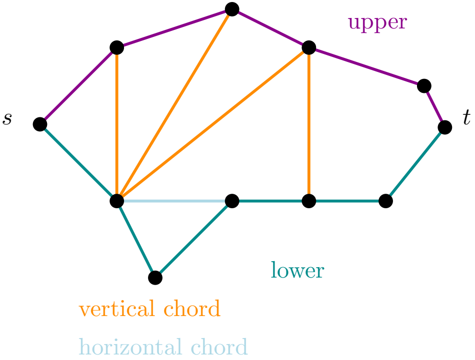

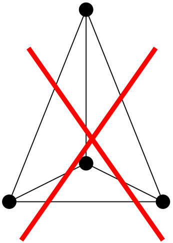

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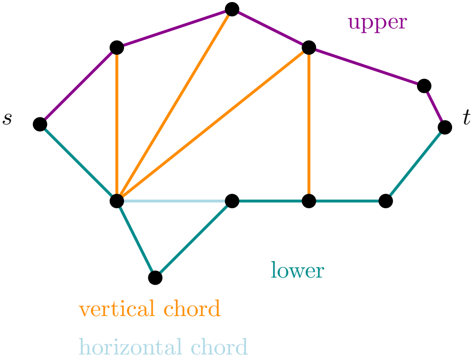
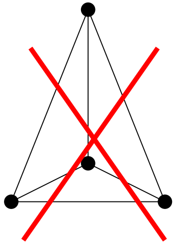
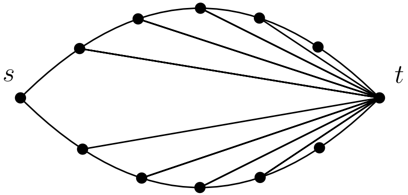




Unit-weighted : $w(e) = 1 \quad \forall e \in E$.

A shell and a cow : Linear Search Problem



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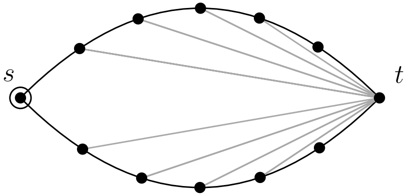

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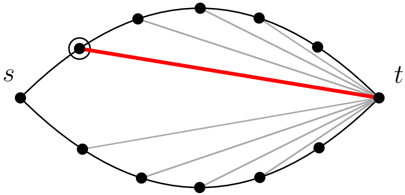

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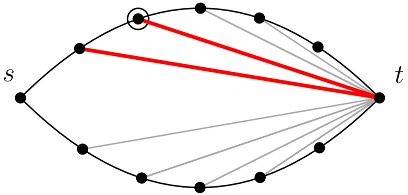

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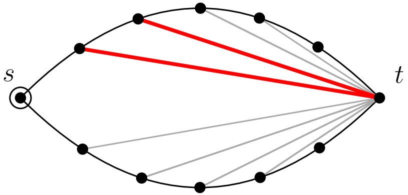

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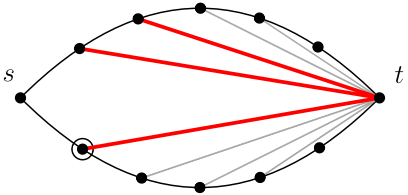

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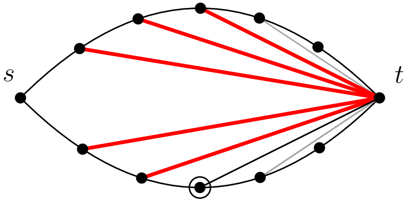

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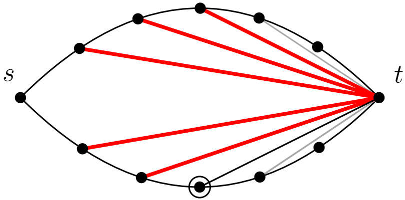
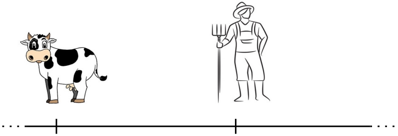


Lost Cow Problem



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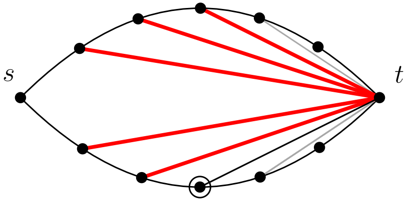

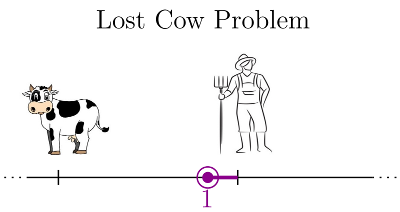

Observation

No competitive ratio < 9 on unit-weighted outerplanar graphs for the *k*-Canadian Traveler Problem.

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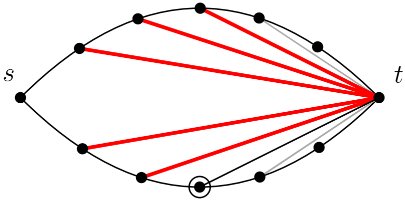

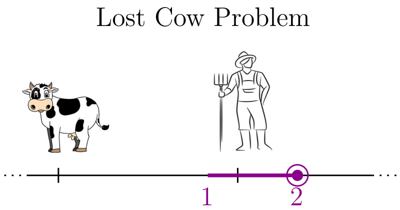

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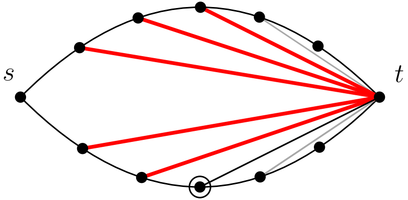

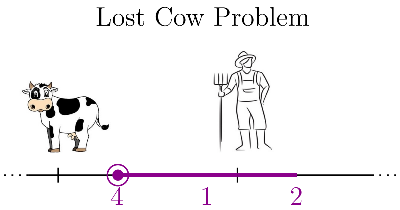

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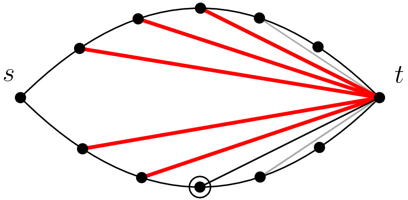

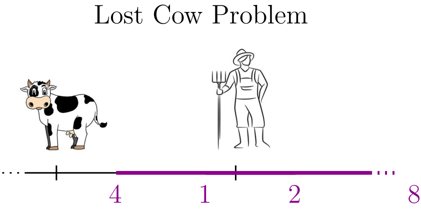

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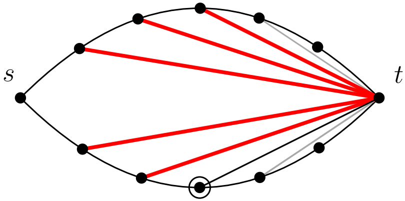

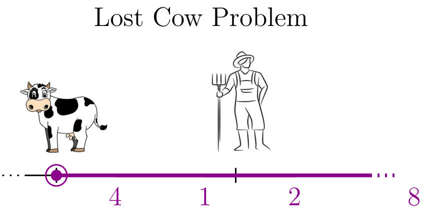

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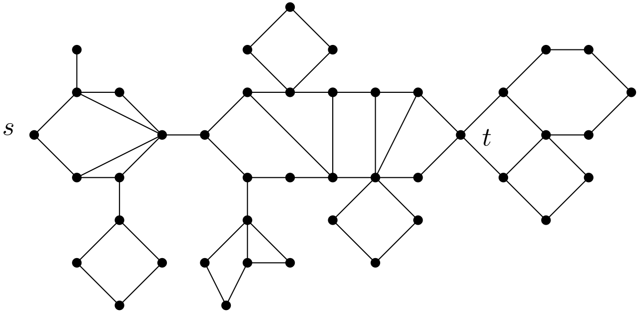
Competitive ratio 9

Theorem [BBCDGLLP24+]

There is a strategy with competitive ratio 9 for the k-Canadian Traveler Problem on all unit-weighted outerplanar graphs.



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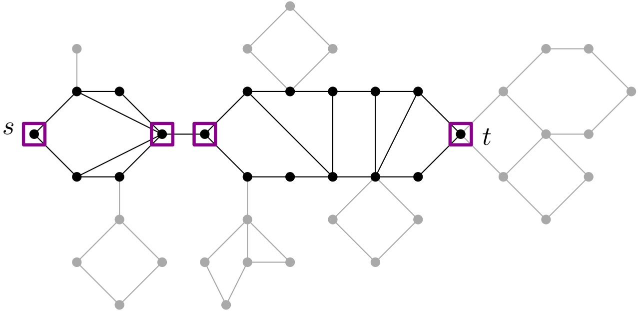
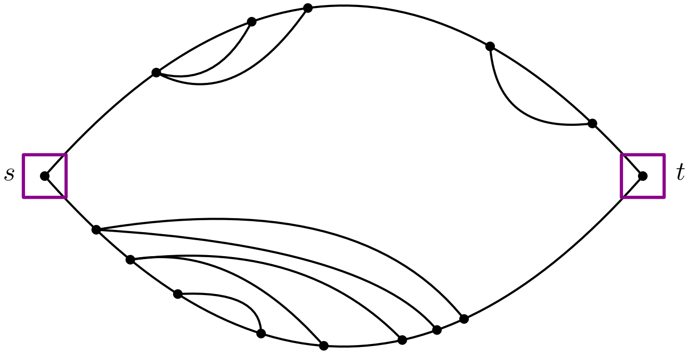


Horizontal chord



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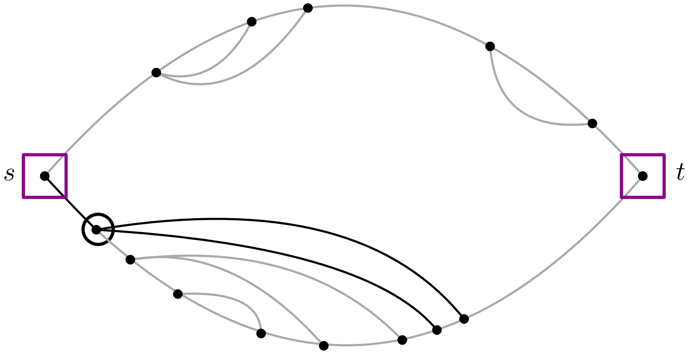
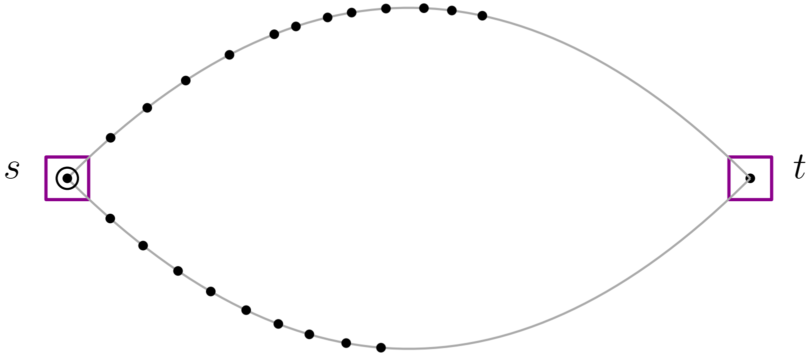


Exponential balancing & Vertical chord



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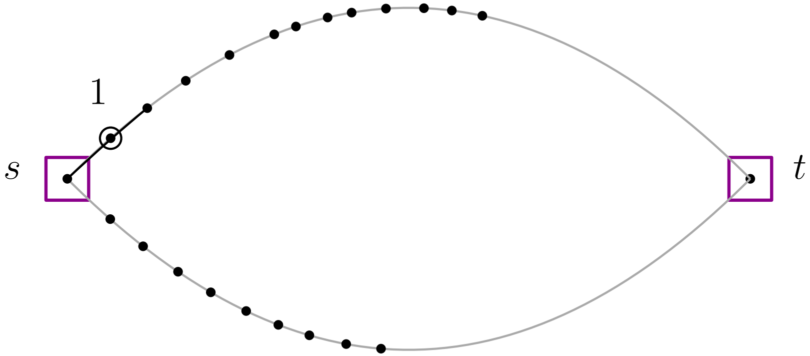

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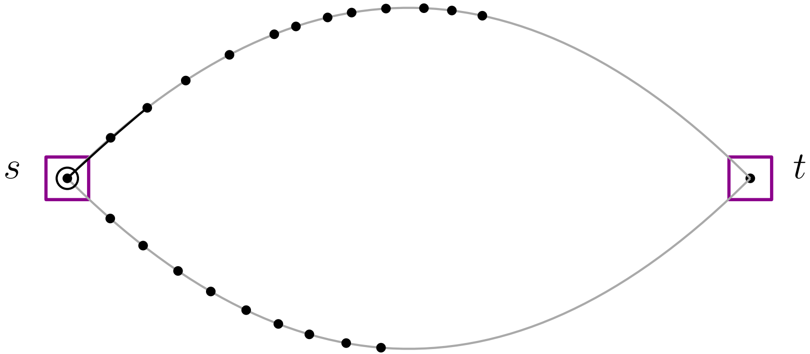

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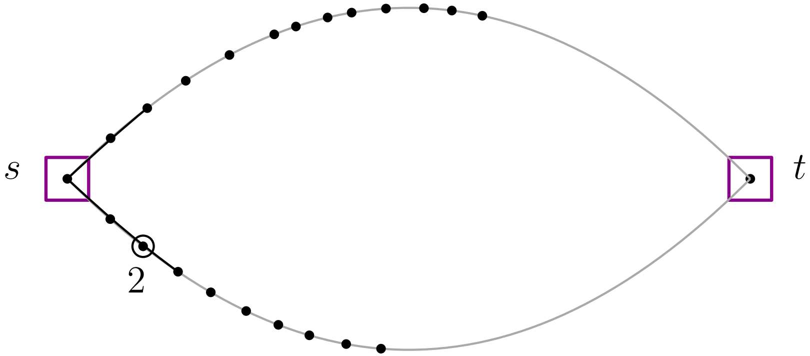

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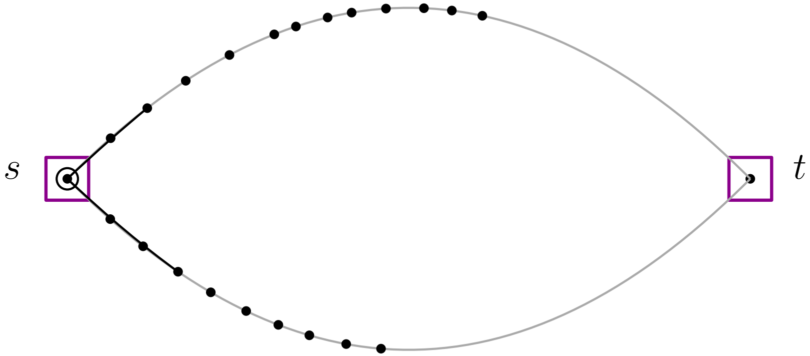

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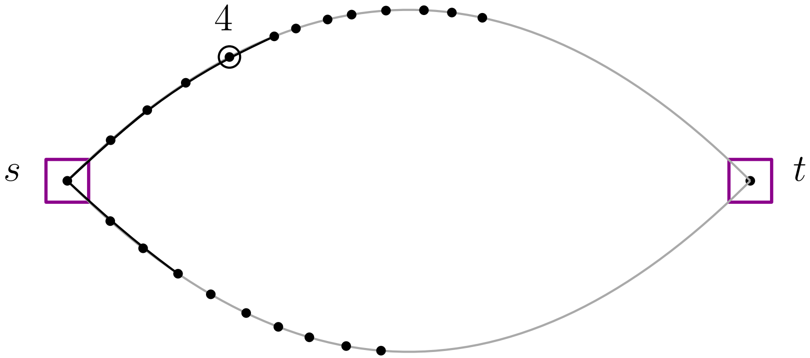

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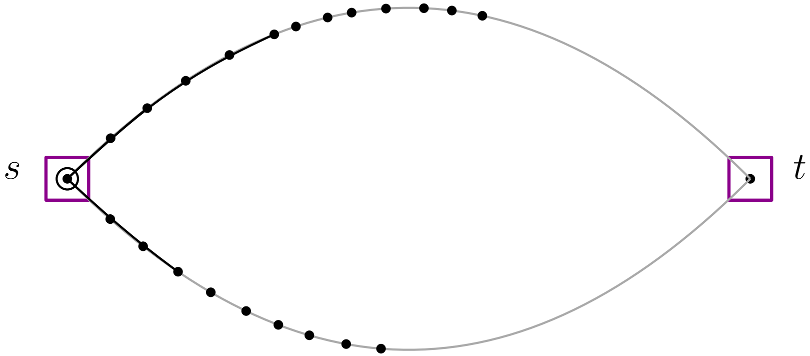

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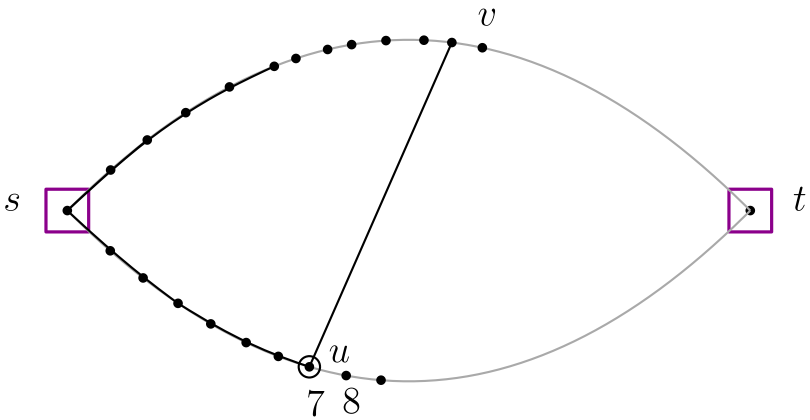

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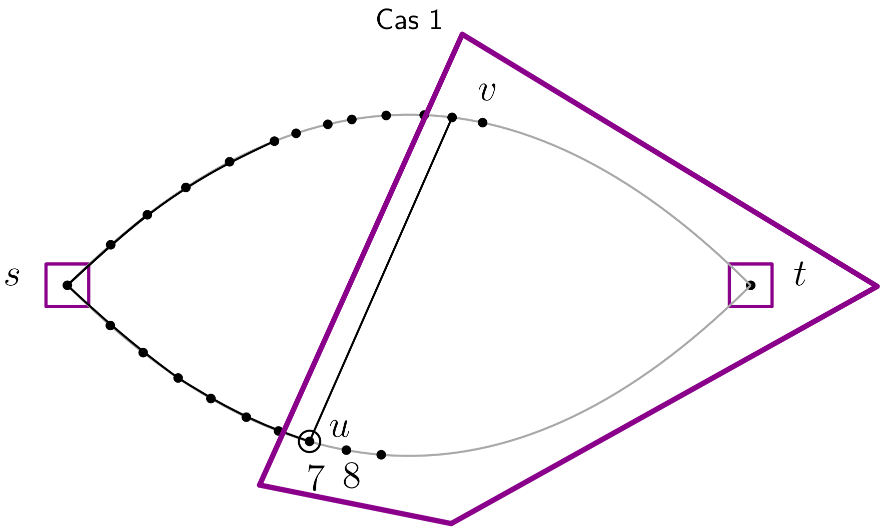

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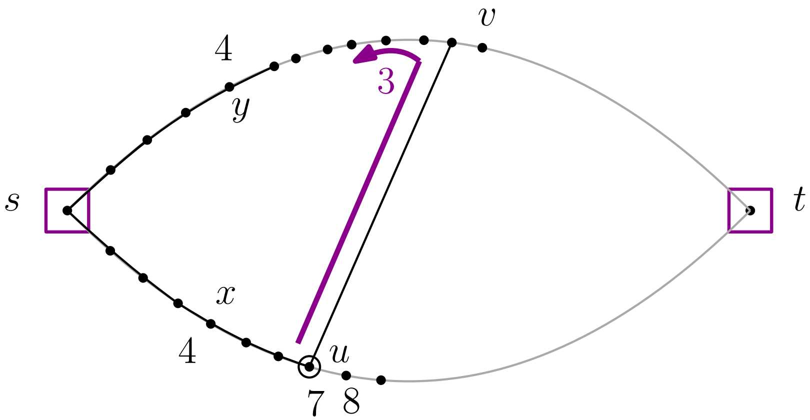

Exponential balancing & Vertical chord

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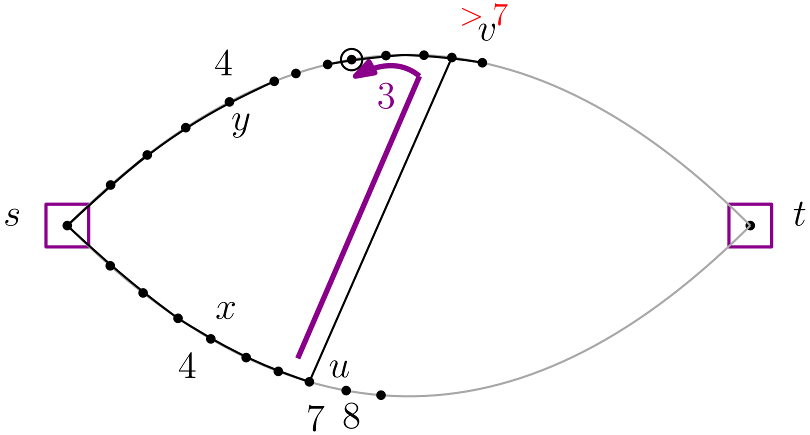
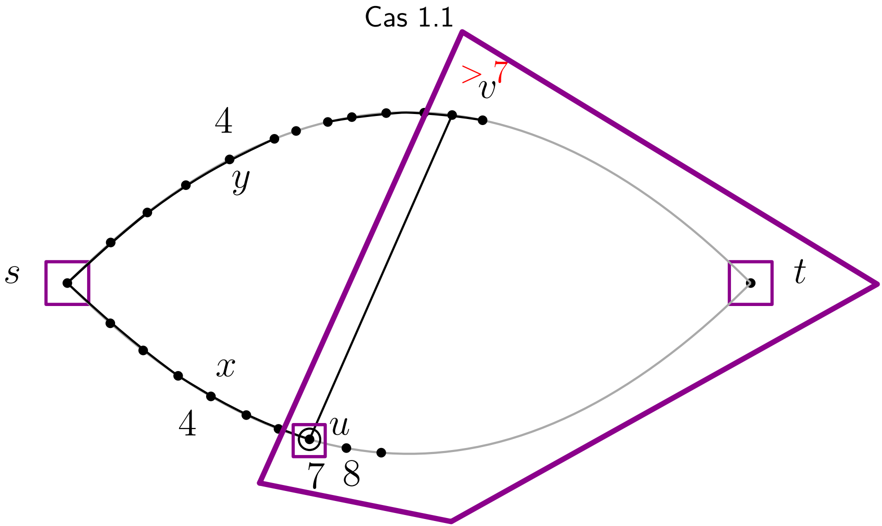


shortest sv-path go through $u \to v$

Exponential balancing & Vertical chord



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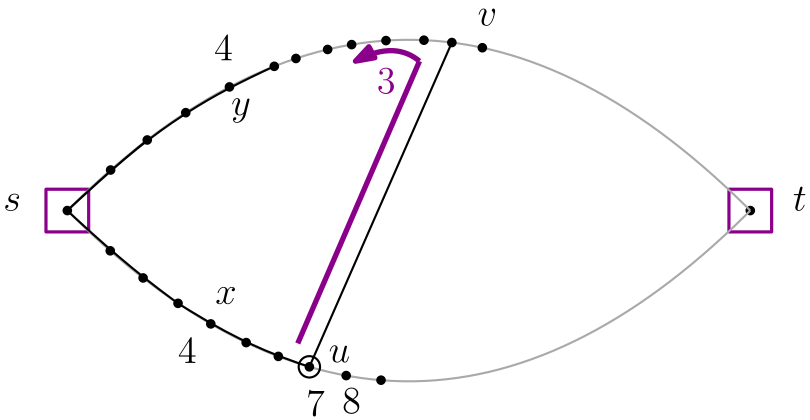
Exponential balancing & Vertical chord

Cas 1.2



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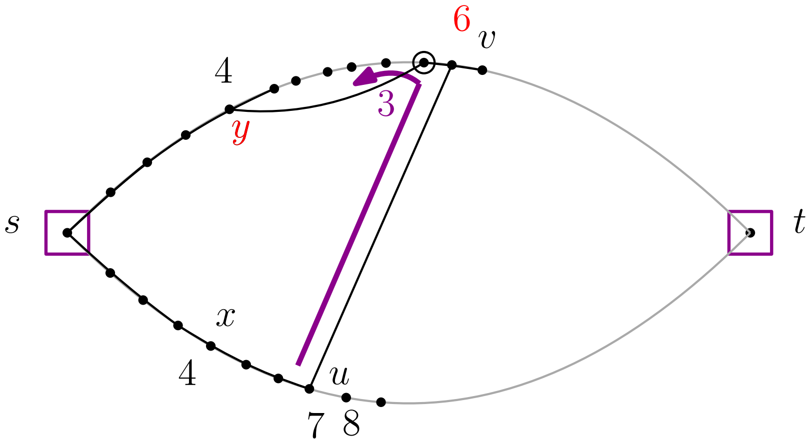

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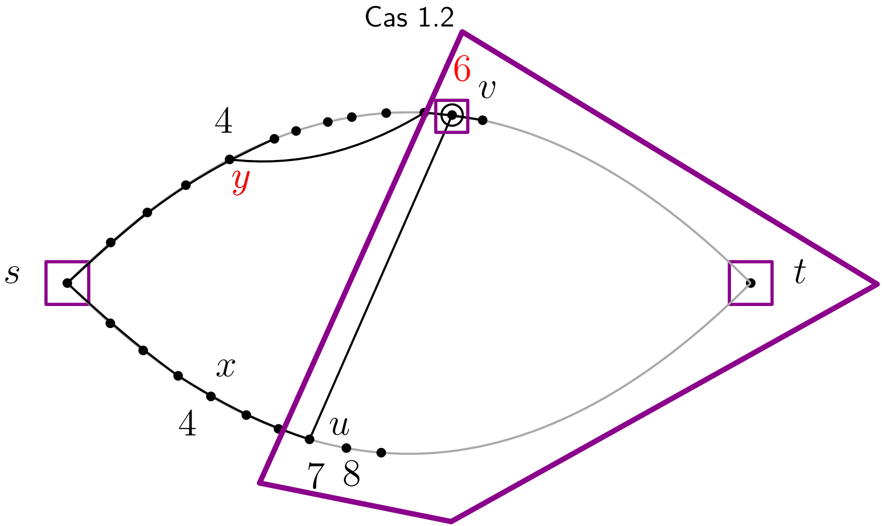
Exponential balancing & Vertical chord



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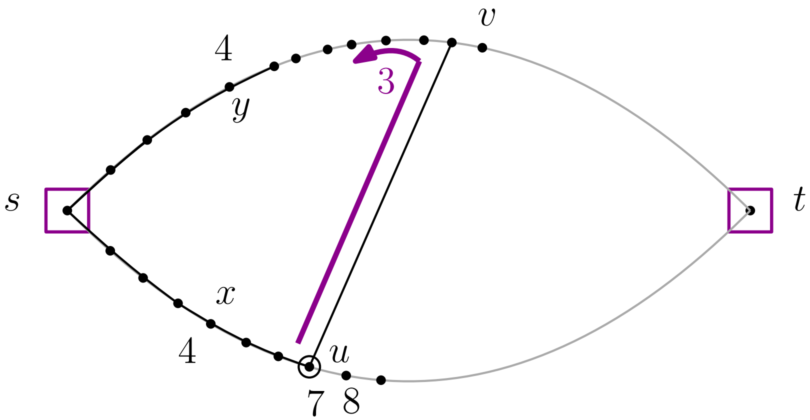
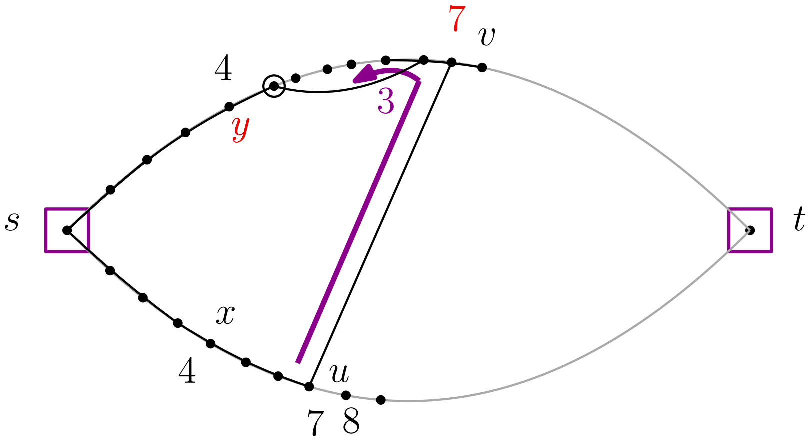


Exponential balancing & Vertical chord

Cas 1.3



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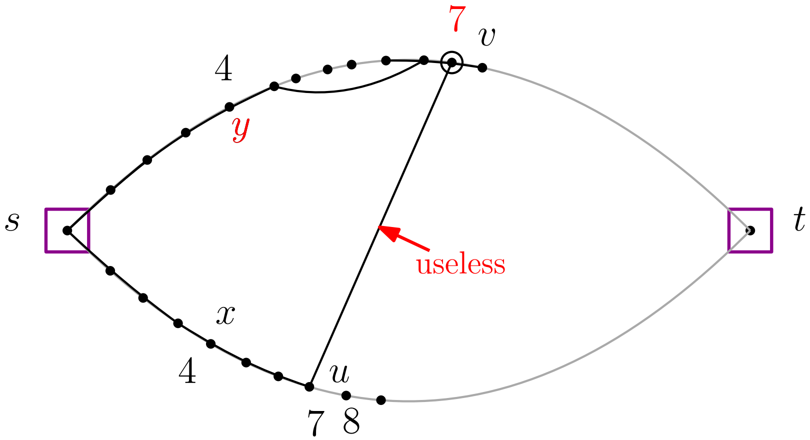

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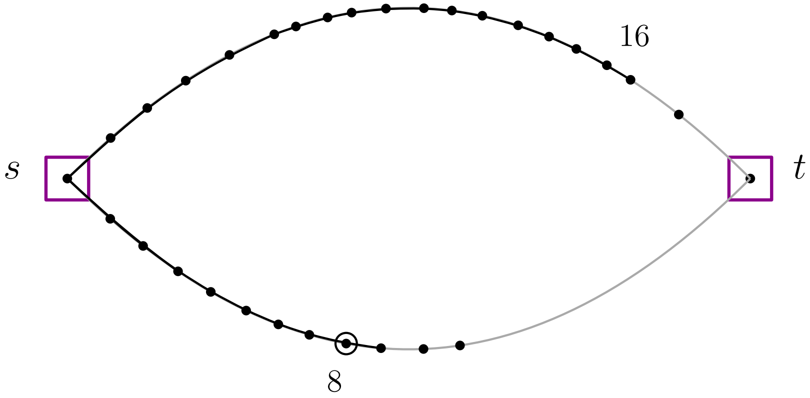
Cas 2



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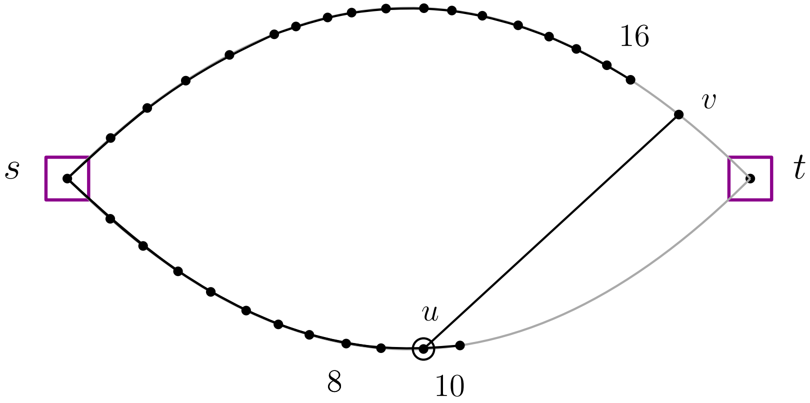

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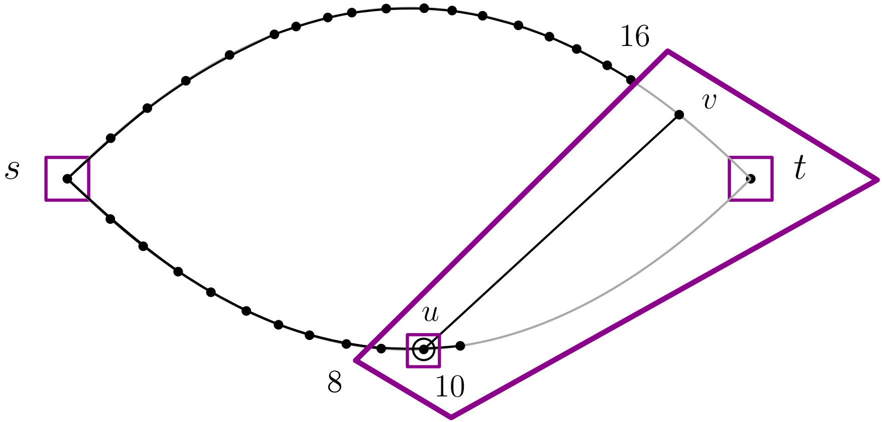
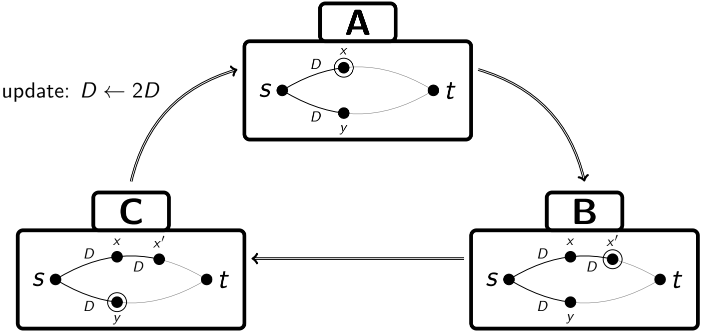


shortest sv-path go through $u \to v$

Proof with 3 steps



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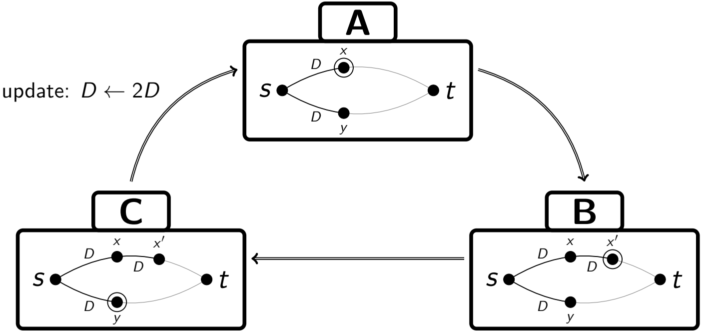

$\rightarrow$ Strategy with ratio 9.

Arbitrary weights

Theorem [BBCDGLLP24+]

There is no constant ratio strategy for the k-Canadian Traveler Problem on all outerplanar graphs.

Here constant = indep. of G and k.

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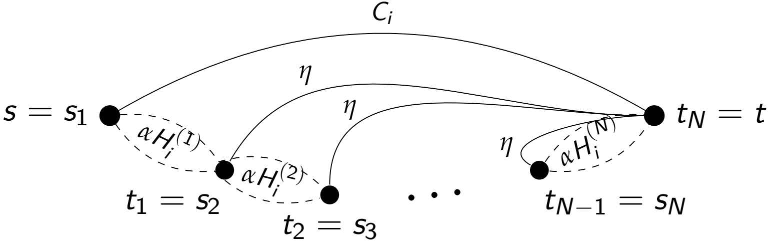
Sketch of proof Build H_i on which we cannot achieve $C_i = \frac{1}{2} + i$.

H_0, cannot achieve $C_0 = \frac{1}{2} + 0$

Build H_{i+1} from H_i



Cannot achieve ratio $C_i + 1$ when α, η small and N big.

Open questions

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Thank you for your attention !