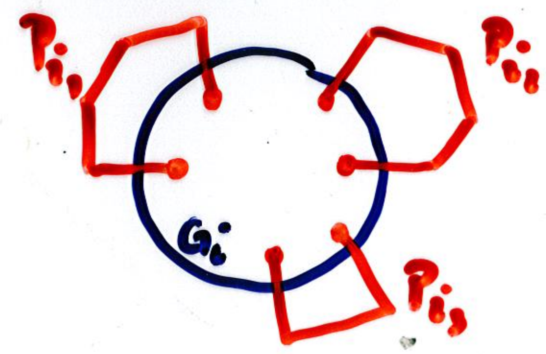
DEFINITION: A GRADED EAR-DECOMPOSITION OF G IS A SEQUENCE $G_0, G_1, \dots, G_k = G$

- G_1 IS A CYCLE, (OF EVEN LENGTH)
- $G_{i+1} = G_i + P_{i_1} + \dots + P_{i_\ell}$, DISJOINT ODD PATHS
- G_i IS MATCHING-COVERED, NICE.



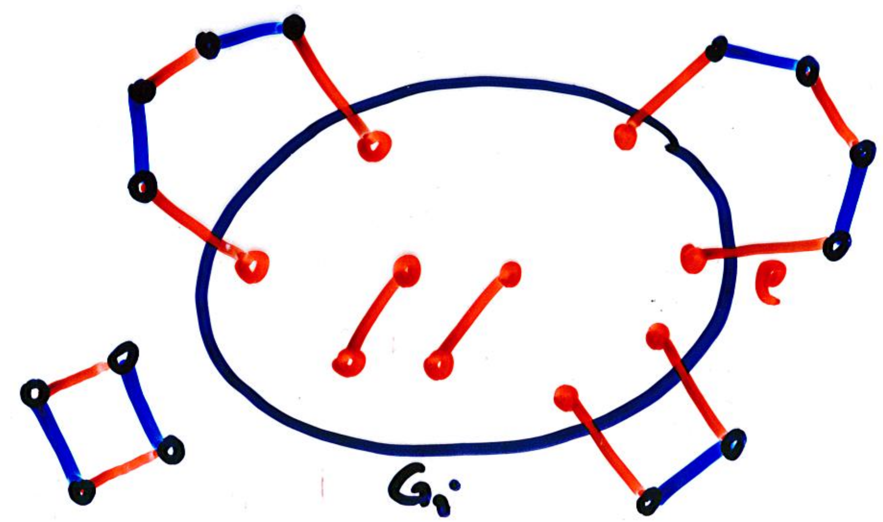
DEFINITION: 2-GRADED EAR-DECOMPOSITION IF IN EACH STEP WE ADD ≤ 2 EARS.

THEOREM: (LOVÁSZ-PLUMMER)

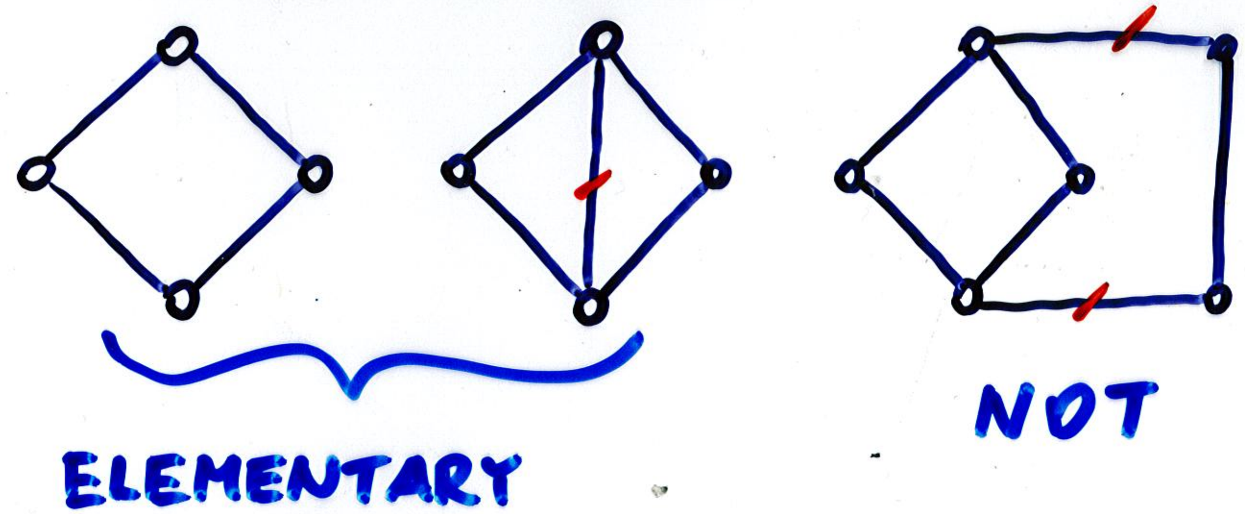
A GRAPH IS MATCHING-COVERED $|V(G)| \geq 4$

$\Updownarrow$

IT HAS A 2-GRADED EAR-DECOMPOSITION.



DEFINITION: G IS **ELEMENTARY** IF THE EDGES WHICH BELONG TO SOME PERFECT MATCHING OF G FORM A CONNECTED SPANNING SUBGRAPH.



$\phi(G)$:= NUMBER OF PERFECT MATCHINGS OF G.

THEOREM (LOVÁSZ-PLUMMER)

G IS ELEMENTARY, $e_1, e_2, \dots, e_k \in E(\bar{G})$.
IF $\phi(G+e_1+e_2+\dots+e_k) > \phi(G)$ THEN
$\exists\, i,j : \phi(G+e_i+e_j) > \phi(G)$.

SHORT PROOF: 2.SZ.

THEOREM (LOVÁSZ - PLUMMER)

G IS MATCHING-COVERED, $e_1, \dots, e_k \in E(\bar{G})$:

$G + e_1 + \dots + e_k$ IS MATCHING-COVERED.

THEN $\exists\, i \leq j$: $G + e_i + e_j$ IS MATCHING-COVERED.

SHORT PROOF : Z. SZ.

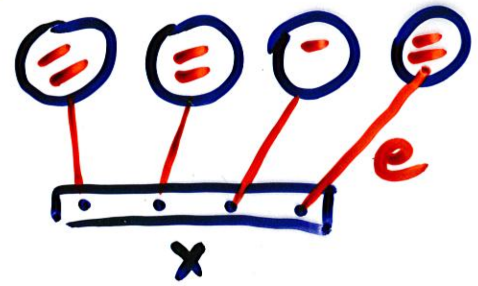
THEOREM: G IS ELEMENTARY, $e_1, e_2, e_3 \in E(\bar{G})$

- $G+e_1+e_2+e_3$ HAS A PERFECT MATCHING M CONTAINING e_1, e_2, e_3.
- $G+e_i$ HAS NO PERFECT MATCHING CONTAINING e_i $(i=1,2,3.)$

THEN $\forall e_i \exists e_j$: $G+e_i+e_j$ HAS A PERFECT MATCHING CONTAINING e_i AND e_j.

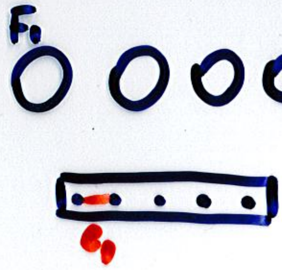
DEFINITION: G IS ELEMENTARY, $X \subset V(G)$.
X IS A STRONG BARRIER IF $G-X$ HAS $|X|$ COMPONENTS AND ALL OF THEM ARE FACTOR-CRITICAL.

LEMMA: IF G IS ELEMENTARY AND X IS A STRONG BARRIER OF G, THEN EACH EDGE LEAVING X BELONGS TO A PERFECT MATCHING OF G.

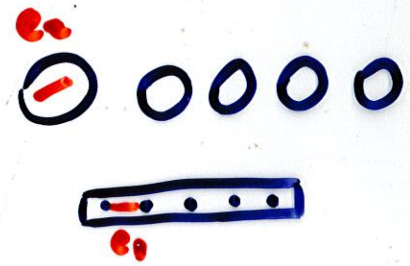


PROOF: SUPPOSE $G' := G + e_1 + e_2$ HAS NO PERFECT MATCHING CONTAINING e_1 AND e_2.

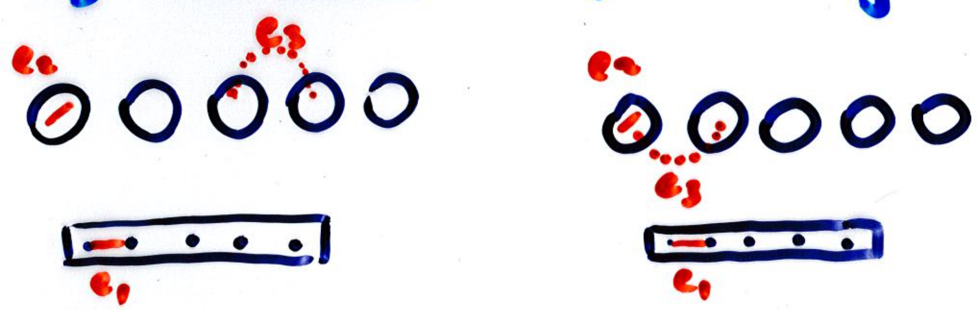
(1) $\exists$ A STRONG BARRIER X IN G' CONTAINING e_1.



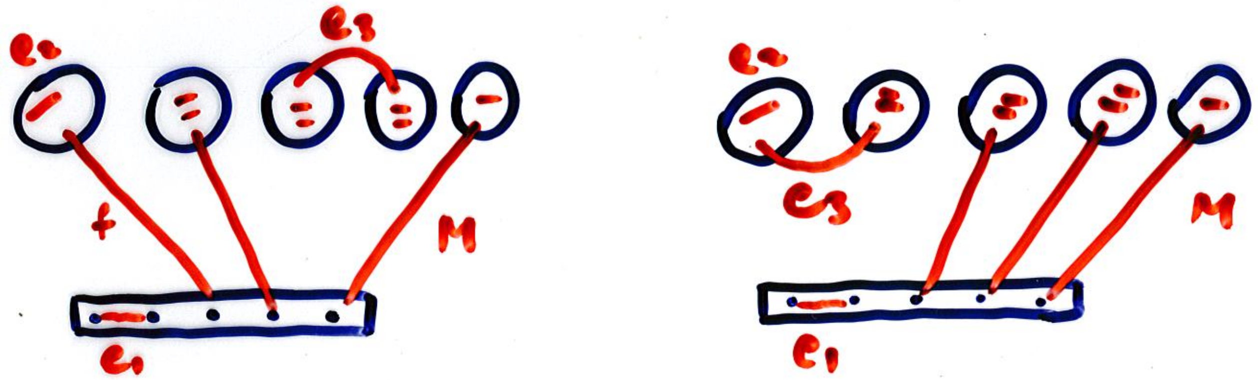
(2) $e_2 \in F_1$.



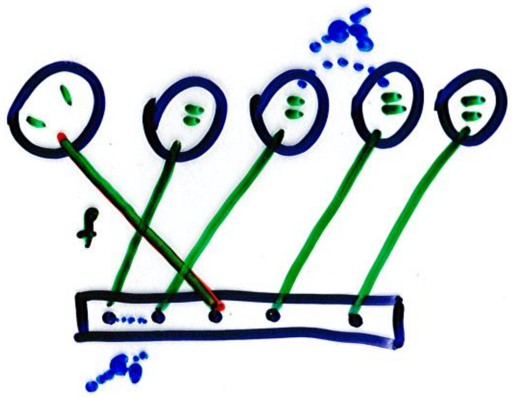
(3) e_3 CONNECTS F_i AND F_j.



(4)

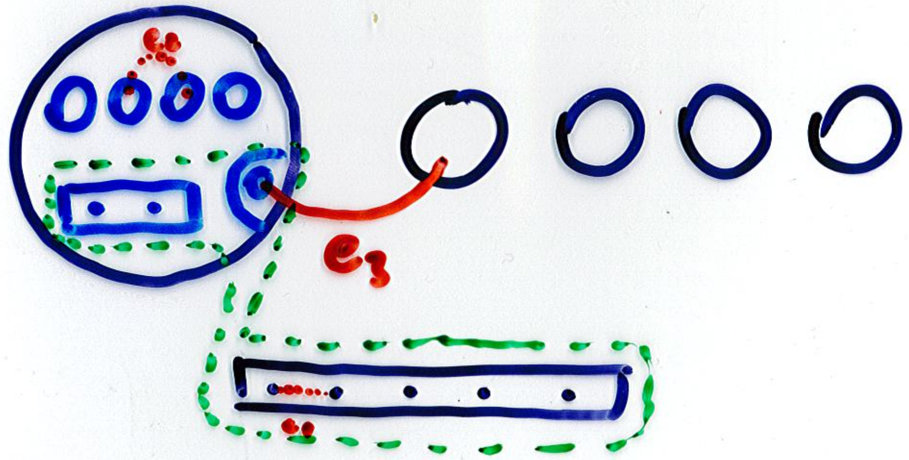


(5)



↯ $G + e_2$ HAS A PERFECT MATCHING CONTAINING e_2.

(6)



(7)