The aircraft ground routing problem: Analysis of industry punctuality indicators in a sustainable perspective

J. Guépet\textsuperscript{a,b}, O. Briant\textsuperscript{a}, J.P. Gayon\textsuperscript{a,*}, R. Acuna-Agost\textsuperscript{b}

\textsuperscript{a}Grenoble-INP / UJF-Grenoble 1 / CNRS, G-SCOP UMR5272 Grenoble, F-38031, France
\textsuperscript{b}Amadeus S.A.S., 485 Route du Pin Montard, 06560 Sophia Antipolis, France

Abstract

The ground routing problem consists in scheduling the movements of aircraft on the ground between runways and parking positions while respecting operational and safety requirements in the most efficient way. We present a Mixed Integer Programming (MIP) formulation for routing aircraft along a predetermined path. This formulation is generalized to allow several possible paths. Our model takes into account the classical performance indicators of the literature (the average taxi and completion times) but also the main punctuality indicators of the air traffic industry (the average delay and the on time performance). Then we investigate their relationship through experiments based on real data from Copenhagen airport (CPH). We show that the industry punctuality indicators are in contradiction with the objective of reducing taxi times and therefore pollution emissions. We propose new indicators that are more sustainable, but also more relevant for stakeholders. We also show that the runway is the main bottleneck of CPH airport and that alternate paths cannot improve the performance indicators.

Keywords: Ground routing, airport operations management, mixed integer programming, sustainable development

1. Introduction

Over the last years, the European air traffic kept growing and Eurocontrol (2012b) predicts an annual increase of the number of flights of 3\% between 2014 and 2018. The traffic is expected to double between 2010 and 2030 (Eurocontrol (2010)). Due to this ceaseless increase of the number of flights in Europe, airports are becoming an important bottleneck of air traffic. Hence, using decision support systems and optimization tools is more and more critical.

The aircraft ground movements play an important role in the airport emissions. London Heathrow (2008-09) airport estimates that 54\% of the airport $NO_x$ emissions are produced by aircraft on the ground. The ground routing is also key component of the airports carbon footprint. Eurocontrol (2009) estimates that 475,000 tonnes of $CO_2$ emissions could be earned if only one
minute of taxi time per flight could be earned in 50 major European airports. It also represents a non negligible part of airlines fuel cost. Ravizza et al. (2013) demonstrate that a better routing optimization allows to earn $9.6 millions of fuel a year in Zurich airport.

The Ground Routing Problem (GRP) consists in scheduling the movements of aircraft between airport facilities without conflicts and in the most effective way. An arriving aircraft has to be routed from its landing runway to its stand or hangar. A departing aircraft has to be routed from its current parking position to its departure runway. The ground movements occur on a network of roads called taxiways which link airport facilities (see Figure 1). In practice this problem is issued by Air Traffic Controllers (ATCs) on an operational window of typically 10 to 40 minutes.

The main constraints of the problem are related to the safety of aircraft: as in airspace, aircraft have to be separated from each other to avoid collisions. Several other routing constraints must also be taken into account such as taxi speeds and acceleration for passengers comfort, turning angle and aircraft / taxiway segment compatibility due to weight or width.

The quality of a routing schedule is defined by several Key Performance Indicators (KPIs). In this paper, we focus on four of them: the average taxi time, the average completion time, the average delay and the On Time Performance (OTP).

The taxi time measures the time an aircraft spends on the ground with engines on, between push back (i.e. leaving the parking position) and take-off for departures and between landing and park-in for arrivals. It includes any waiting time (e.g. runway queuing time) and not just the time spent moving, as engines cannot be turned off once started up. Pollution emission is directly related to the fuel consumption. The aircraft fuel consumption is not accurately known for the taxi process nowadays, but various statistical studies conclude that it mainly depends on
the taxi time (see e.g. Khadilkar and Balakrishnan (2011), Nikoleris et al. (2011) and Ravizza et al. (2012)). Other influencing factors have been identified, such as the number of stops, turns and accelerations but their effects are less clear and of minor importance in comparison to the taxi time.

We are also interested in minimizing the completion times, i.e. the take-off times for departing flights and the park-in times for arriving flights. In peak hours, the runway is often the main bottleneck of the airport (Idris et al. (1998)). Minimizing take-off times reduces the risk of runway starving and ensures a good use of its capacity. Minimizing park-in times reduces the passengers debarkation time, which increases the quality of service.

In the air traffic industry, the main indicators of punctuality are the average delay per flight and the OTP (see Eurocontrol (2012a) and Eurocontrol (2013)). The delay is measured with respect to the scheduled times of park-in and push back, which are both printed on passenger tickets. For example, an arriving flight is 5 minutes late if the aircraft arrives at the stand 5 minutes after the scheduled time and a departing flight is 5 minutes late if the aircraft is pushed back from its stand 5 minutes after the scheduled time. The OTP $L$ is the percentage of flights having a delay less than $L$ minutes. The most common value of $L$ in the industry is 15 minutes and OTP 15 is simply called OTP.

A common practice in airports is to push back aircraft as soon as possible and to taxi to the runway (see Atkin et al. (2011a) for London Heathrow airport (LHR) and Smeltink et al. (2004) for Amsterdam Schiphol airport (AMS)). It reduces the risk of runway starting and is beneficial for the departure delay and OTP. However, especially during peak hours, the runway capacity is often exceeded and a push back as soon as possible policy results in a take-off queue (see Figure 2) in which aircraft engines remain turned on. Pollution emissions can be reduced by transferring the runway queuing time (with engines on) to the stand (with engines off). This process is called stand holding. Nevertheless, if an aircraft is held too long, it may not reach the runway in time for take-off and some runway capacity can be wasted. It may also prevent an
arriving aircraft from using the stand (stand blockage).

Accurate estimations of aircraft ready times and taxi times are required to schedule push back adequately, i.e. holding stand as much as possible in order to reduce taxi times, but without wasting the runway capacity. Accurate estimations of ready times are not always available in airports: ATCs are often informed of an aircraft ready time only when the pilot calls the control tower for push back and start up approval. That is why Eurocontrol designed the Collaborative Decision Making (CDM) project. The main goal of CDM project is precisely to improve predictability and information sharing between all stakeholders. In a CDM airport, airlines and ground handlers are required to communicate and update an accurate ready time (typically 30 to 40 minutes in advance). The ready time is called the Target Off-Block Time (TOBT) and corresponds to an estimation of the time at which the aircraft will be ready to push back (all doors closed, boarding bridge removed, etc). In the air traffic industry, the push back scheduling is called often the pre-departure sequencing.

In the literature, it has been shown that stand holding can reduce taxi times significantly without impacting the runway capacity (see Section 2). Nevertheless, the impact of stand holding on the KPIs of the industry (OTP and delay) has not been investigated. In this paper, we propose a model including the OTP and delay indicators. Our model allows several possible paths in the taxiways. We then address the following questions through a numerical study based on realistic instances from Copenhagen airport (CPH). How do the performance indicators compete? Are the key indicators of the industry consistent with a sustainable development? Can we propose better indicators? Can we reduce taxi times by considering alternate paths? What is the bottleneck in ground routing operations? This work is the result of a collaboration with Amadeus company.

The remainder of this paper is organized as follows. A review of the literature and a summary of our contributions are presented in Section 2. In Section 3, we propose a model for the GRP and formulate it as a Mixed Integer Program (MIP). We provide details on the data set and instances from CPH in Section 4. Then the results of our numerical study are given in Section 5. Finally, a conclusion and discussion of our results are presented in Section 6.

2. Literature review

In this section, we present a literature review of the GRP and related works. More details can be found in a survey by Atkin et al. (2010). We also present our contributions.

2.1. The Ground Routing Problem

The main differences between the GRP models are the routing options, the link with the take-off schedule and whether the time is discretized or not.

Three different routing options exists: single path, alternate paths and path free. In the single path approach, each aircraft can be routed along one and only one predetermined route. In the alternate paths approach, each aircraft can be routed along a route chosen between a set of predetermined routes. In the path free approach, any route can potentially be assigned to an aircraft. In most of papers, a take-off schedule is given as an input and the objective is to
minimize its deviation. The CFMU slot compliance\(^1\) is also often considered with a high penalty for missed slots and linear cost for deviation inside the slot.

**Single path.** The single path was first studied by Smeltink et al. (2004) who propose a mixed integer program formulation. The (average) taxi time is minimized and the respect of take-off schedule is forced by constraints. Rathinam et al. (2008) simplify this formulation and improve safety by adding separation constraints.

**Alternate path.** Gotteland and Durand (2003) address the alternate path approach with a genetic algorithm whose solutions are evaluated with a Branch & Bound algorithm. Aircraft are allowed to stop once while taxiing and stand holding is not possible. Their objective is to minimize the routing time. Gotteland et al. (2003) adapt this approach to include take-off predictions and CFMU slots. They linearly penalize deviations from take-off predictions and add a large penalty for missed slots. Balakrishnan and Jung (2007) propose a discrete time integer program. Their model minimizes the taxi time and the delay to the take-off schedule. A large penalty is added for each aircraft reaching the runway after its target time. Deau et al. (2008) and Deau et al. (2009) propose a two stages method for optimizing runway scheduling and ground routing in order to minimize the delay. Firstly, the runway scheduling problem is solved, which provides Target Take-Off Times (TTOTs) for departing aircraft. They are used as an input for the ground routing which is solved with the approach of Gotteland et al. (2003) in the second phase.

**Path free.** Marin (2006) models the path free approach with an integer program formulation of the capacitated multi-commodity flow problem in a space-time network (time is discretized). Aircraft are assumed to taxi with a constant speed. The weighted routing time (completion time) is minimized. The model is extended by Marin and Codina (2008) to include other objectives such as the number of controller interventions, the worst taxi time, the delay and the airport throughput. Keith and Richards (2008) propose an integer program for the path free approach optimizing both the runway scheduling and the ground routing. They minimize a weighted combination of the makespan, the average taxi time and the average distance. The model is slightly adapted by Clare and Richards (2011) to improve computational efficiency. Nevertheless, computation times are still too important: their model needs more than a minute to handle instances with 8 aircraft on a small network with only the runway holding point. Lesire (2010) presents an iterative algorithm for the path free approach. Their method is based on an A* algorithm and aims to minimize the completion time. Liu et al. (2011) present a hybrid genetic algorithm and simulated annealing algorithm based on the multi-commodity flow model of Marin (2006). The idea of their method is to replace usual selection criteria by a simulated annealing temperature principle. Atkin et al. (2011b) and Ravizza et al. (2013) propose an iterative approach. Aircraft are routed by a Quickest Path Problem with Time Window (QPPTW) algorithm. Take-off

---

\(^1\)When an air sector of an aircraft flight plan is congested or when the destination airport is facing adverse conditions, the Central Flow Management Unit (CFMU) assigns a Calculated Take-Off Time (CTOT). It generally results in delaying the take-off to prevent the situation from getting worse in the perturbed sector. The take-off is allowed within the interval \([CTOT - 5 \text{ min}; CTOT + 10 \text{ min}]\), called a CFMU slot. Otherwise, the aircraft has to wait for another slot from the CFMU.
predictions are taken into account by using a backward version of QPPTW algorithm. The
method is tested on Zurich airport and show a potential taxi time reduction of up to 23.6% for
Atkin et al. (2011b) and 30.3% Ravizza et al. (2013). The method offers computation times of
less than 50ms by aircraft. Ravizza et al. (2013) propose a swap based method for changing
the assignment order, which allows to slightly improve the results while still offering very short
computation times.

2.2. The Push Back Scheduling Problem

The literature on the ground routing problem reveals that most of inefficiencies of the taxi
process come from runway congestion and can be reduced by stand holding (see e.g. Balakrishnan
and Jung (2007) or Ravizza et al. (2013)), i.e. by scheduling push back time latter. Based on this
result, some papers focus on the scheduling of push back times and do not consider a detailed
routing.

Deterministic models. Malik et al. (2010), Jung et al. (2010, 2011) and Atkin et al. (2012) use a
similar decomposition to Deau et al. (2009): they first optimize the take-off sequence and then
allocate push back times to meet this take-off sequence, while trying to absorb as much delay as
possible at the stand.

In fact, Malik et al. (2010), Jung et al. (2010, 2011) spot release times are issued and not push
back times\(^2\), which requires a further collaboration with ramp area to transfer the delay to the
gate. Gupta et al. (2012) present the concepts of such a collaboration. The main drawback of this
practice is that the complexity of the ramp area structure is hidden and some more appropriate
spot release sequences may be missed. On the contrary, Atkin et al. (2012) explicitly consider
ramp area contention. A feasible push back time is assigned to aircraft while computing the
take-off sequence. A sequence is rejected if such push back times can not be found. Push back
times are then re-optimize to improve stand holding.

Note that the problem of considering only push back times is that it requires an accurate taxi
time estimation in order not to waste the runway capacity and not to excessively wait at the
runway. The ground routing problem can be used for solving the push back scheduling problem,
which may lessen the impact of the taxi time prediction.

Queuing models. An airport can be seen as a network of queues and the push back scheduling
problem has been first modeled as a queuing problem by Pujet et al. (1999). The whole taxiway
network is modeled as a server of infinite capacity with stochastic processing times. They propose
a simple threshold policy that regulates the number of aircraft taxiing or waiting at the runway.
Carr et al. (2002) adapt this model to take into account departure fix closures. Burgain et al.
(2012) refine the modeling of the taxiway network and propose a sequential queues network. It
allows to take advantage of recent surface surveillance technology. A comparison to the threshold
policy of Pujet et al. (1999) highlights that this technology can be beneficial when the runway

\(^2\)In most US airports, particularly in Dallas Fort Worth International airport (DFW) which is considered in
these studies, the push back is not managed by the ground controller. Indeed, the ramp area, i.e. the areas around
the gates, is managed by flight operators (airlines) or a ramp controller (airport authorities). Their responsibility
is to push back aircraft and bring them to predetermined spots (exit point of the ramp area and entry points of
the taxiway area), where responsibility is handed over to the ground ATC.
is operated at intermediate capacity. Finally, Simaiakis et al. (2014) propose a push back rate control policy, which advises push back frequency for the next 15 minutes. The method was tested in real situation in Boston Logan airport (BOS) over 16 demo period and 8 periods with significant gate-holds were kept for the analysis. They estimate an average earning of one minute and a half in taxi-out times.

2.3. Our contributions

We present a MIP formulation of the single path GRP that follows the continuous time models of Smeltink et al. (2004) and Rathinam et al. (2008). We extend their approach to include alternate paths. This requires to formulate separation constraints temporally as in Clare and Richards (2011).

Our model differs in several other aspects. First, we include the punctuality indicators of the industry (OTP and delay) in the objective function, in addition to the taxi time and completion time indicators. A second difference is the link with the runway. The input of our model is the take-off sequence while the input of most of the models is the take-off schedule with targeted take-off times. We made this choice because manipulating sequences is more convenient than schedules for ATCs. In that way, our model can be used as a tool for supporting runway sequencing decisions: it provides optimal take-off times from a sequence, while accurately taking into account routing considerations. Finally we add stand blockage constraints.

To the best of our knowledge, the pertinence of the OTP and delay indicators have not been questioned in the context of the GRP. In a numerical study, we analyze the impact of including these KPIs in the optimization. We show that they are in contradiction with the objective to reduce taxi times and pollution emissions in airports. We propose new indicators that are more sustainable, but also more relevant for stakeholders.

Our experiments confirm that the runway is the main bottleneck in CPH airport, particularly during departure peaks. It explains why the alternate paths approach does not succeed in improving the KPIs significantly.

3. Ground routing problem formulation

In this section, we introduce the main notations and formulate the GRP as a MIP. We first present the model with a single route for each flight. Then the model is generalized to consider a set of possible paths for each flight (alternate paths approach).

For both models, the main inputs are the runway allocation, the take-off sequence, the landing schedule and the stand allocation plan (including the sequence of aircraft for every stand).

3.1. Single path model

**Taxiway network.** The taxiway network is modeled as a graph \( G = (V, E) \) with \( V \) the set nodes and \( E \) the set of edges. There is a node for each taxiway intersection and additional nodes for stands. An edge represents an elementary segment of taxiway.

The set of arriving and departing flights is \( \mathcal{F} = \mathcal{F}_{\text{arr}} \cup \mathcal{F}_{\text{dep}} \). For a flight \( i \), the single path from its origin \( o_i \) to its destination \( d_i \) is \( P_i = (o_i, u_2, \ldots, u_{|P_i|-1}, d_i) \). Let \( V_i \subset V \) and \( E_i \subset E \) the set of nodes and edges that flight \( i \) can use. Note that the origin \( o_i \) and the destination \( d_i \) are fixed by the runway and stand allocation.
**Flight characteristics.** A flight $i$ is ready to leave its origin $o_i$ at time $T_0$. For an arriving flight $i \in F_{arr}$, it corresponds to the Target LanDing Time ($TLDT$) estimated by ATCs. For a departing flight $i \in F_{dep}$, it corresponds to the Target Off-Block Time ($TOBT$) estimated by the airline and the ground handlers.

The scheduled time for flight $i$ is $SB_i$, this time is used to measure the delay and the OTP. For an arriving flight, it is the Scheduled In-Block Time ($SIBT$), i.e. the time the aircraft is supposed to arrive at its stand. For a departing flight, it corresponds to the Scheduled Off-Block Time ($SOBT$), i.e. the time the aircraft is supposed to push back from its stand.

A flight $i$ can spend a minimum (maximum) time $T_{min}^{iu} (T_{max}^{iu})$ on edge $uv \in E_i$. These times can be directly computed from the minimum and maximum speeds allowed on edge $uv$ for aircraft $i$ and from the edge length (see Section 4).

The take-off sequence is an input of our model. The position of departure $i \in F_{dep}$ in the take-off sequence is $\Gamma(i)$.

**Interactions between flights.** Flights $i$ and $j$ must have a minimum separation time at each node $u \in V_i \cap V_j$; if flight $i$ arrives first at node $u$ at time $t$, then $j$ cannot cross node $u$ before $t + S_{iju}$.

Let $G \subset F_{dep} \times F_{arr}$ the set of possible stand blockages. A pair of flights $(i, j)$ belongs to $G$ if departure $i$ and arrival $j$ are assigned to the same stand and $i$ is scheduled before $j$ (in the stand allocation plan). In this case, departure $i$ must leave the stand before arrival $j$ parks in.

**Decision variables.** In the single path approach, the main decisions are the time that aircraft reach each node of their path. Our formulation uses the following variables:

- $t_{iu}$: the time when flight $i$ reaches node $u \in V_i$. The origin time $t_{io}$ corresponds to the landing time for an arrival and to the push back time for a departure. The destination time $t_{id}$ corresponds to the take-off time for a departure and to the park-in time for an arrival.

- $\delta_i$: the delay of flight $i$ to its scheduled reference time $SB_i$. The delay is $\max(0, t_{id_i} - SB_i)$ for an arrival and $\max(0, t_{io_i} - SB_i)$ for a departure.

- $\beta_i = 1$ if flight $i$ is delayed by more than $L \geq 0$ with respect to the scheduled reference time $SB_i$ (if $\delta_i > L$), 0 otherwise.

- $z_{iju} = 1$ if flight $i$ arrives before flight $j$ in node $u \in V_i \cap V_j$, 0 otherwise.

**Objective function.** We are interested in minimizing the following performance indicators.

- $\sum_{i \in F} \beta_i$: Number of flights delayed by more than $L$

- $\sum_{i \in F} \delta_i$: Total delay

- $\sum_{i \in F} (t_{id_i} - t_{io_i})$: Total taxi time

- $\sum_{i \in F} (t_{id_i} - T_{oi})$: Total completion time
Note that minimizing the number of flights delayed by more than $L$ is equivalent to maximizing the OTP $L$ (percentage of flights with a delay less than $L$).

Our objective function is a linear combination of these four indicators using non-negative coefficients $c_{OTP}$, $c_{delay}$, $c_{taxi}$ and $c_{ct}$. It can be extended to include the CFMU slot compliance objective (see Appendix A).

**MIP formulation.** The single path problem can be formulated by MIP 1.

$$
\begin{align}
\min \quad & c_{OTP} \sum_{i \in F} \beta_i + c_{delay} \sum_{i \in F} \delta_i + c_{taxi} \sum_{i \in F} (t_{id_i} - t_{io_i}) + c_{ct} \sum_{i \in F} (t_{id_i} - T_{oi}) \\
\text{subject to} \quad & t_{io_i} \geq T_{oi} \quad \forall i \in F_{dep} \\
& t_{io_i} = T_{oi} \quad \forall i \in F_{arr} \\
& t_{io_i} \leq t_{jd_j} \quad \forall (i, j) \in G \\
& T_{iuv}^{\min} \leq t_{iv} - t_{iu} \leq T_{iuv}^{\max} \quad \forall i \in F, \forall uv \in E_i \\
& z_{iju} + z_{jiu} = 1 \quad \forall i, j \in F, \forall u \in V_i \cap V_j \\
& z_{iju} = 1 \quad \forall i, j \in F_{dep}, u = d_i = d_j, \Gamma(i) < \Gamma(j) \\
& t_{ju} \geq t_{iu} + S_{iju} - M(1 - z_{iju}) \quad \forall i, j \in F, \forall u \in V_i \cap V_j \\
& z_{iju} = z_{jiu} \quad \forall i, j \in F, \forall uv \in E_i \cap E_j \\
& \delta_i \geq t_{io_i} - SB_i \quad \forall i \in F_{dep} \\
& \delta_i \geq t_{id_i} - SB_i \quad \forall i \in F_{arr} \\
& \delta_i \leq L + M\beta_i \quad \forall i \in F_{dep} \\
& t_{iu} \geq 0 \quad \forall i \in F, \forall u \in V_i \\
& \delta_i \geq 0 \quad \forall i \in F \\
& \beta_i \in \{0, 1\} \quad \forall i \in F \\
& z_{iju} \in \{0, 1\} \quad \forall i \neq j \in F, \forall u \in V_i \cap V_j 
\end{align}
$$

MIP 1: Single path approach

Constraints (2) ensure that departures cannot push back before they are ready to and Constraints (3) ensure that arrivals start taxiing as soon as they land, in order to free the runway. Constraints (4) ensure that an arrival does not park-in until the previous departure has left (stand blockage constraints). Constraints (5) ensure the respect of minimum and maximum time spent on every edge (speed constraints). The maximum time spent on an edge allows to prevent aircraft from stopping in certain taxiway segments (e.g. runway crossing). It also ensures that the capacity of the edge is not exceeded (i.e. no more aircraft that its length allows it). Note that in all the solutions in our experiments, aircraft never taxi at the minimal speed. Constraints (6) ensure the definition of sequencing variables $z_{iju}$, i.e. either flight $i$ arrives before flight $j$ in node $u \in V_i \cap V_j$ or the opposite. Constraints (7) ensure that the take-off sequence is respected. Constraints (8) and (9) prevent the three kinds of conflict illustrated in Figure 3. Constraints (8) prevent aircraft from bumping into each other at every node (see Figure 3(a)), where $M$
is supposed to be a high enough value (e.g. 10 times the time window is largely sufficient, it remains in forcing every aircraft to end taxiing in less than 10 times the time window which is reasonable). Constraints (9) prevent two aircraft from using an edge in opposite directions simultaneously (see Figure 3(b)). Constraints (9) also prevent an aircraft from overtaking another one on an edge, which is physically impossible (see Figure 3(c)). Constraints (10) to (12) ensure the definition of delay variables $\delta_i$ and OTP variables $\beta_i$. Finally, constraints (13) to (16) specify the domains of the variables.

Note that it is possible to differentiate flights for all objectives through a slight change in the objective function (indexing the coefficients):

$$\sum_{i \in F} c_{i}^{TP} \beta_i + \sum_{i \in F} c_{i}^{delay} \delta_i + \sum_{i \in F} c_{i}^{taxi} (t_{id_i} - t_{io_i}) + \sum_{i \in F} c_{i}^{ct} (t_{id_i} - T_{io_i})$$

It particularly makes sense for the taxi time as a big aircraft will consume more fuel (and thus pollute more) than a small one.

3.2. Alternate paths model

In this section, we generalize the previous model in order to allow multiple paths. For each flight $i$, the path can be chosen in a set of alternate paths $\mathcal{P}_i$. For each flight $i$ and each path $p \in \mathcal{P}_i$, we denote $V_i^p$ and $E_i^p$ the set of nodes and edges used in $p$. The set of nodes that can be used by flight $i$ is now $V_i = \cup_{p \in \mathcal{P}_i} V_i^p$. Similarly, the set of edges that can be used by flight $i$ is $E_i = \cup_{p \in \mathcal{P}_i} E_i^p$. The aim of this model is to choose a path for every flight and to schedule the moves along this path. Consequently, we need to define the following path selection variables and scheduling variables:

- $x_i^p = 1$ if aircraft $i$ uses path $p \in \mathcal{P}_i$, 0 otherwise.
the time flight \( i \) reaches node \( u \in V_i \) through path \( p \in P_i \) if flight \( i \) uses path \( p \), 0 otherwise.

Variables \( \beta_i, \delta_i, z_{iju}, t_{ioi} \) and \( t_{oid} \) are defined as previously. The alternate paths model can be formulated by MIP 2.

Bounding constraints (2) and (3), stand blockage constraints (4), runway sequencing constraints (7) and variables definition constraints (10) to (12) and (14) to (16) are still valid in this formulation. Other constraints need to be adapted.

Note that \( \sum_{p \in P_i, u \in V_i} x_{pi} = 1 \) if flight \( i \) uses node \( u \), 0 otherwise. Similarly, \( \sum_{p \in P_i, uv \in E_i} x_{pi} = 1 \) if flight \( i \) uses edge \( uv \).

Constraints (17) ensure that one and only one path is chosen for every flight. Constraints (18) to (22) ensure the definition of scheduling variables. Speed constraints (5) must be replaced by constraints (23). Separation constraints (8) must be replaced by constraints (24). Note that if flight \( i \) or \( j \) does not use the node \( u \in V_i \cap V_j \) then \( z_{iju} = 0 \) and the constraints is disabled. Sequencing constraints (6) must be replaced by constraints (25) to (28). Note that constraints (25) and (26) are equivalent to constraints (6) if flight \( i \) and \( j \) uses node \( u \). Otherwise these constraints have no effect. Also note that constraints (27) to (28) are not necessary as the optimization will naturally verify them to disable associated separation constraints if necessary. Overtake and head-on constraints (9) are equivalent to constraints (29) and (30) if flight \( i \) and \( j \) uses edge \( uv \). Otherwise, the right hand side is less or equal to -1 or -2 and the constraints have no effect.

Including the option of choosing a path increases drastically the number of constraints and binary variables, especially if a high number of paths is possible. Clare and Richards (2011) had the same problem in their model. They designed an iterative approach, close to a constraints generation process, to tackle it. It consists is solving the MIP without a subset of constraints (and the binary variables that appear only in these constraints). If the optimal solution of the obtained MIP does not satisfy some constraints not taken into account, they are added to the MIP (with the involved binary variables), which is then solved again. The process is repeated until the current optimal solution satisfies all constraints of the problem, even those that have been relaxed. As shown by Clare and Richards (2011), for this kind of problem, solving smaller and simpler MIP, even several times, is more efficient than solving the whole MIP. It is also the case for our formulation. The constraints that we have chosen to relax are those involving sequencing variables \( z_{iju} \), i.e. separation constraints (24), sequencing constraints (25) to (28) and overtaking / head-on constraints (29) and (30). This method reduces the computation times significantly, the maximum computation time was divided by approximately 3. We do not present a comparison as it has already been shown by Clare and Richards (2011). We tried to apply the same approach for the single path model, but no improvement were observed.

4. Data set and instances

In this section we present our instances and how they were generated. Each instance represents an operational day in Copenhagen airport (CPH).
\[
\begin{align*}
\min & \quad c^{OTP} \sum_{i \in \mathcal{F}} \beta_i + c^{\text{delay}} \sum_{i \in \mathcal{F}} \delta_i + c^{\text{taxi}} \sum_{i \in \mathcal{F}} (t_{id_i} - t_{io_i}) + c^{t} \sum_{i \in \mathcal{F}} (t_{id_i} - T_{oa}) \\
\text{s.t.} & \quad (2 - 4), (7), (10 - 12), (14 - 16) \\
& \quad \sum_{p \in \mathcal{P}_i} x^p_i = 1 \quad \forall i \in \mathcal{F} \\
& \quad t_{io_i} = \sum_{p \in \mathcal{P}_i} t^p_{io_i} \quad \forall i \in \mathcal{F} \\
& \quad t_{id_i} = \sum_{p \in \mathcal{P}_i} t^p_{id_i} \quad \forall i \in \mathcal{F} \\
& \quad t^p_{io_i} \geq T_{oa} x^p_i \quad \forall i \in \mathcal{F}_{\text{dep}}, \forall i \in \mathcal{P}_i \\
& \quad t^p_{io_i} = T_{oa} x^p_i \quad \forall i \in \mathcal{F}_{\text{arr}}, \forall i \in \mathcal{P}_i \\
& \quad t^p_{iu} \leq M x^p_i \quad \forall i \in \mathcal{F}, \forall p \in \mathcal{P}_i, \forall u \in V^p_i \\
& \quad T^\text{min} x^p_i \leq t^p_{iu} - t^p_{io_i} \leq T^\text{max} x^p_i \quad \forall i \in \mathcal{F}, \forall p \in \mathcal{P}_i, \forall uv \in E^p_i \\
& \quad \sum_{p \in \mathcal{P}_j \atop u \in V^p_j} t^p_{ju} \geq \sum_{p \in \mathcal{P}_i \atop u \in V^p_i} t^p_{iu} + S_{iju} - M (1 - z_{iju}) \quad \forall i, j \in \mathcal{F}, \forall u \in V_i \cap V_j \\
& \quad z_{iju} + z_{jiu} \geq \sum_{p \in \mathcal{P}_i \atop u \in V^p_i} x^p_i + \sum_{p \in \mathcal{P}_j \atop u \in V^p_j} x^p_j - 1 \quad \forall i, j \in \mathcal{F}, \forall u \in V_i \cap V_j \\
& \quad z_{iju} + z_{jiu} \leq 1 \quad \forall i, j \in \mathcal{F}, \forall u \in V_i \cap V_j \\
& \quad z_{iju} \leq \sum_{p \in \mathcal{P}_j \atop u \in V^p_j} x^p_i \quad \forall i, j \in \mathcal{F}, \forall u \in V_i \cap V_j \\
& \quad z_{iju} \leq \sum_{p \in \mathcal{P}_j \atop u \in V^p_j} x^p_j \quad \forall i, j \in \mathcal{F}, \forall u \in V_i \cap V_j \\
& \quad z_{iju} - z_{jiu} \geq \sum_{p \in \mathcal{P}_i \atop uv \in E^p_i} x^p_i + \sum_{p \in \mathcal{P}_j \atop uv \in E^p_j} x^p_j - 2 \quad \forall i, j \in \mathcal{F}, \forall uv \in E_i \cap E_j \\
& \quad z_{iju} - z_{jiu} \geq \sum_{p \in \mathcal{P}_i \atop uv \in E^p_i} x^p_i + \sum_{p \in \mathcal{P}_j \atop uv \in E^p_j} x^p_j - 2 \quad \forall i, j \in \mathcal{F}, \forall uv \in E_i \cap E_j \\
& \quad t^p_{iu} \geq 0 \quad \forall i \in \mathcal{F}, \forall p \in \mathcal{P}_i, \forall u \in V^p_i \\
& \quad t_{io_i}, t_{id_i} \geq 0 \quad \forall i \in \mathcal{F} \\
& \quad x^p_i \in \{0, 1\} \quad \forall i \in \mathcal{F}, \forall p \in \mathcal{P}_i
\end{align*}
\]
**Runway configuration.** The most frequent runway configuration in CPH is the 22 mode (220° with the north), with take-offs on runway 22R (R=Right) and landings on runway 22L (L=Left). In the month of September 2012, we have selected 8 busy days (with more than 700 flights) in which more than 98% of flights were operated in this runway configuration. The 8 instances have a similar traffic profile. The average number of arrivals and departures by hour is presented in Figure 4. Minimum runway separation times used at CPH are presented in Table 1.

![Figure 4: Average profile of instances](image)

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Leading aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

Table 1: Minimum runway separation time at CPH airport (H = heavy, M = medium and L = light)

**Taxiway network.** Figure 5 presents the graph of the taxiway network with the 22 runway configuration. An edge represents an elementary taxiway segment. A node needs to be defined for each taxiway intersection. There is also a node for each stand. The graph is composed of 93 nodes and 235 edges. The standard path between each stand and each runway was provided by the airport, as well as the standard push back scheme and its duration, for each stand. We observed on ground radar data that standard paths were used for more than 83% of flights.

Ground radar data also provides an estimation of the maximal speeds. Based on these data, we assume a maximal speed of 15 m/s for the taxiways around the runway (in blue in Figure 5), of 5 m/s for the taxiways around the stands (in red in Figure 5) and of 10 m/s for the other taxiways. A minimum speed of 2 m/s is assumed on every edges. The minimum and maximum times spent on an edge ($T_{\text{min}}$ and $T_{\text{max}}$) can be directly computed from the minimum and maximum speeds allowed on this edge and from the edge length. The minimum separation time ($S_{iju}$) between two aircraft is assumed to be 40 seconds for every nodes (except the runways, see Table 1), which guarantees a minimum separation of 80 meters between aircraft.

**Flights.** The ground radar data does not contain information on flights but only records composed of aircraft identifier, position in the airport and time stamp. The airport operational database provides other useful data for each flight.
It gives the Scheduled In-Block Time (SIBT) or the Scheduled Off-Block Time (SOBT), denoted by $SB_i$ in our model. For each arriving flight, it also provides the Actual Landing Time (ALDT) which can be used to define the release date $T_{o_i}$. However, we did not have access to the release date (or ready time) for departing flights, as CPH was not a CDM airport in 2012. As a push back as soon as possible policy is used in CPH most of the time, we have decided to take the Actual Off-Block Time (AOBT) to define the release date $T_{o_i}$ for departing flights. Finally, the take-off sequence is the actual one which can be derived from the Actual Take-Off Times (ATOTs).
Average performance indicators. All results are averaged among the 8 days and the aircraft. For instance, an average taxi time of 8 minutes means that it takes on average 8 minutes for an aircraft to taxi, among the 8 days. We choose the OTP 15 indicator \((L = 15)\) which is one of the main punctuality indicators in the air traffic industry.

5. Numerical results

The results are presented for the single path problem, except in Section 5.3 where we study the effect of the number of paths. We set \(c^{\text{taxi}} = 1\) without loss of generality, as we vary the other weights \(c^t, c^{\text{delay}}\) and \(c^{\text{OTP}}\). Results of mixed integer programs were obtained with Cplex 12.4 solver using default parameter tuning on a personal computer (Intel Core i5-2400 3.10Ghz, 4Go RAM) under Ubuntu 12.04 LTS operating system. Java Concert API was used to define the models.

5.1. Sliding window optimization

As there are many stochastic events, it is not possible to schedule the movements of aircraft for the entire day. Hence the GRP is usually solved dynamically with a sliding window approach, in both literature and practice. The longer the time window is the better the solutions are, but the higher the computation times are.

The optimization does not need to be performed continuously but only when a new aircraft enters the system. Once an aircraft has started taxiing, changing its schedule is not allowed in the next time windows, but it has to be taken into account to ensure a conflict free routing.

In the rest of the numerical study we set a time window of 30 minutes. This assumption seems reasonable in the context of CDM project in which airlines and ground handlers are required to communicate accurate ready times 30 to 40 minutes in advance.

With a 30 minutes time window, computation times were always below two seconds for the single path approach. It appears that a time window of 15 minutes is sufficient in our test case, i.e. longer time windows do not provide better solutions. This value may be airport dependent and cannot be generalized without experiments in other airports.

5.2. Including the punctuality key performance indicators

The average delay and the OTP are the main punctuality indicators for airlines and airports. However the literature focuses on taxi time and completion time indicators. In this section, we study the impact of including the average delay and the OTP in the optimization.

5.2.1. Average delay

Figure 6 presents the effect of the weight \(c^{\text{delay}}\) on all KPIs for arrivals (dashed lines) and departures (solid lines) and for different values of \(c^t\).

For arrivals, we observe that KPIs are not much impacted by \(c^{\text{delay}}\), which can be explained as follows. The contribution of a delayed arriving flight \(i \in \mathcal{F}_{\text{arr}} (t_{\text{id}_i} > SB_i)\) to objective function (1) is, within a constant and because the landing time \(t_{\text{id}_i}\) is fixed (see constraints (3))

\[
(c^{\text{delay}} + c^{\text{taxi}} + c^t)t_{\text{id}_i} + c^{\text{OTP}}1_{\{t_{\text{id}_i} > SB_i + L\}}
\]

(34)
It clearly appears that KPIs are based only on variables $t_{id_i}$. Therefore including the delay adds redundancy and there is no trade-off to be done between the taxi time and the delay.

On the contrary for departures, increasing $c_{delay}$ reduces delays but increases taxi times. One can even observe a threshold effect between the delay and the taxi time when $c_{delay} = c_{taxi}$. It is explained by the contribution of a delayed departure $i \in \mathcal{F}_{dep}$ ($t_{io_i} > SB_i$) to the objective function, which is, within a constant,

$$
(c_{delay} - c_{taxi})t_{io_i} + (c_{taxi} + c^t) t_{id_i} + c^{OTP} 1\{t_{io_i} > SB_i + L\}
$$

(35)

It clearly highlights an opposition of the taxi time and the delay for departures. When $c_{delay} - c_{taxi} > 0$, pushing back aircraft earlier (which reduces $t_{io_i}$) is preferable as it reduces delays. But it leads to longer taxi times when the runway is congested. When $c_{delay} - c_{taxi} < 0$, holding aircraft at stands as much as possible (which increases $t_{io_i}$) is more profitable and avoids runway queuing. It consequently decreases taxi times, but implies larger delays.
To further illustrate the opposition between the taxi time and the delay for departures, Figure 7 details the results along the day with a 30 minutes time window and a 5 minutes step. For instance, at 6h05, 7(a) plots the number of departures in the time interval [6h05,6h35]. Figure 7(b) plots the additional taxi time and the additional delay when we set $c_{\text{delay}} = 2$ instead of $c_{\text{delay}} = 0$.

We observe differences mainly during the peaks. In lows, aircraft push back as soon as possible, go to the runway in the shortest time and take-off immediately. Hence, all performance indicators are optimized. In peak hours, the runway is saturated and flights cannot take-off as soon as they reach the runway. They must either wait at stands or at the runway. When $c_{\text{delay}} = 0$, stand holding is preferred since it reduces the taxi time. When $c_{\text{delay}} = 2$, pushing-back earlier is preferred in order to reduce delays to the scheduled push back time.

The number of departures in the next 30 minutes and the additional taxi time and delay are plotted in Figure 7. The results show that delays cannot be significantly reduced without degrading taxi times in peak hours.

5.2.2. OTP 15

Figure 8 presents the effect of the weight $c_{\text{OTP}}$ on all KPIs for arrivals (dashed lines) and departures (solid lines) and for different values of $c_{\text{ct}}$. The results have some similarities with the previous section, as the OTP 15 is highly correlated to the average delay. However, there are some differences to be highlighted.

In conclusion, delays cannot be significantly reduced without degrading taxi times in peak hours.
We observe that it is possible to significantly improve the OTP without degrading much the taxi time. Moreover, there is no threshold. Here again, the issue is to decide for each departure if the waiting time, due to runway congestion, may be spent at the stand or at the runway. OTP 15 is flexible as it provides a margin of 15 minutes, contrary to the delay for which every minute matters. This 15 minutes margin can be used to hold aircraft at stands and consequently to reduce the waiting time at the runway.

5.2.3. Effect of reducing the network capacity

Figure 7(a) reveals that the maximal runway capacity is not exceeded even in departure peaks ($\approx 20$ aircraft vs a maximal capacity of 30 aircraft), which means that the airport is not over-congested.

To simulate a higher congestion, we focus on the morning peak (from 5 to 7) and reduce network capacity by increasing the minimum separation times at each node, including the runway.
All separation times are multiplied by a factor $\gamma$. Figure 9 presents the effect of $\gamma$ on the performance indicators, for three different objective functions (without delay and OTP indicators, with delay indicator and with OTP indicator). We observe that the larger $\gamma$ is, the more there is an opposition between the taxi time and the punctuality indicators. For instance, the departure taxi time is increased by 7 minutes (resp. 2 minutes) if we include the delay (resp. OTP) in the objective function, when $\gamma = 2$.

Figure 9: Effect of increasing separation times during the morning peak (from 5 to 7 am) for arrivals (dashed lines) and departures (solid lines) ($c_{\text{ct}} = 2$ and $c_{\text{taxi}} = 1$)

5.2.4. New departure punctuality indicators

Stand holding succeeds in reducing the taxi time significantly by transferring runway queuing time with engines on to the stands with engines off. We observed in figures 7 and 9 that it is particularly efficient during departure peaks. Nevertheless, our analysis also shows that this practice degrades the punctuality indicators. Hence airports and airlines may be reluctant to use stand holding and may prefer a push back ASAP policy to ensure good departure indicators. In
this section, we question the relevance of OTP and delay indicators for airlines and airports and propose new punctuality indicators.

British Airways (2008-09) propose to base the measure of departure punctuality for airlines on the ready time and not on the push back time. It seems more appropriate since airlines are not accountable for the delay between the ready time and the push back time.

Measuring the punctuality with respect to push back times is also very artificial for airports as additional delays occur during the taxi process and particularly in the runway queue. Thus, it would be much more natural to base the measure of the punctuality on take-off times. However, a Scheduled Take-Off Time (STOT) does not exist neither in CDM, nor (to our knowledge) in the industry. We propose to define STOT as SOBT plus a constant depending on the airport, for instance the average departure completion time \( \text{ATOT} - \text{TOBT} \). Our models can easily be adapted to measure the delay and the OTP with respect to STOT. Constraints (10) can be merged with constraints (11) as follows:

\[
\begin{align*}
 t_{id_i} & \leq SB'_i + L + M \beta_i \quad \forall i \in F \\
 \delta_i & \geq t_{id_i} - SB'_i \quad \forall i \in F
\end{align*}
\]

where \( SB'_i \) is the Scheduled In-block Time (SIBT) for arrivals and the Schedule Take-Off Time (STOT) for departures. OTP constraints (12) are unchanged.

Figure 10 presents the effect of the congestion factor \( \gamma \) with the new definition of the delay and the OTP. We observe that including the delay to SIBT / STOT in the objective function does not impact the KPIs. It can be explained by the contribution of a delayed departure \( i \in F_{dep} \) \( t_{id_i} > SB'_i \) to the objective function (within a constant):

\[
c_{\text{taxi}} t_{io_i} + (c_{\text{taxi}} + c_{\text{ct}} + c_{\text{delay}}) t_{id_i} + c_{\text{OTP}} I \{ t_{id_i} > SB'_i + L \}
\]

Both the completion time and the delay are now based on take-off times and are thus redundant. The contradiction between the taxi time and the delay is tackled. From a practical point of view, it means that optimizing the completion times and the taxi times is sufficient to optimize the new delay. We performed the same study for the OTP and the same conclusions were obtained.

The new definition of the departure punctuality indicators will be used in the rest of this paper, i.e. the delay and the OTP will be measured with respect to STOT.

5.3. Effect of the number of paths

In this section, we compare the single path approach to the alternate paths one through an analysis of the effect of the number of paths. The alternate paths were generated with a \( k \)-shortest paths algorithm (as proposed by Gotteland et al. (2001)) with \( k = 10 \). Then unrealistic paths were filtered to respect basic routing logic (no cycles, turning-angles, etc...). The final number of paths between a stand and a runway is between 3 and 9 in our data. The first path of the set is always the one used in the single path approach (provided by the airport).

Figure 11 shows that the number of paths has a limited effect on KPIs but increases a lot the computation time. One can also notice, that considering more than two paths does not improve the results. Consequently, computation times can be reduced by restricting the number of paths,
which brings the maximum computation time from 61 seconds to 27, but the method is still difficult to implement in peak hours. Nevertheless, experiments were performed with a standard computer and simply increasing computing power may be enough to fit industrial requirements.

The lack of gain provided by the alternate path can be explained by the structure of the taxiway network: there are two main parallel taxiways serving the stands and each one is used in a different direction in the single path approach, consequently avoiding most of head-on conflicts between departures and arrivals. On the contrary, the area around the stands is much more intricate and generally offers a single taxiway. This means that the alternate paths do not allow to avoid much more conflicts that the single path approach. This intuition is further explored in the next section through an analysis of the airport bottlenecks.
5.4. Bottleneck analysis

The taxiway network can be divided in three distinct parts: the runway, the ramp area (the area around the stands) and the taxiway area. In this section, we evaluate the impact of each area on the routing, by relaxing its constraints in the optimization.

Figure 12 presents the results of this analysis. All constraints means that all constraints are taken into account. Taxiway means that safety constraints of the taxiway area are relaxed. Ramp means that safety constraints of the ramp area and stand blockages constraints are relaxed. Runway means that separation constraints of the runway are relaxed. Besides no take-off sequence is forced. No constraint is the case where all the above constraints are relaxed and aircraft taxi at maximal speeds without stopping anywhere. The delay and the OTP are measured with respect to SIBT / STOT (as defined in Section 5.2.4).

Figure 12 shows that the runway is the main bottleneck: all indicators are consequently impacted, particularly the completion time, the delay and the OTP. It also shows that the taxiway area has a limited impact on indicators, which join the conclusion of previous section. We also observe that the ramp area impacts all indicators in a non negligible way.

6. Conclusion

In this paper, we formulate the ground routing problem as a MIP. We present a formulation with a single path and generalize it to include alternate paths. We consider an objective function that includes the punctuality indicators used in industry (average delay and OTP 15).

In a numerical study based on data from CPH, we first show how the industry punctuality indicators are in contradiction with a sustainable management of airports. The punctuality of departures is currently measured with respect to push back times, which encourages to push back as soon as possible and results in large taxi times in peak hours because of runway congestion. Including the delay in the objective function leads to a taxi time increase of 1 minute in average for departures at CPH. In more congested situations, this increase can reach 6 minutes. Including
the OTP 15 in the objective function has less impact in current traffic situations. However, in more congested situations, it also leads to longer taxi times.

We propose to measure the punctuality of airports with respect to take-off times and not with respect to push back times. We show that this new measure of punctuality do not prevent stand holding and limit pollution emissions. Besides they are more appropriate since they capture additional delays between push back and take-off.

We also show that the runway is the main bottleneck of CPH airport and that considering alternate paths do not improve the performance indicators significantly.

Numerical experiments were performed in CPH airport and we may wonder to what extent our results can be generalized to other airports. In congested airports, the delay and OTP indicators will intuitively not be adequate to measure punctuality, as they encourage to push back as soon as possible and lead to long taxi times. In non congested airports, it will not matter as a push back as soon as possible policy should be nearly optimal.

The main parallel taxiways serving the ramp areas in CPH prevent most of head-on conflicts in the taxiway area. Such a structure is very common and is present in the five most frequented airports in the world\(^3\). In such configurations, the alternate path approach will probably not bring much with respect to the single path approach. However, the alternate path approach is certainly more beneficial in airports with more complex taxiway layout, typically with runway crossing as shown by Balakrishman and Jung (2007).

---

\(^3\)In 2014 by passenger traffic according to Wikipedia, Hartsfield-Jackson Atlanta International Airport (ATL), Beijing Capital International International airport (PEK), London Heathrow airport (LHR), Tokyo Haneda airport (HND) and Los Angeles International airport (LAX).
Appendix A. Other objectives

- The CFMU slot compliance can easily be included in our model. Maximizing the CFMU slot compliance is equivalent to minimize the number of slots missed. We define new binary variables $\beta'_i$ indicating if the slot is missed for every regulated departure $i \in F_{dep}^{reg}$. The CFMU slot compliance objective is

$$\sum_{i \in F_{dep}^{reg}} c_{CFMU}^i \beta'_i$$

where $c_{CFMU}^i \geq 0$. Following constraints are necessary to ensure the definition of variables $\beta'_i$.

$$t_{id_i} \leq CTOT_i + 10 \times 60 + M \beta'_i \quad \forall i \in F_{dep}^{reg}$$
$$t_{id_i} \geq CTOT_i - 5 \times 60 - M \beta'_i \quad \forall i \in F_{dep}^{reg}$$
$$\beta'_i \in \{0, 1\} \quad \forall i \in F_{dep}^{reg}$$

The deviation inside the slot can also be considered, it can be modeled in a similar way that the deviation to the take-off schedule.

- We chose not to use the deviation to TTOT as an objective but it can easily be modeled as it is often used in the literature. We need to define new continuous variables $\delta'_i$ for every departure $i \in F_{dep}$ which will measure the deviation. The deviation to TTOTs is

$$\sum_{i \in F_{dep}} c_{TTOT}^i \delta'_i$$

where $c_{TTOT}^i \geq 0$. Following constraints are necessary to ensure the definition of variables $\delta'_i$.

$$\delta'_i \geq t_{id_i} - TTOT_i \quad \forall i \in F_{dep}$$
$$\delta'_i \geq TTOT_i - t_{id_i} \quad \forall i \in F_{dep}$$
$$\delta'_i \geq 0 \quad \forall i \in F_{dep}$$

References


