Characterizations of non-Seymour graphs

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Outline

1. Motivation
2. Definitions : joins, complete packing of cuts
3. Seymour graphs
4. Characterizations of non-Seymour graphs
5. Ingredients from Matching Theory
6. Equivalent forms
7. Proof ideas
8. Algorithmic aspects
9. Open problem

Motivation

Edge-disjoint paths problem

Given a graph $H = (V, E)$ and k pairs of vertices $\{s_i, t_i\}$, decide whether there exist k edge-disjoint paths connecting the k pairs s_i, t_i.

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Applications :

1. Real-time communication,
2. VLSI design,
3. Transportation networks,

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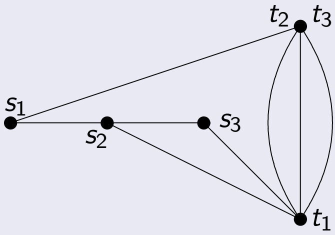
Given a graph $G = (V', E' + F')$, decide whether there exist $|F'|$ edge-disjoint cuts in G, each containing exactly one edge of F'.

An example

Edge-disjoint paths problem



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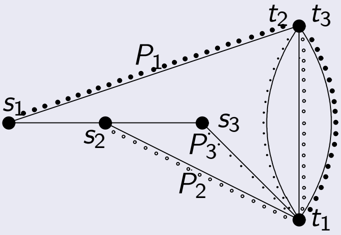

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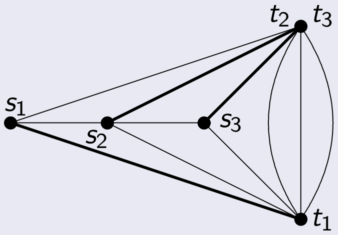
Adding the edges $s_i t_i$



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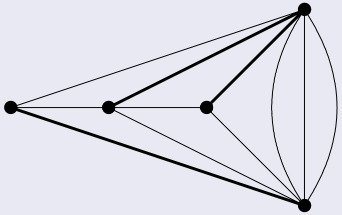
The graph H'



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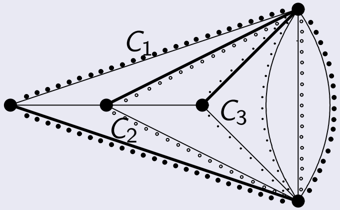

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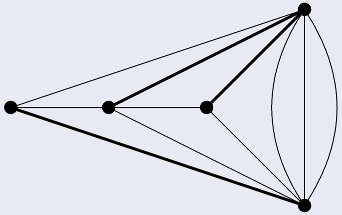
H' is planar



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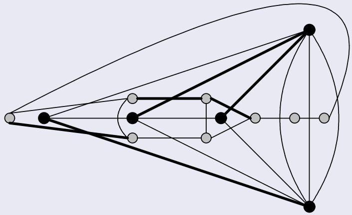
H' and his dual G



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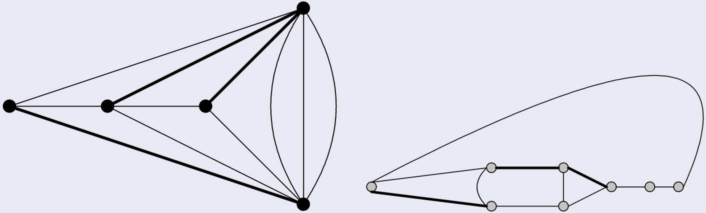

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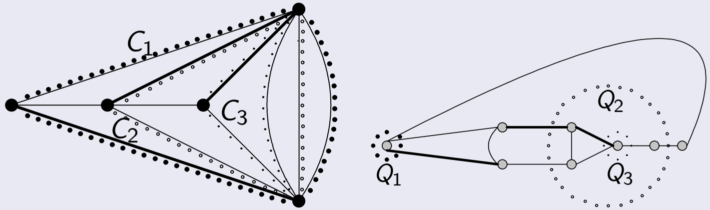
Complete packing of cycles and cuts



Complete packing of cuts

The graphs are not planar anymore!

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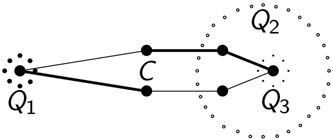

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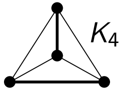

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Theorem (Middendorf, Pfeiffer '93)

Given a join in a graph, decide whether there exists a complete packing of cuts is an NP-complete problem.

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Around Seymour graphs

Subclasses

1. Seymour '77 : Graphs without subdivision of K_4,
2. Seymour '81 : Graphs without odd cycle,
3. Gerards '92 : Graphs without odd K_4 and without odd prism,
4. Szigeti '93 : Graphs without non-Seymour odd K_4 and without non-Seymour odd prism.

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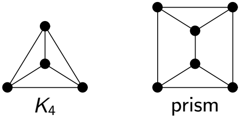

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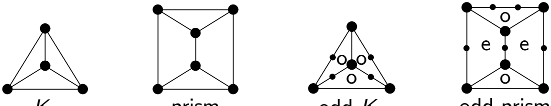

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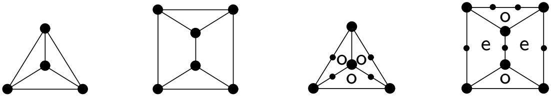

K_4 prism odd K_4 odd prism

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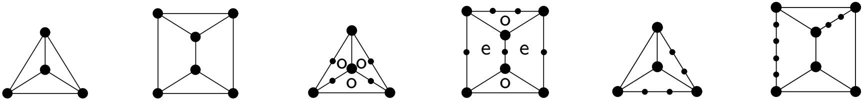
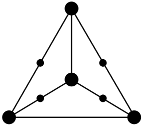


K_4 prism odd K_4 odd prism even subdivisions

Superclass

Seymour graph $\Longrightarrow$ no even subdivision of K_4 and of prism.

Preliminaries



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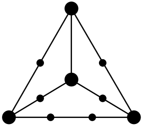

non-Seymour
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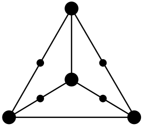

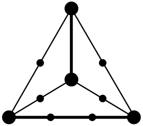

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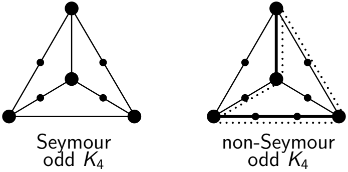

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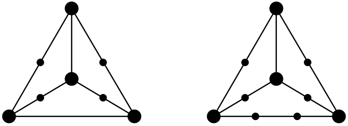

Seymour odd K_4 non-Seymour odd K_4

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Lemma (Sebő '92)

If for a join F of G there exist two F-tight cycles whose union is not bipartite, then G is not Seymour.

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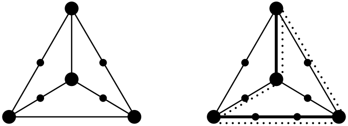

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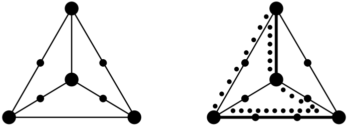

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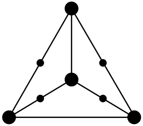

Seymour
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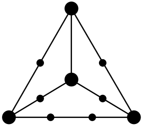

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Conjecture (Sebő '92)

G is not Seymour if and only if G admits a join F and two F-tight cycles whose union is an odd K_4 or an odd prism.

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Theorem (Ageev, Kostochka, Szigeti '97)

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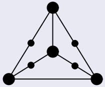

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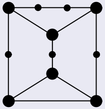

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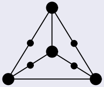

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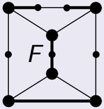

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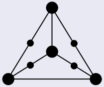

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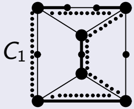

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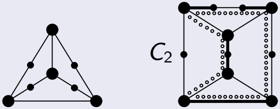

Seymour odd K_4

non-Seymour odd prism

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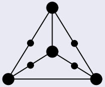

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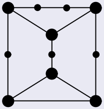

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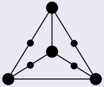

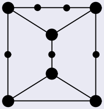

Theorem (Ageev, Benchetrit, Sebő, Szigeti '11)

G is non-Seymour if and only if contracting stars and odd cycles it contains an even subdivision of K_4.

Matching Theory : Graphs

Definitions

1. Matching-covered = connected and any edge belongs to a perfect matching,
2. Elementary = edges belonging to a perfect matching form a connected subgraph,
3. Barrier of elementary graph G = vertex set X such that the number of odd components of $G - X$ is $|X|$.

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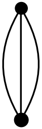

K_2^3

Definitions

1. Matching-covered = connected and any edge belongs to a perfect matching, examples : K_2^3, K_4, prism and their even subdivisions.
2. Elementary = edges belonging to a perfect matching form a connected subgraph,
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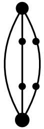

even subdivision of K_2^3

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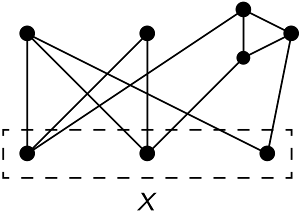

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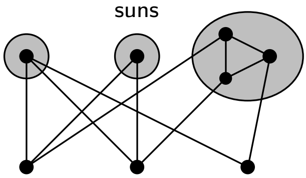

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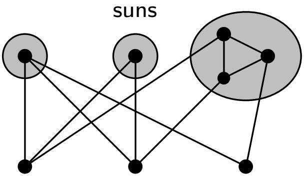

Matching Theory : Results

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1. Lovász '75 : A graph is factor-critical if and only contracting odd cycles it can be reduced to a vertex.
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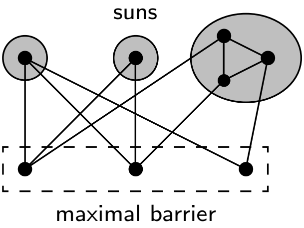
Remarks

1. Each connected component of an elementary graph minus a maximal barrier is factor-critical, and hence provides a sun.
2. Let H be obtained by gluing G_1 and G_2 in a vertex set Y. If H/G_2 is elementary then H/G_1 can be obtained from H by contracting suns.



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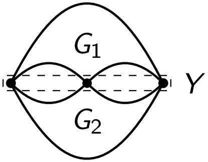

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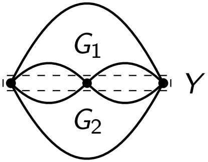

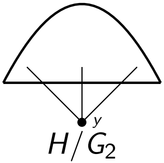

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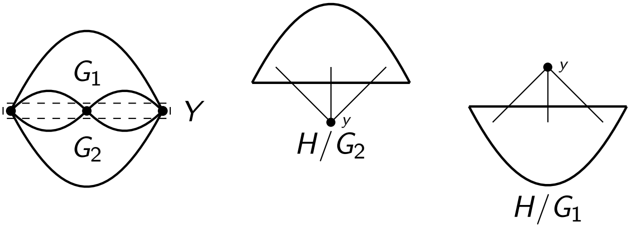

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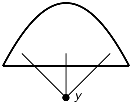

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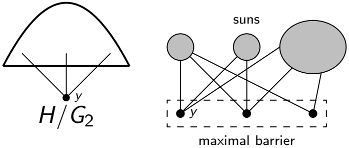

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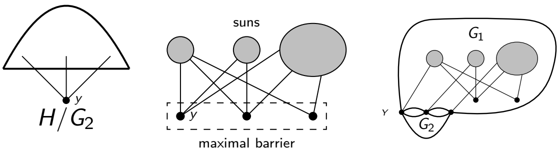

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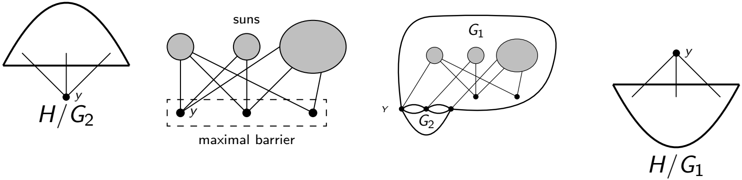

Equivalent forms

Theorem (Ageev, Benchetrit, Sebő, Szigeti '11)

The following conditions are equivalent for any graph G :

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(5) $\Longrightarrow$ (6) : Contract an odd cycle of the even subdivision of the prism to get an even subdivision of K_4. (6) $\Longrightarrow$ (5) : OK

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To see that (6) $\Longrightarrow$ (3), we need (7).

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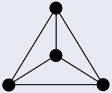
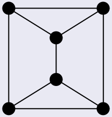
Graphs

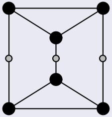
3 graphs



K_4



prism



bi-prism

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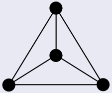

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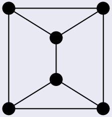

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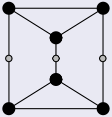
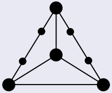
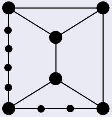
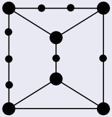


bi-prism

and their even subdivisions







Graphs

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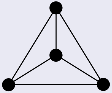
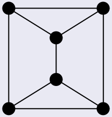
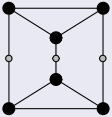


K_4

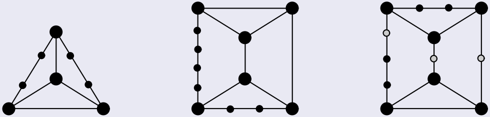


prism



bi-prism

and their even subdivisions



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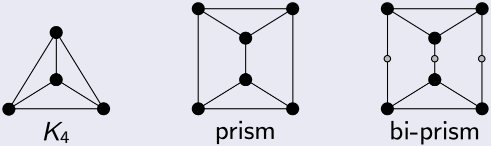

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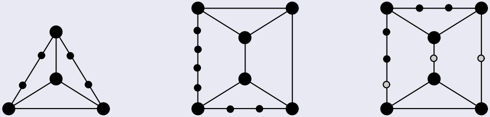
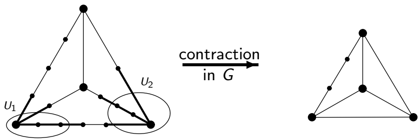


Core-contraction to K_4

K_4-obstruction

An odd K_4 subgraph H of G with disjoint sets $U_i \subseteq V(H)$ such that

1. $H[U_i \cup N_H(U_i)]$ is an even subdivision of a 3-star,
2. contracting each $U_i \cup N_G(U_i)$, H transforms into an even subdivision of K_4.

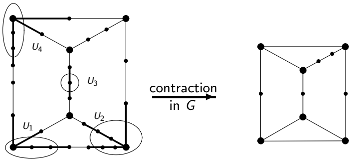


Core-contraction to the prism or to the biprism

Prism- or biprism-obstruction

An odd prism subgraph H of G with disjoint sets $U_i \subseteq V(H)$ such that

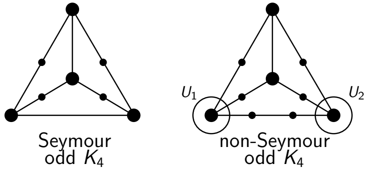
1. $H[U_i \cup N_H(U_i)]$ is an even subdivision of a 2- or 3-star,
2. contracting each $U_i \cup N_G(U_i)$, H transforms into an even subdivision of the prism or of the biprism (no edge of G connects the two connected components of the biprism minus its separator).



About obstructions

Remark :

1. The contraction of a core in an obstruction changes the parity of the three paths of the obstruction that contain the core.
2. Their main role is to be able to change the odd K_4 (or odd prism) into an even subdivision of K_4 (or of the prism).



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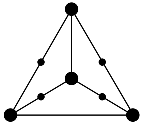
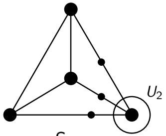


Seymour odd K_4



non-Seymour odd K_4

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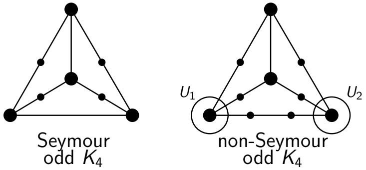

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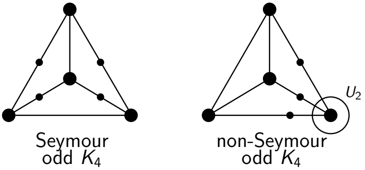

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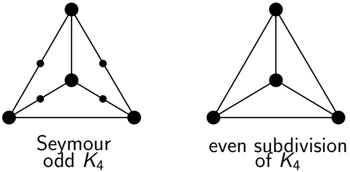

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(6) Contracting stars and odd cycles it contains an even subdivision of K_4,

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Lemma

If G/C (C : star or odd cycle) contains an obstruction then so does G.

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(7) Contracting cores it contains an even subdivision of K_4 or of the prism or of the biprism.

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Lemma

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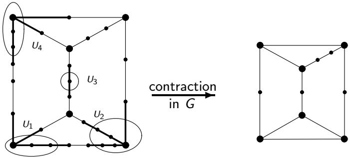
Both are implied by the lemma about the contraction of elementary graphs because an even subdivision of K_2^3 (and of K_4) is matching-covered.



Returning to non-Seymour graphs

Equivalence to non-Seymour graphs

1. Non-Seymour graph implies (1) : by structure theorem of Sebő '90.
2. (7) implies non-Seymour graph : by lemma of Sebő '92 : a join of G and two tight cycles whose union is an odd K_4 or an odd prism can be easily found in an obstruction.

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Algorithmic aspects

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2. Given a graph G and a join F in G, decide whether there exists an F-complete packing of cuts.

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Open problem

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Find a construction for Seymour graphs !

Thanks !