Clique-Stable set Separation

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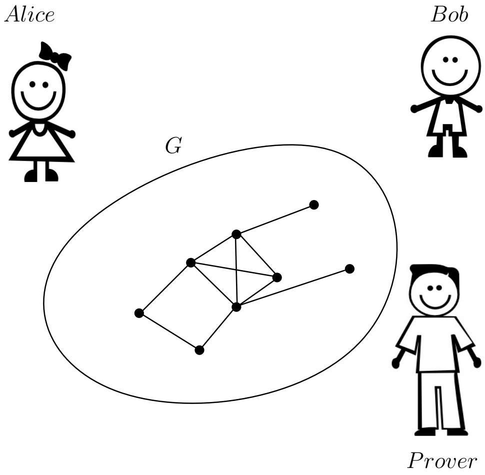
LIP, ENS Lyon

Princeton Discrete Mathematics Seminar - Oct.15, 2015

Joint work with N. Bousquet, S. Thomassé and T. Trunck

Clique vs Independent Set Problem



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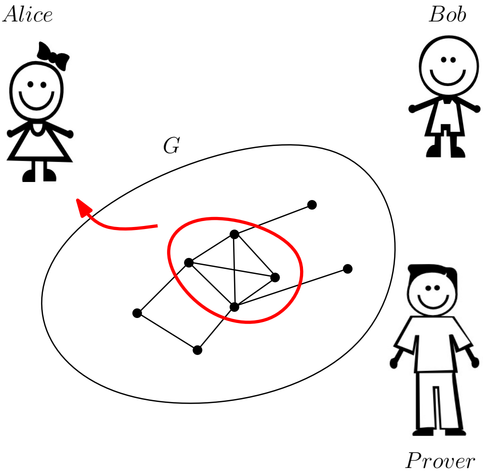

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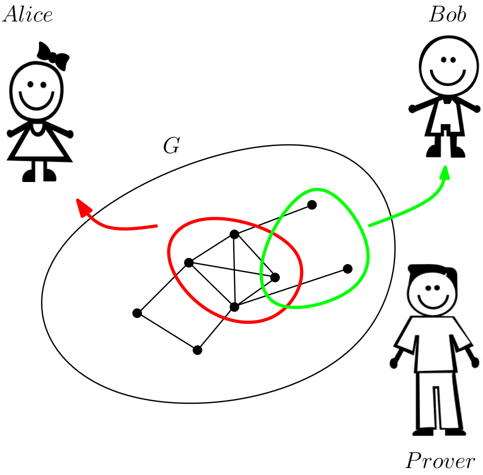

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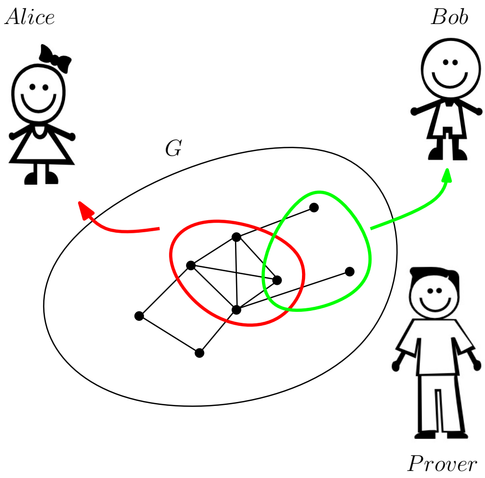

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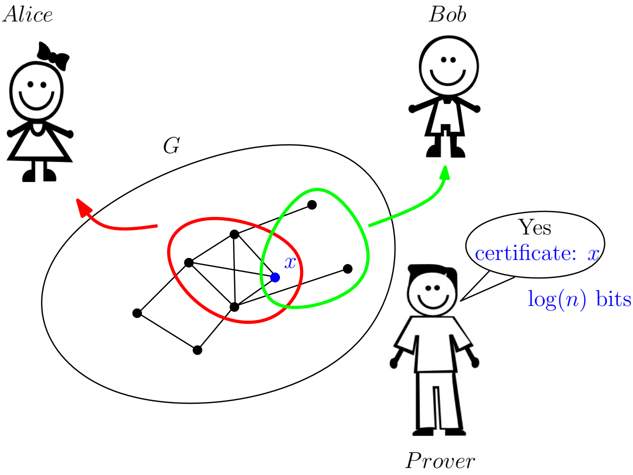

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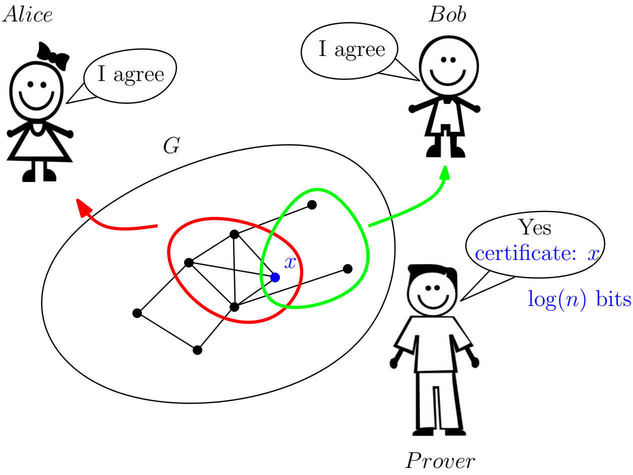

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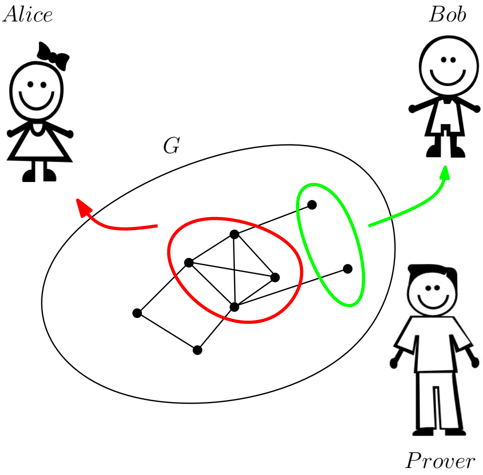

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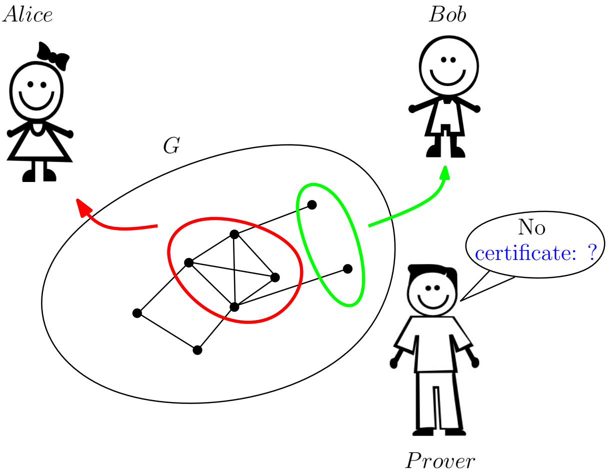

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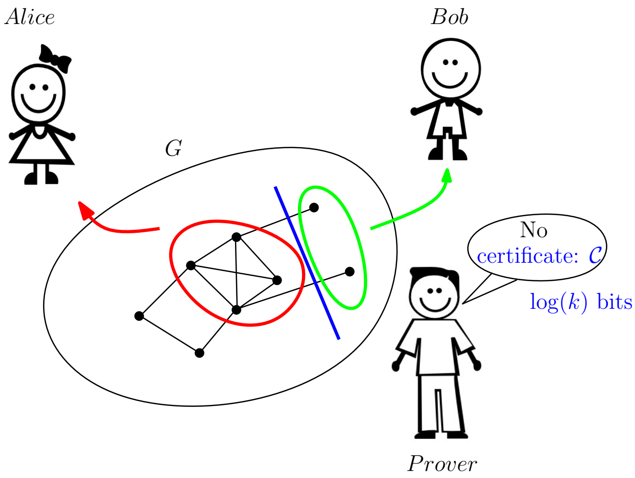

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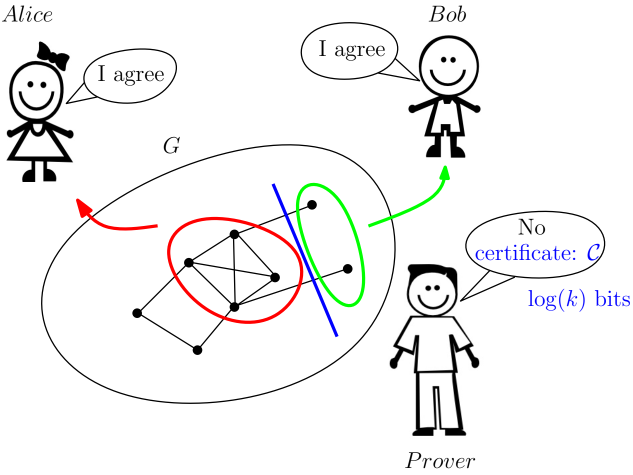

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Non-det communication complexity $\leftrightarrow$ min. size of a CS-Separator.

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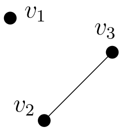
For which classes of graphs does there exist a polynomial CS-Separator?

Motivation

Stable Set polytope

$STAB(G) = \mathrm{conv}(\chi^S \in \mathbb{R}^n \mid S \subseteq V$ is a stable set of $G)$
where χ^S denotes the characteristic vector of $S \subseteq V$.



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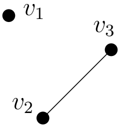

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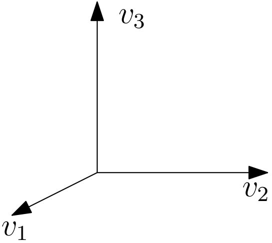

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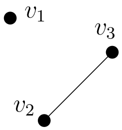

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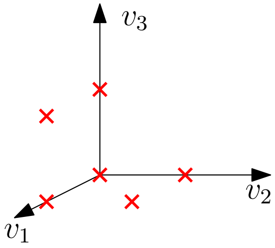

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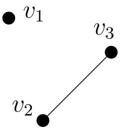

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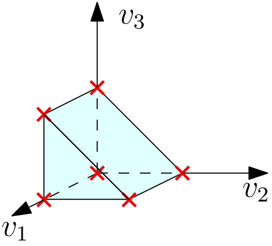
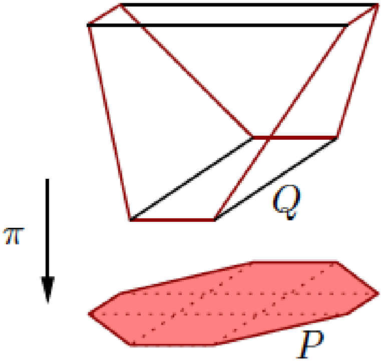


We like polytopes with a **small number of facets**.

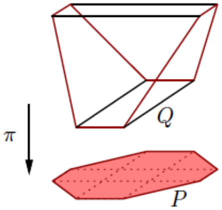


P: polytope in $\mathbb{R}^2$ we want to optimize on (8 facets)

Q: polytope in $\mathbb{R}^3$ which projects to P (6 facets)

$\Rightarrow$ Easier to optimize on Q and project the solution!

Extended formulation



Extension complexity of P

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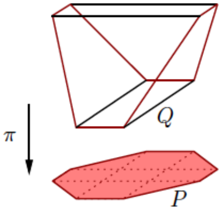

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we can describe $STAB(G)$ with clique inequalities.

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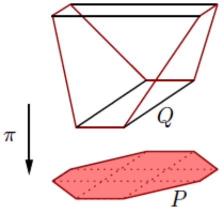

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Reminder: definition of a CS-Separator

A set of cuts such that for every clique K and stable set S disjoint from K, there is a cut that separates K from S.
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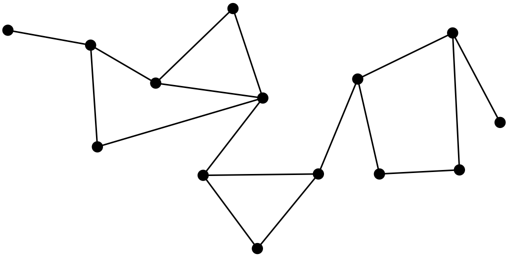

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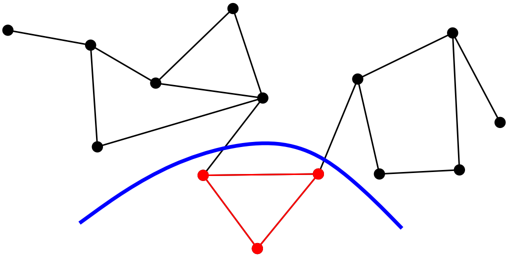

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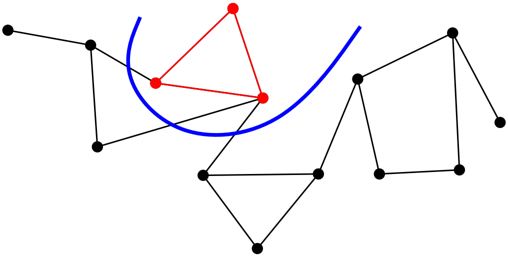

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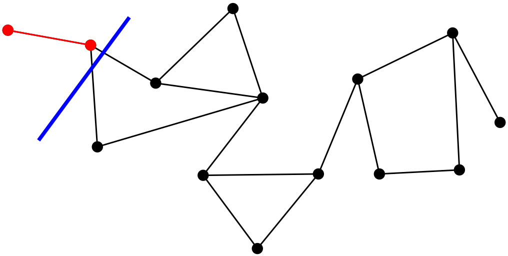

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Even stronger: ① ② and ④ have polynomial extension complexity.

Tools and results about the CS-Separation

Comparability graphs
LP to compute maximum weighted stable set.

When α or ω is bounded, chordal graphs, C_4-free, ...
Polynomially many maximal cliques or stable sets.

P_5-free graphs
In-depth study of potential maximal cliques.

H-free graphs when H is split
Can define cuts with a constant nb of neighborhoods.

$(P_k, \overline{P_k})$-free graphs
Strong Erdős-Hajnal property.

Perfect graphs with no BSP
Decomposition by 2-joins.

k-windmill-free
Easy neighborhood property.

Homogeneous set

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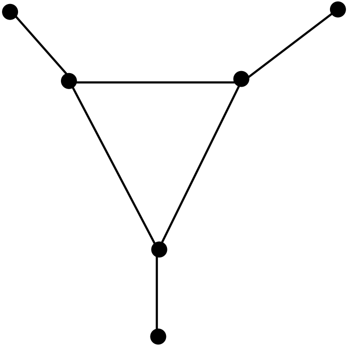

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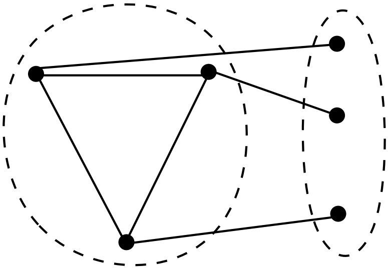

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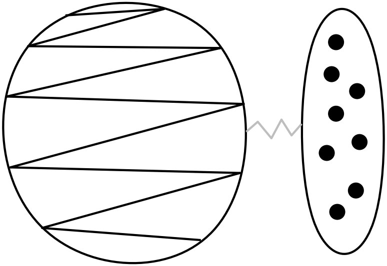

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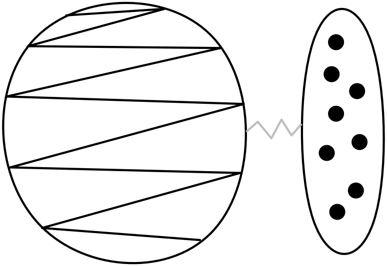

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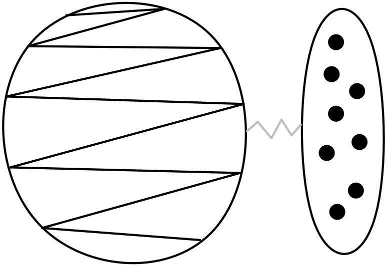

Split-free [Bousquet, L., Thomassé]

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Key Lemma (using VC-dimension)

Let G be a H-free graph, K be a clique of G and S be a stable set disjoint from K. Then one of the following holds:

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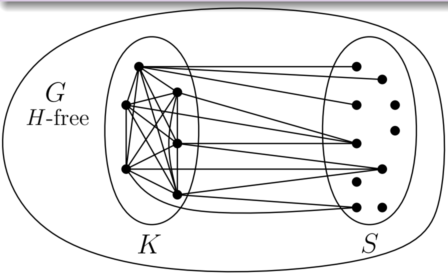

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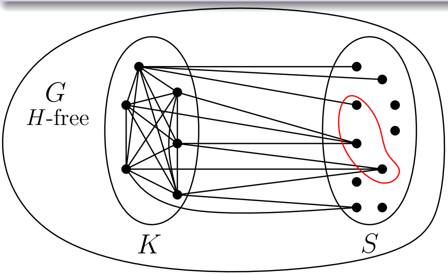

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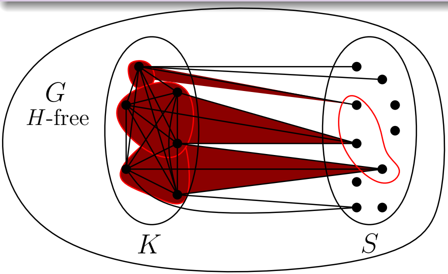

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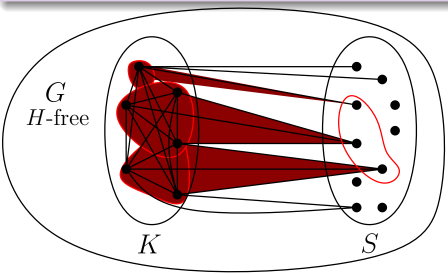

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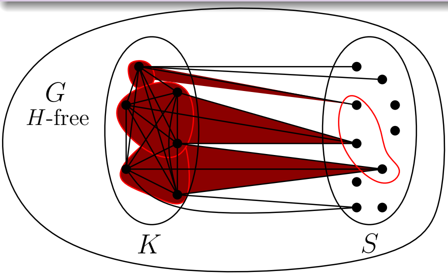

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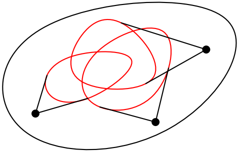

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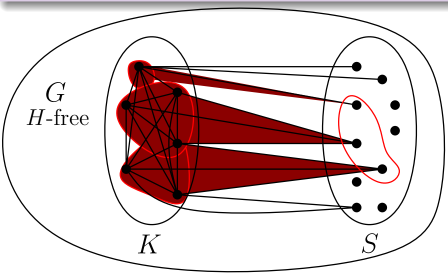

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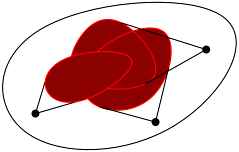

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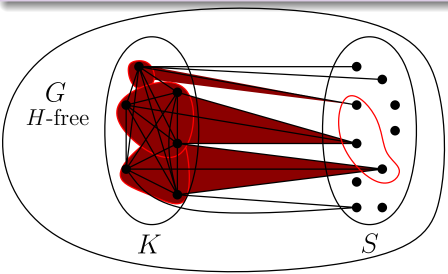

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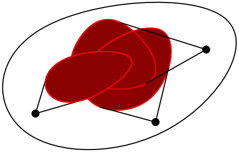

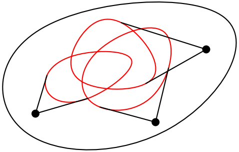

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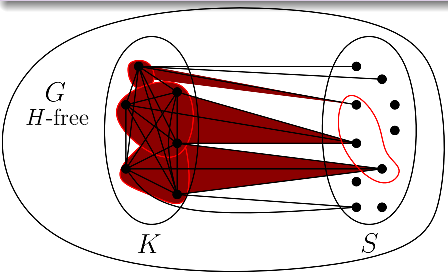

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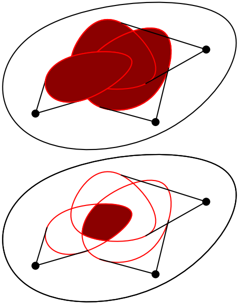

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When α or ω is bounded, chordal graphs, C_4-free, ...

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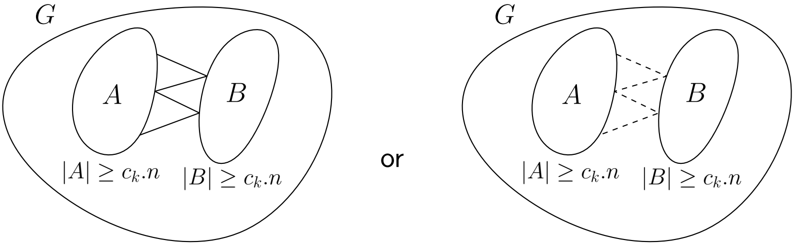

Sketch of proof

Theorem (Rödl 1986, Fox, Sudakov 2008)

$\forall k$, every graph G satisfies one of the following:

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Sketch of proof (continued)

Connected P_k-free subgraph G' on n' vertices, max degree $\leq \varepsilon \cdot n'$.

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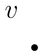

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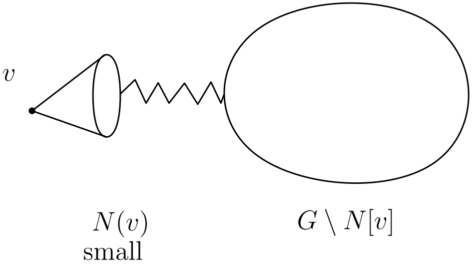

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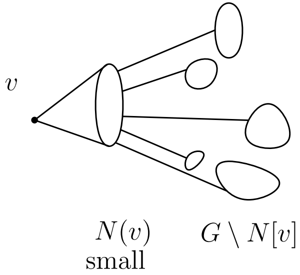

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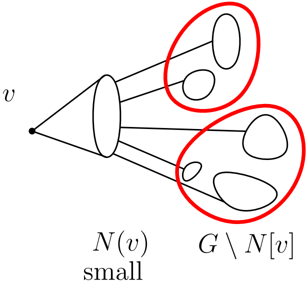

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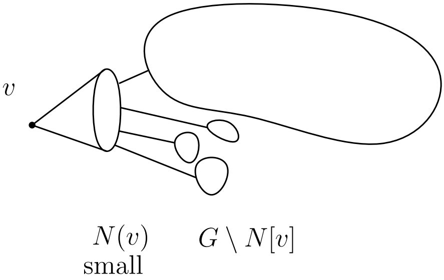

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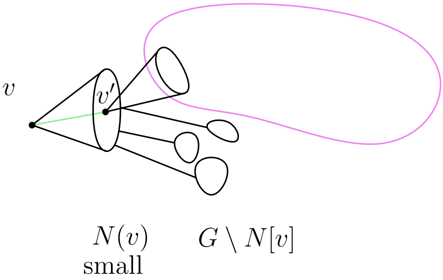

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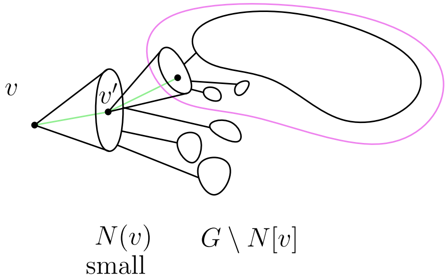

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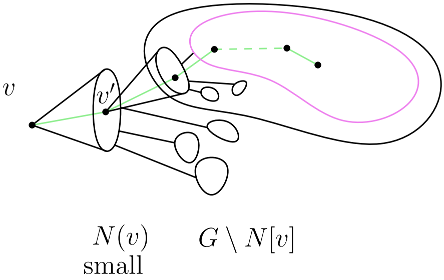

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Main step in the proof of the Strong Perfect Graph Theorem:

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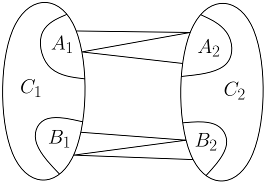

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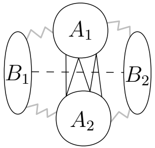

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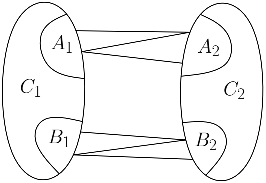

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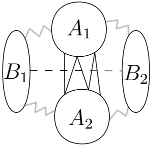


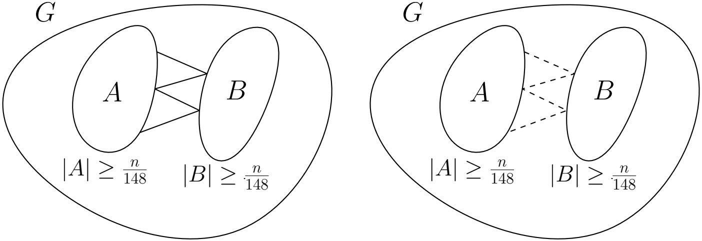
Skew Partition

Theorem [L., Trunck]

Let G be a perfect graph with no balanced skew partition, then there exists a CS-separator for G of size $\mathcal{O}(n^2)$.

Strong Erdős-Hajnal property [L., Trunck]

Every perfect graph with no balanced skew partition admits a biclique or complement biclique of size at least $n/148$.



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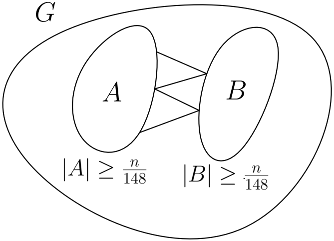

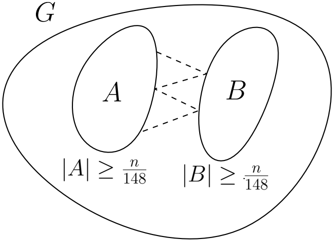

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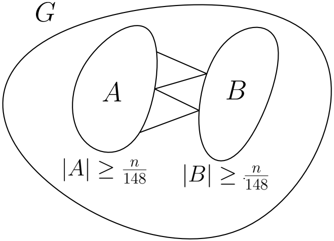

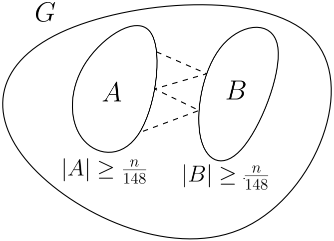

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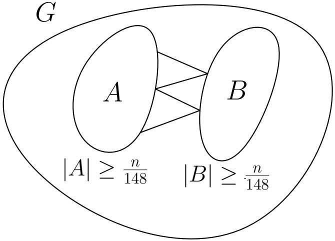

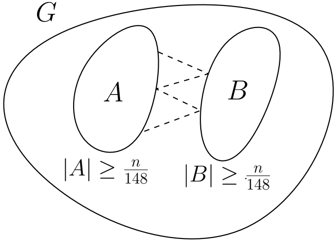

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But there exist perfect graphs where the Strong Erdős-Hajnal property does not hold [Fox, Pach 2009] $\Rightarrow$ Evidence of some special structure.

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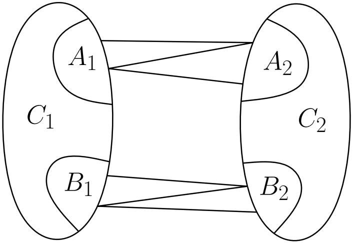
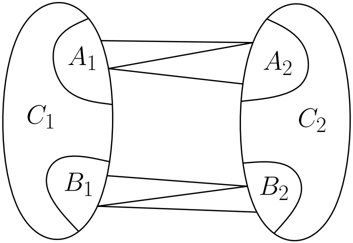

A_1
A_2
C_1
C_2
B_1
B_2


A_1
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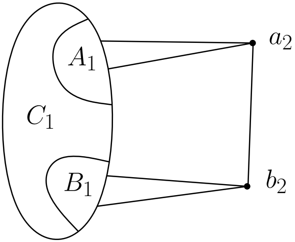
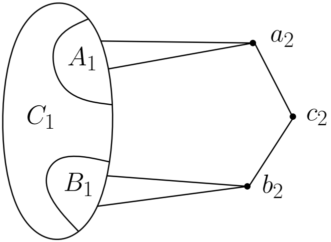
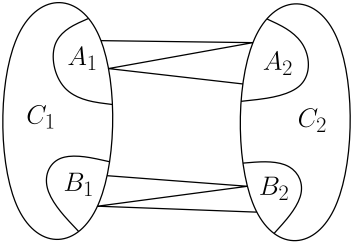
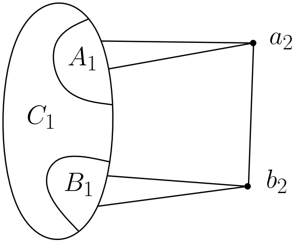
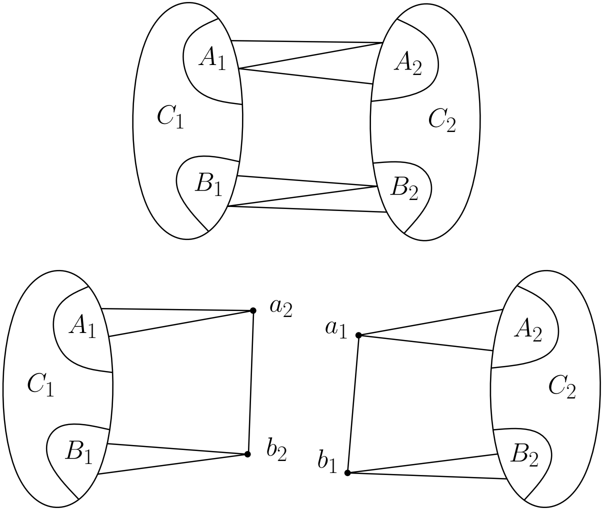
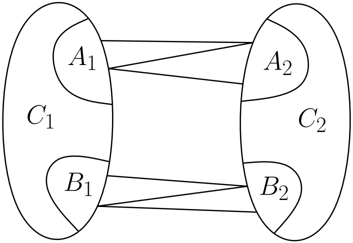

A_1
a_2
C_1
B_1
b_2

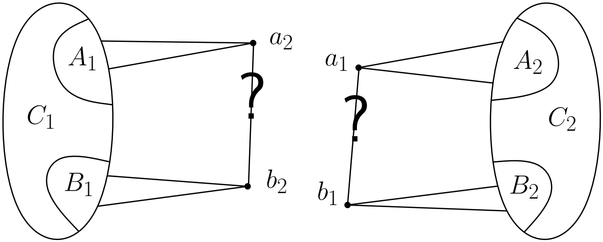

A_1
a_2
C_1
c_2
B_1
b_2

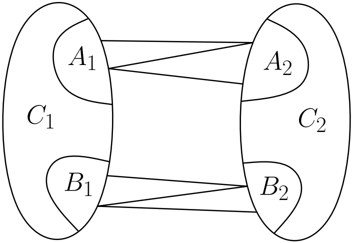

A_1
A_2
C_1
C_2
B_1
B_2

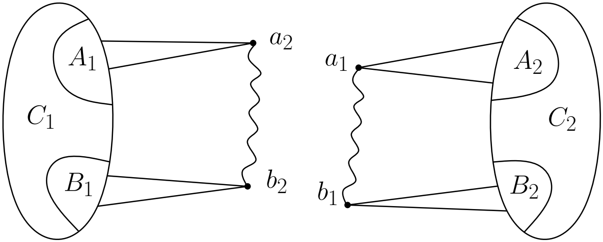

A_1
a_2
C_1
B_1
b_2

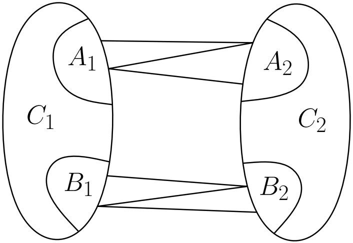

A_1
A_2
C_1
C_2
B_1
B_2
a_2
a_1
A_1
A_2
C_1
C_2
B_1
B_2
b_2
b_1

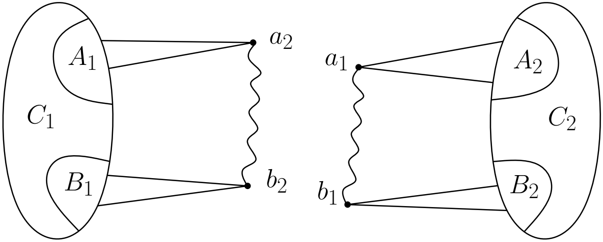

A_1
A_2
C_1
C_2
B_1
B_2


A_1
a_2
C_1
B_1
b_2
?
a_1
A_2
C_2
b_1
B_2
?

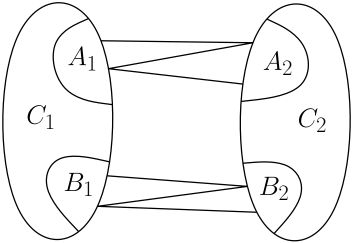

A_1
A_2
C_1
C_2
B_1
B_2

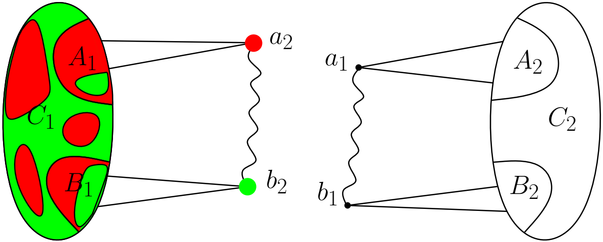

a_2
a_1
A_1
A_2
C_1
C_2
B_1
b_2
b_1
B_2


A_1
A_2
C_1
C_2
B_1
B_2


A_1
a_2
a_1
A_2
C_1
C_2
B_1
b_2
b_1
B_2

Red=
What is put on the left (clique side)
Green=
What is put on the right (stable set s.)





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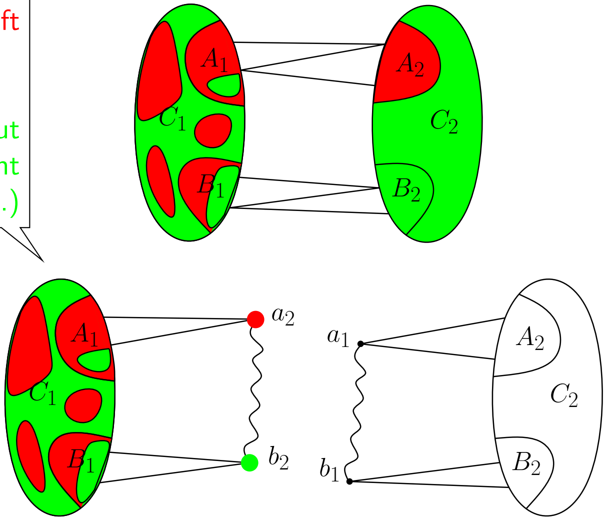

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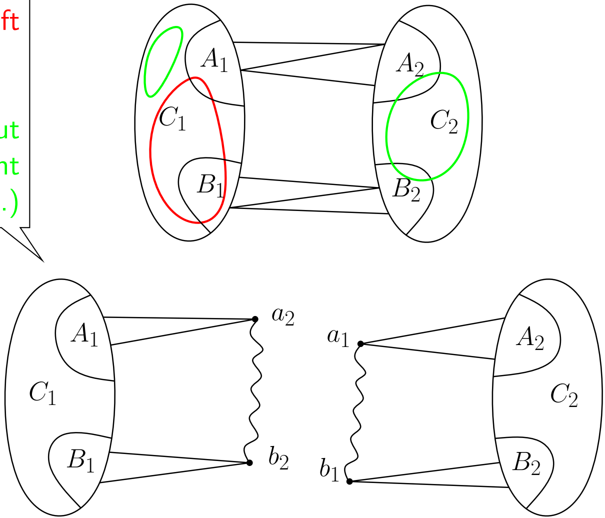

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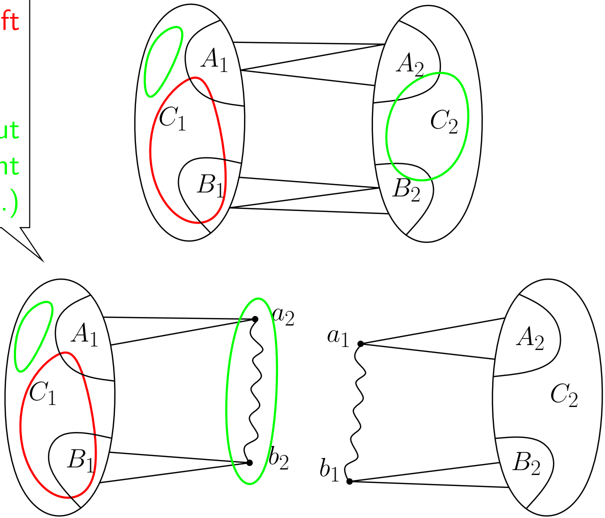

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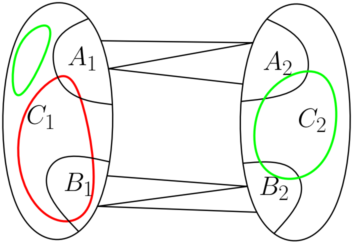

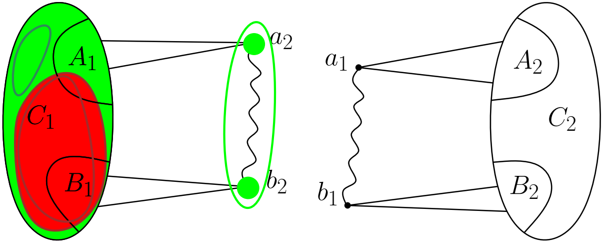

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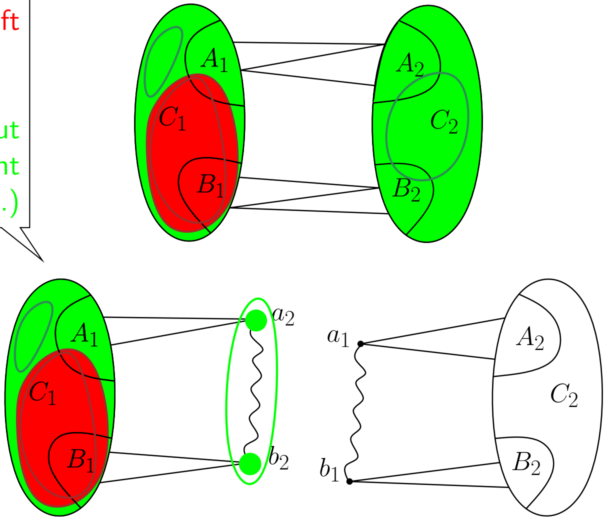

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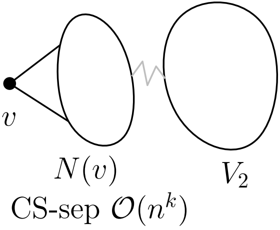

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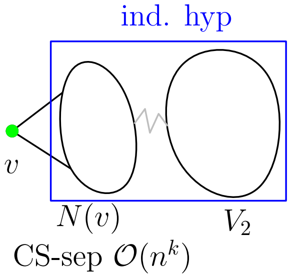

v in stable set side

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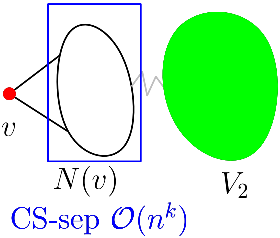

v in clique side

V_2 in stable set side

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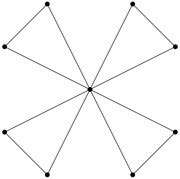
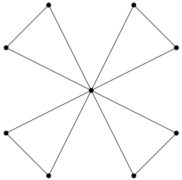
Windmill-free graphs



A 4-windmill

Windmill-free graphs



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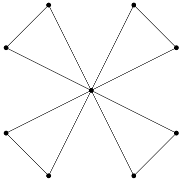

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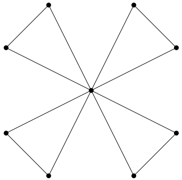

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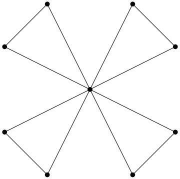

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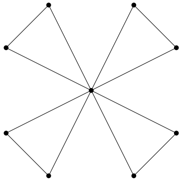

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Relationship with other problems

Erdős-Hajnal property

Can we always find a *large* clique or a *large* stable set?

Ramsey Theory

Every graph has a clique or a stable set of logarithmic size.

$\Rightarrow$ Logarithmic order is best possible (Erdős 1947).

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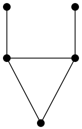

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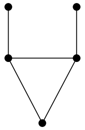

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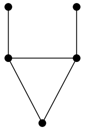

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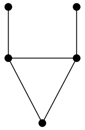

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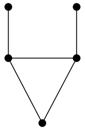

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Towards P_5-free graphs?

- $(P_5, \overline{P_5})$-free graphs (Fouquet 1993)
- $(P_5, \overline{P_6})$-free graphs (Chudnovsky, Zwols 2012)
- $(P_5, \overline{P_7})$-free graphs (Chudnovsky, Seymour 2012)

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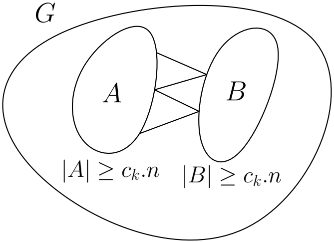
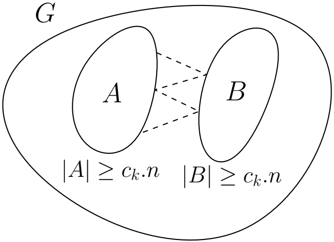
For every k, every graph G with no P_k nor $\overline{P_k}$ has a linear-size biclique or antibiclique (A, B).



or



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Thank you for your attention!