Decomposition and learning for a hard real time task allocation problem

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Abstract. We present a cooperation technique using an accurate management of nogoods to solve a hard real-time problem which consists in assigning periodic tasks to processors in the context of fixed priorities preemptive scheduling. The problem is to be solved off-line and our solving strategy is related to the logic based Benders decomposition. A master problem is solved using constraint programming whereas subproblems are solved with schedulability analysis techniques coupled with an *ad hoc* nogood computation algorithm. Constraints and nogoods are learnt during the process and play a role close to Benders cuts.

1 Introduction

Real-time systems are at the heart of embedded systems and have applications in many industrial areas: telecommunication, automotive, aircraft and robotics systems, etc. Today, applications (*e.g.* cars) involve many processors to serve different demands (cruise control, ABS, engine management, etc.). These systems are made of specialized and distributed processors (interconnected through a network) which receive data from sensors, process appropriate answers and send it to actuators. Their main characteristics lie in functional as well as non-functional requirements like physical distribution of the resources and timing constraints. Timing constraints are usually specified as deadlines for tasks which have to be executed. Serious damage can occur if deadlines are not met. In this case, the system is called a *hard* real-time system and timing predictability is required. In this field, some related works are based on off-line analysis techniques that compute the response time of the constrained tasks. Such techniques have been initiated by Liu and al. [15] and consist in computing the worst-case scenario of execution. Extensions have been introduced later to take into account shared resources, distributed systems [23] or precedence constraints [7].

Our problem consists in assigning periodic and preemptive tasks with fixed priorities (a task is periodically activated and can be preempted by a higher

priority task) to distributed processors. A solution is an allocation of the tasks on the processors which meets the schedulability requirements. The problem of assigning a set of hard preemptive real-time tasks in a distributed system is NP-Hard [14]. It has been tackled with heuristic methods [6, 17], simulated annealing [22, 4] and genetic algorithms [6, 19]. However, these techniques are often incomplete and can fail in finding any feasible assignment even after a large computation time. New practical approaches are still needed.

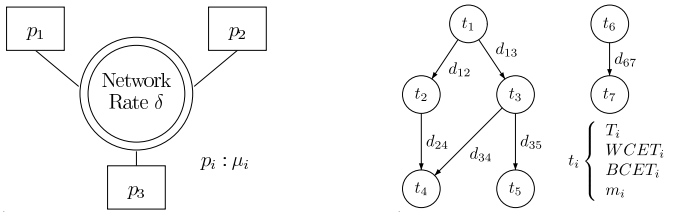
We propose here a decomposition based method which separates the allocation problem itself from the scheduling one. It is related to the Benders decomposition and especially to the logic Benders based decomposition. On the one hand, constraint programming offers competitive tools to solve the assignment problem, on the other hand, real-time scheduling techniques are able to achieve an accurate analysis of the schedulability. Our method uses Benders decomposition as a way of generating precise nogoods in constraint programming.

This paper is organized as follows: Section 2 introduces the problem. Related work and solving strategies are discussed in Section 3. The logical Benders decomposition scheme is briefly introduced and the links with our approach are put forward. Section 4 is dedicated to the master/subproblems and communication between them thanks to nogoods. Experimental results are presented in Section 5 and finally, a discussion of the technique is made in Section 6.

2 Problem description

2.1 The real-time system architecture

The hard real-time system we consider can be modeled with a software architecture (the set of tasks) and a hardware architecture (the physical execution platform for the tasks). Such a model is used by Tindell [22].



(a) Hardware architecture (b) Software architecture

Fig. 1. Main parameters of the problem

Hardware architecture. The hardware architecture is made of a set $\mathcal{P} = \{p_1, \ldots, p_k, \ldots, p_m\}$ of m identical processors with a fixed memory capacity μ_k, connected to a network. All the processors from $\mathcal{P}$ have the same processing speed. They are connected to a network with a transit rate of δ and a token ring protocol. A token travels around the ring allowing processors to send data only if they hold the token. It stays at the same place during a fixed maximum period of time large enough to ensure all messages waiting on processors are sent.

Software architecture. To model the software architecture, we consider a valued, oriented and acyclic graph $(\mathcal{T}, \mathcal{C})$. The set of nodes $\mathcal{T} = \{t_1, ..., t_n\}$ corresponds to the tasks whereas the set of edges $\mathcal{C} \subseteq \mathcal{T} \times \mathcal{T}$ refers to message sending between tasks.

A task t_i is defined by its temporal characteristics and resource needs: its period, T_i (a task is periodically activated); its worst-case execution time without preemption, $WCET_i$ and its memory need, m_i. Edges $c_{ij} = (t_i, t_j) \in \mathcal{C}$ are valued with the amount of exchanged data: d_{ij}. Communicating tasks have the same activation period. Moreover, they are able to communicate in two ways: a local communication with no delay using the memory of the processor (requiring the tasks to be located on the same processor) and a distant communication using the network. In any case, we do not consider precedence constraints. Tasks are periodically activated in an independent way, they read and write data at the beginning and the end of their execution.

Finally, each processor is scheduled with a fixed priority strategy. A priority, $prio_i = i$ is given to each task. t_j has priority over t_i if and only if $prio_j < prio_i$ and a task execution may be pre-empted by higher priority tasks.

2.2 The allocation problem

An allocation is an application $A : \mathcal{T} \rightarrow \mathcal{P}$ mapping a task t_i to a processor p_k:

$$t_i \mapsto A(t_i) = p_k \tag{1}$$

The allocation problem consists in finding the application A which respects the constraints described below.

Timing constraints. They are expressed by the means of deadlines for the tasks. Timing constraints enforces the duration between the activation date of any instance of the task t_i and its completion time to be bounded by its deadline D_i (the constraint on D_i is detailed in 4.2).

Resource constraints. Three kinds of constraints are considered:

- **Memory capacity**: The memory use of a processor p_k cannot not exceed its capacity (μ_k):

$$\forall k = 1..m, \sum_{A(t_i)=p_k} m_i \leqslant \mu_k \tag{2}$$

- **Utilization factor**: The utilization factor of a processor cannot exceed its processing capacity. The ratio $r_i = WCET_i/T_i$ means that a processor is used $r_i\%$ of the time by the task t_i. The following inequality is a simple necessary condition of schedulability:

$$\forall k = 1..m, \sum_{A(t_i)=p_k} WCET_i/T_i \leqslant 1 \tag{3}$$

– **Network use**: To avoid overload, the amount of data carried along the network per unit of time cannot exceed the network capacity:

$$\sum_{\substack{c_{ij} = (t_i, t_j) \\ A(t_i) \neq A(t_j)}} d_{ij}/T_i \leqslant \delta \tag{4}$$

Allocation constraints. Allocation constraints are due to the system architecture. We distinguish three kinds of constraints: residence, co-residence and exclusion.

– **Residence**: A task sometimes needs a specific hardware or software resource which is only available on specific processors (*e.g.* a task monitoring a sensor has to run on a processor connected to the input peripheral). It is a couple (t_i, α) where $t_i \in \mathcal{T}$ is a task and $\alpha \subseteq \mathcal{P}$ is the set of available processors for the task. A given allocation A must respect:

$$A(t_i) \in \alpha \tag{5}$$

– **Co-residence**: This constraint enforces several tasks to be placed on the same processor (they share a common resource). Such a constraint is defined by a set of tasks $\beta \subseteq \mathcal{T}$ and any allocation A has to fulfil:

$$\forall (t_i, t_j) \in \beta^2, A(t_i) = A(t_j) \tag{6}$$

– **Exclusion**: Some tasks may be replicated for fault tolerance and therefore cannot be assigned to the same processor. It corresponds to a set $\gamma \subseteq \mathcal{T}$ of tasks which cannot be placed together. An allocation A must satisfy:

$$\forall (t_i, t_j) \in \gamma^2, A(t_i) \neq A(t_j) \tag{7}$$

An allocation is said to be *valid* if it satisfies allocation and resource constraints. It is said to be *schedulable* if it satisfies timing constraints. A solution for our problem is a valid and schedulable allocation of the tasks.

3 About related decomposition approaches

Our approach is based to a certain extent on a Benders decomposition [2] scheme. We will therefore introduce it to highlight the underlying concepts. Benders decomposition can be seen as a form of *learning from mistakes*. It is a solving strategy that uses a partition of the problem among its variables: x, y. The strategy can be applied to a problem of this general form:

$$\begin{aligned} &\text{P} : \text{Min } f(x) + cy \\ &\text{s.t} : g(x) + Ay \geq a \text{ with} : x \in D, y \geq 0 \end{aligned}$$

A master problem considers only a subset of variables x (often integer variables, D is a discrete domain). A subproblem (SP) tries to complete the assignment on y and produces a Benders cut added to the master problem. This cut

has the form $z \geq h(x)$ and constitutes the key point of the method, it is inferred by the dual of the subproblem. Let us consider an assignment x^* given by the master, the subproblem (SP) and its dual (DSP) can be written as follows:

$$\begin{array}{ll} \text{SP} : \text{Min } cy & \text{DSP} : \text{Max } u(a - g(x^*)) \\ \text{s.t } Ay \geq a - g(x^*) \text{ with} : y \geq 0 & \text{s.t } uA \leq c \text{ with} : u \geq 0 \end{array}$$

Duality theory ensures that $cy \geq u(a - g(x^*))$. As feasibility of the dual is independent of x^*, $cy \geq u(a - g(x))$ and the following inequality is valid: $f(x) + cy \geq f(x) + u(a - g(x))$. Moreover, according to duality, the optimal value of u^* maximizing $u(a - g(x^*))$ corresponds to the same optimal value of cy. Even if the cut is derived from a particular x^*, it is valid for all x and excludes a large class of assignments which share common characteristics that make them inconsistent. The number of solutions to explore is reduced and the master problem can be written at the I^{th} iteration:

$$\begin{array}{ll} \text{PM} : \text{Min } z & \\ \text{s.t} : z \geq f(x) + u_i^*(a - g(x)) & \forall i < I \end{array}$$

From all of this, it can be noticed that dual variables need to be defined to apply the decomposition. However, [8] proposes to overcome this limit and to enlarge the classical notion of *dual* by introducing an *inference dual* available for all kinds of subproblems. He refers to a more general scheme and suggests a different way of thinking about duality: a Benders decomposition based on *logic*. Duality now means to be able to produce a proof, the logical proof of optimality of the subproblem and the correctness of inferred cuts. In the original Benders decomposition, this proof is established thanks to duality theorems.

For a discrete satisfaction problem, the resolution of the dual consists in computing the infeasibility proof of the subproblem and determining under what conditions the proof remains valid. It therefore infers valid cuts.
The success of the decomposition depends on both the degree to which decomposition can exploit structures and the quality of the cuts inferred. [8] suggests to identify classes of structured problems that exhibit useful characteristics for the Benders decomposition. Off-line scheduling problems fall into such classes and [10] demonstrates the efficiency of such an approach on a scheduling problem with dissimilar parallel machines.

Our approach is strongly connected to Benders decomposition and the related concepts. It is inspired from methods used to integrate constraint programming into a Benders scheme [21, 3]. The allocation and ressource problem will be considered on one side and schedulability on the other side. The subproblem checks the schedulability of an allocation, finds out why it is unschedulable and design a set of constraints (both symbolic and arithmetic) which rule out all assignments that are unschedulable for the same reason. Our approach concurs therefore the Benders decomposition on this central element: the Benders cut. The proof proposed here is based on off-line analysis techniques from real-time scheduling. One might think that a fast analytic proof could not provide enough relevant information on the inconsistency. As the speed of convergence and the

success of the technique greatly depends on the quality of the cut, a conflict detection algorithm will be coupled with analytic techniques: QuickXplain [11]. Moreover, the master problem will be considered as a dynamic problem to avoid redundant computations as much as possible.

4 Solving strategy

The solving process requires a tight cooperation between master and subproblem(s). Both problems share a common model introduced in the next section in order to easily exchange nogoods. They will be presented before examining the cooperation mechanisms and the incremental resolution of the master.

4.1 Master problem

The master problem is solved using constraint programming techniques. The model is based on a redundant formulation using three kinds of variables: x, y, w. At first, let us consider n integer variables x (our decision variables) corresponding to each task and representing the processor selected to process the task: $\forall i \in \{1..n\},\ x_i \in [1..m]$. Secondly, boolean variables y indicate the presence of a task onto a processor: $\forall i \in \{1..n\}, \forall p \in \{1..m\},\ y_{ip} \in \{0,1\}$. Finally, boolean variables w are introduced to express the fact that a pair of tasks exchanging a message are located on the same processor or not: $\forall c_{ij} = (t_i, t_j) \in \mathcal{C},\ w_{ij} \in \{0,1\}$. Integrity constraints (*channeling constraints*) are used to enforce the consistency of the redundant model. Links between x, y and w are made using *element* constraints. One of the main objectives of the master problem is to efficiently solve the assignment part. It handles two kinds of constraints: allocation and resources.

- **Residence (*cf.* eq (5)):** it consists of forbidden values for x. A constraint is added for each forbidden processor p of t_i: $x_i \neq p$
- **Co-residence (*cf.* eq (6)):** $\forall (t_i, t_j) \in \beta^2, x_i = x_j$
- **Exclusion (*cf.* eq (7)):** $alldifferent(x_i | t_i \in \gamma)$
- **Memory capacity (*cf.* eq (2)):** $\forall p \in \{1..m\},\ \sum_{i \in \{1..n\}} y_{ip} \times m_i \leq \mu_p$
- **Utilization factor (*cf.* eq (3)):** Let *lcm(T)* be the least common multiple of periods of the tasks. The constraint can be written as follows:

$$\forall p \in \{1..m\}, \qquad \sum_{i \in \{1..n\}} lcm(T) \times WCET_i \times y_{ip}/T_i \leq lcm(T)$$

- **Network use (*cf.* eq (4)):** The network capacity is bounded by δ. Therefore, the size of the set of messages carried on the network cannot exceed this limit:

$$\sum_{i \in \{1..n\}} lcm(T) \times d_{ij} \times w_{ij}/T_i \leq lcm(T) \times \delta$$

Utilization factor and network use are reformulated with the *lcm* of tasks periods because our constraint solver cannot currently handle constraints with real coefficients and integer variables.

4.2 Subproblem(s)

An assignment provided by the master problem is a valid allocation of tasks. The problem is here to rule on its schedulability to determine why it may be unschedulable.

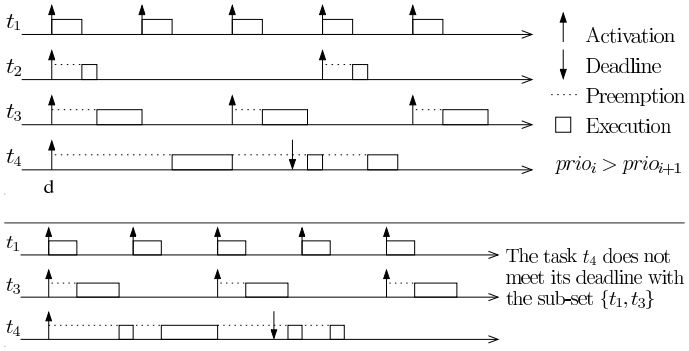


Fig. 2. Illustration of a schedulability analysis. The task t_4 does not meet its deadline. The sub-set $\{t_1, t_3, t_4\}$ is identified to explain the unschedulability of the system.

Independent tasks. The first schedulability analysis has been initiated by Liu and Layland [15] for mono-processor real-time systems with independent and fixed priority tasks. The analysis consists in computing for each task t_i its worst response time, $WCRT_i$. The aim is to build the worst execution scenario which penalizes as much as possible the execution of t_i.
For independent tasks, it has been proved that the worst execution scenario for a task t_i happens when all tasks with a higher priority are awoken simultaneously (date d on Figure 2). The worst-case response time of t_i is:

$$WCRT_i = WCET_i + \sum_{t_j \in hp(A,t_i)} \lceil WCRT_i / T_j \rceil \, WCET_j \tag{8}$$

$hp(A, t_i)$ corresponds to the set of tasks with a higher priority than t_i and located on the processor $A(t_i)$ for a given allocation A. $WCRT_i$ is easily obtained by looking for the fix-point of equation (8). Then, it is sufficient to compare for each task its worst case response time with its deadline D_i to know if the system is schedulable. In this case, the deadline of a task is equal to its period ($D_i = T_i$).

Communicating tasks on a token ring. The result computed by a task must be made available before its next period to ensure regular data refreshment between tasks. The messages must reach their destination within the time allowed. With the token ring protocol, the maximum delay of transmission on the network is bounded and the TRT is proved to be an upper bound. This duration is computed by taking into account all the messages to be sent on the network:

$$TRT = \sum_{\substack{\{c_{ij} = (t_i, t_j) | \\ A(t_i) \neq A(t_j)\}}} d_{ij}/\delta \tag{9}$$

The deadline for tasks sending data to non co-located tasks becomes $D_i = T_i - TRT$. A sufficient condition of scheduling is written:

$$\forall i = 1..n, WCET_i + \sum_{t_j \in hp(A,t_i)} \lceil D_i/T_j \rceil \, WCET_j \leqslant D_i \tag{10}$$

4.3 Cooperation between master and subproblem(s)

A complete or partial assignment of variables x, y, w will be now considered. The key point is to find an accurate explanation that encompasses all values of x for which the infeasibility proof (obtained for particular values of x) remains valid. We know at least that the current assignment is contradictory, in other words, a *nogood* is identified. The links between the concept of *nogood* [20] introduced in constraint programming and the Benders cut are underlined in [9].

Independent tasks. m independent subproblems for each processor are solved. The schedulability of a processor k is established by applying equation (8) to each task t_i located on k ($x_i = k$) in a descendent order of priority until a contradiction occurs. For instance, in Figure 2, the set (t_1, t_2, t_3, t_4) is unschedulable. It explains the inconsistency but is not minimal. However the set (t_1, t_3, t_4) is sufficient to justify the contradiction. In order to compute more precise explanations (*i.e.* achieve a more relevant learning), a conflict algorithm, *QuickXplain* [11], has been used to determine the minimal involved set of tasks (*w.r.t.* inclusion). The *propagation* algorithm considered here is equation (8). Tasks are added from t_1 until a contradiction occurs on t_c, the last added task t_c belongs to the minimal conflict c. The algorithm re-starts by initially adding the tasks involved in c. When c is inconsistent, it represents the minimal conflict among the initial set $(t_1, \ldots, t_c)$. The subset of tasks $T \subset \mathcal{T}$ corresponds to a *NotAllEqual*[3] on x:

$$NotAllEqual(x_i | t_i \in T)$$

It is worth noting that the constraint could be expressed as a linear combination of variables y. However, *NotAllEqual(x_1,x_3,x_4)* excludes the solutions that contain the tasks 1,2,3 gathered on *any* processor.

Communicating tasks on a token ring. The difficulty is to avoid incriminating the whole system:

1. At first, the network is simply not considered. If a processor is unschedulable without taking additional latency times due to the exchange of messages, it is still true in the general case. We can again infer: $NotAllEqual(x_i | t_i \in T)$.

[3] A *NotAllEqual* on a set V of variables ensures that at least two variables among V take distinct values.

2. Secondly, we only consider the network. When the sending tasks have a period less than TRT, the token does not come back early enough to allow the end of their execution. In this case, equation (10) will never be satisfied. A set of inconsistent messages $M \subset \mathcal{C}$ is obtained:

$$\sum_{c_{ij} \in M} w_{ij} < |M|$$

3. The last test consists in checking equation (10). A failure returns a set $T \subseteq \mathcal{T}$ of tasks which is inconsistent with a set of messages $M \subseteq \mathcal{C}$. It corresponds to a *nogood*. We use a specific constraint to take advantage of symmetries and to forbid this assignment as well as permutations of tasks among processors. It can be written as a disjunction between the two previous cuts:

$$nogood(x_i|t_i \in T, w_{ij}|c_{ij} \in M) = NotAllEqual(x_i|t_i \in T) \bigvee \sum_{c_{ij} \in M} w_{ij} < |M|$$

QuickXplain has been used again to refine information given in point 2 and 3. Let us now continue with the question of how information learnt from the previous failures can be integrated efficiently ? [21] outlines this problem and notices a possible significant overhead with redundant calculations. To address this issue, we considered the master problem as a dynamic problem.

Incremental resolution. Solving dynamic constraint problems has led to different approaches. Two main classes of methods can be distinguished: proactive and reactive methods. On the one hand, proactive methods propose to build robust solutions that remain solutions even if changes occur. On the other hand, reactive methods try to reuse as much as possible previous reasonings and solutions found in the past. They avoid restarting from scratch and can be seen as a form of learning. One of the main methods currently used to perform such learning is a justification technique that keeps trace of inferences made by the solver during the search. Such an extension of constraint programming has been recently introduced [12]: explanation-based constraint programming (*e-constraints*).

Definition 1 *An explanation records information to justify a decision of the solver as a reduction of domain or a contradiction. It is made of a set of constraints C' (a subset of the original constraints of the problem) and a set of decisions dc_1, ..., dc_n taken during search. An explanation of the removal of value a from variable v will be written: $C' \wedge dc_1 \wedge dc_2 \wedge \cdots \wedge dc_n \Rightarrow v \neq a$.*

When a domain is emptied, a contradiction is identified. An explanation for this contradiction is computed by uniting each explanation of each removal of value of the variable concerned. At this point, dynamic backtracking algorithms that only question a relevant decision appearing in the conflict are conceivable. By keeping in memory a relevant part of the explanations involved in conflicts, a learning mechanism can be implemented [13].

Here, explanations allow us to perform an incremental resolution of the master problem. At each iteration, the constraints added by the subproblem generate a contradiction. Instead of backtracking to the last choice point as usual, the current solution of the master problem is *repaired* by removing the decisions that occur in the contradiction as done by the MAC-DBT algorithm [12]. Tasks assigned at the beginning of the search can be moved without disturbing the whole allocation. In addition, the model reinforcement phase tries to transform a learnt set of elementary constraints that have been added at previous iterations into higher level constraints. Explanations offer facilities to easily dynamically add or remove a constraint from the constraint network [12].

Notice that the master problem is never re-started. It is solved only once but is gradually *repaired* using the dynamic abilities of the explanation-based solver.

Model reinforcement. Pattern recognition among a set of constraints that expresses specific subproblems is a critical aspect of the modelisation step. Constraint learning deals with the problem of automatically recognizing such patterns. We would like to perform a similar process in order to extract global constraints among a set of elementary constraints. For instance, a set of difference constraints can be formulated as an all-different constraint by looking for a maximal clique in the induced constraint graph. It is a well-known issue to this question in constraint programming and a version of the Bron/Kerbosh algorithm [5] has been implemented to this end (difference constraints occur when $NotAllEquals$ involve only two tasks). In a similar way, a set of $NotAllEqual$ constraints can be expressed by a *global cardinality constraint* (gcc) [18]. It corresponds now to a maximal clique in a hypergraph (where hyperarcs between tasks are $NotAllEquals$). However, it is still for us an open question that could significantly improve performances.

5 First experimental results

For the allocation problem, specific benchmarks are not provided in real-time scheduling. Experiments are usually done on didactic examples [22, 1] or randomly generated configurations [17, 16]. We opted for this last solution. Our generator takes several parameters into account:

- n, m, mes: the number of tasks, processors (experiments have been done on a fixed size: $n = 40$ and $m = 7$) and messages;
- $\%_{global}$: the global utilization factor of processors;
- $\%_{mem}$: the over-capacity memory, *i.e.* the amount of additionnal memory avalaible on processors with respect to the memory needs of all tasks;
- $\%_{res}$: the percentage of tasks included in residence constraints;
- $\%_{co-res}$: the percentage of tasks included in co-residence constraints;
- $\%_{exc}$: the percentage of tasks included in exclusion constraints;
- $\%_{msize}$: the size of a message is evaluated as a percentage of the period of the tasks exchanging it.

Task periods and priorities are randomly generated. However, worst-case execution time are initially randomly chosen and evaluated again to respect: $\sum_{i=1}^{n} WCET_i/T_i = m\%_{global}$. The memory need of a task is proportional to its worst-case execution time. Memory capacities are randomly generated but must satisfy: $\sum_{k=1}^{m} \mu_k = (1 + \%_{mem}) \sum_{i=1}^{n} m_i$.

The number of tasks involved in allocation constraints is given by the parameters $\%_{res}$, $\%_{co-res}$, $\%_{exc}$. Tasks are randomly chosen and their number (involved in co-residence and exclusion constraints) can be set through specific levels. Several classes of problems have been defined depending on the difficulty of both allocation and schedulability problems. The difficulty of schedulability is evaluated using the global usage factor $\%_{global}$ which varies from 40 to 90 %. Allocation difficulty is based on the number of tasks included in residence, co-residence and exclusion constraints ($\%_{res}$, $\%_{co-res}$, $\%_{exc}$). Moreover, the memory over-capacity, $\%_{mem}$ has a significant impact (a very low capacity can lead to solve a *packing* problem, sometimes very difficult). The presence of messages impacts on both problems and the difficulty has been characterized by the ratios mes/n and $\%_{msize}$. As we consider precedence chains, we can not have more than one message per task and the ratio mes/n is always less than 1. $\%_{msize}$ reflects the impact of messages on schedulability analysis by linking periods and message sizes.

The table 1 describes the parameters and difficulty class of the considered problems. For instance, a class 2-1-4 indicates a problem with an allocation difficulty in class 2, a schedulability difficulty in class 1 and a network difficulty in class 4.

Table 1. Details on classes of difficulty

Alloc.	$\%_{mem}$	$\%_{res}$	$\%_{co-res}$	$\%_{exc}$	Sched.	$\%_{global}$	Mes.	mes/n	$\%_{msize}$
1	80	0	0	0	1	40	1	0.5	40
2	40	15	15	15	2	60	2	0.5	70
3	30	25	25	25	3	75	3	0.75	70
4	15	35	35	35	4	90	4	0.875	150

5.1 Independent tasks

Table 2 summarizes the results of our experiments. *Iter* is the number of iterations between master and subproblems, *NotAllEq* and *Diff* are the number of *NotAllEqual* and difference constraints inferred. *CPU* is the resolution time in seconds and *Xplain* expresses if the QuickXplain algorithm has been used. Finally % Success gives the number of instances successfully solved (a schedulable solution has been found or the proof of inconsistency has been done) within the time limit of 10 minutes per instance. The data are obtained in average (on instances solved within the required time) on 100 instances per class of difficulty with a pentium 4, 3 GigaHz and the Java version of PaLM [12].

Table 2. Average results on 100 instances randomly generated into classes of problems

Cat(Alloc/Sched)	Xplain	Iter	NotAllEq	Diff	CPU (s)	% Success
1-1	N	46,35	91,29	4,45	0,58	100%
1-1	Y	10,59	39,79	12,41	0,28	100%
1-2	Y	26,75	96,93	28,50	3,46	99%
1-3	Y	65,23	213,87	39,21	28,70	94%
1-4	**Y**	**100,88**	**373,08**	**57,82**	**93,40**	**40%**
2-2	Y	46,00	168,27	23,13	34,51	91%
2-3	Y	58,89	233,63	37,06	71,18	81%
3-4	Y	138,29	131,22	40,65	62,12	91%

The class 1-4 represents the hardest class of problem. Without the allocation problem, the initial search space is complete and everything has to be learnt. Moreover, these problems are close to inconsistency due to the hardness of the schedulability. Limits of our approach seem to be reached in such a case without an efficient re-modeling of *NotAllEquals* constraints into *gcc* (see 4.3). The cuts generated seem actually quite efficient. A relevant learning can be made in the case of independent tasks by solving m independent subproblems. Of course, if the symetry of the processors does not hold, this could be questionnable.
The execution of a particular and hard instance of class 2-3 is outlined on Figure 3. Resolution time and learnt constraints at each iteration are detailed. The master problem adapts the current solution to the cuts due to its dynamic abilities and the learning process is very quick at the beginning. The number of cuts decreases until a hard satisfaction problem is formulated (a-b in Fig. 3). The master is then forced to question a lot of choices to provide a valid allocation (b). The process starts again with a quick learning of nogoods (b-c, c-d).

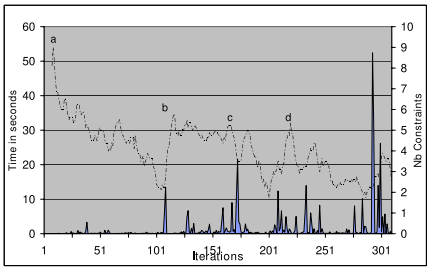


Fig. 3. Execution of a hard instance of class 2-3. Resolution time and a floating average of step 10 of the number of cuts (in dotlines) inferred at each iteration are shown. (310 iterations, 1192 *NotAllEqual*, 75 *differences* partially re-modeled into 12 *alldifferent*)

5.2 Communicating tasks on a token ring.

We chose to experiment the technique on a well-known instance of real-time scheduling: the Tindell instance [22], solved thanks to simulated annealing. This instance exhibits a particular structure: the network plays a critical part and feasible solutions have a network utilization almost minimal. We were forced to

specialize our generic approach on this particular point through the use of an allocation heuristic that try to gather tasks exchanging messages. One can obtain the solution of Tindell very quickly (less than 10 seconds) if minimizing the network at each iteration. Moreover, we experimented our approach on random problems involving messages:

Table 3. Average results on 100 instances randomly generated into classes of problems

Cat(A/S/M)	Iter	NotAllEq	Diff	NetCuts	Nogoods	CPU (s)	%Succ
2-1-1	34,7	47,7	24	23,5	8,6	24,7	98%
2-1-2	40,1	56,9	25,4	36,2	8,4	18,6	93%
2-1-3	**91,9**	**64,2**	**23,5**	**134,3**	**27,2**	**106,6**	**56%**
2-2-1	58,9	118,4	47,5	11,2	2,7	72,7	82%
2-2-2	55,3	116,5	46,9	45,2	9,2	60,5	74%
2-2-3	**77,6**	**97,3**	**39,1**	**96,2**	**43,1**	**142,1**	**38%**

One can see on the table 3 that when several hardness aspects compete on the problem, the difficulty increases (2-2-3 compared to 1-1-3). The presence of messages make the problem much more complex for our approach because independency of subproblems (a key point of Benders) is lost and the network cut is a weak one. Determining what tasks should be or not together becomes a difficult question when a tigth overall memory is combined to a difficult schedulability and a lot of medium size messages. However, simple heuristics approachs have received a lot of attention from the real-time community and could be used to guide the search efficiently in CP. We hope to achieve better results with a more efficient heuristic inspired from the best one designed in real-time systems and coupled with the learnt information of the cuts. More experiments have to be carried out to clearly establish the difficulty frontier.

6 Discussion on the approach

Our approach tries to use logic based Benders as a mean of generating relevant nogoods. It is not far from the hybrid framework *Branch and Check* of [21] which consists in checking the feasibility of a delayed part of the problem in a subproblem. In our case, the schedulability problem is gradually converted into the assignment problematic. The idea is that the first problem could be dealt with efficiently with constraint programming, and especially, with an efficient re-modeling process. In addition, it avoids thrashing on schedulability inconsistencies. As with explanation based algorithms (MAC-DBT or Decision-repair [13]), it tries to learn from its mistakes.

The technique is actually complete but it could be interesting to relax its completeness (from this point, we step back from Benders). One current problem is the overload of the propagation mechanism because of the accumulation of low power filtering constraints. We could use a tabu list of benders cuts and decide to keep permanently in memory the most accurate nogoods or only those contributing to a stronger model (a fine management of memory can be implemented due to dynamic abilities of the master problem).

One could also think building a filtering algorithm on equation (8). However, the objective is to show how precise nogoods could be used and to validate an approach we intend to implement on complex scheduling models. As analysis techniques quickly become very complex, a contradiction raised by a constraint encapsulating such an analysis seems to be less relevant than a precise explanation of failure.

The idea is to take advantage of the know-how of real-time scheduling community in a decomposition scheme such as the Benders one where constraint programming could efficiently solve the allocation problem.

7 Conclusion and future work

We propose in this paper, a decomposition method built to a certain extent on a logic Benders decomposition as a way of generating nogoods. It implements a *logical* duality to infer nogoods, tries to enforce the constraint model and finally performs an incremental resolution of the master problem. It is also strongly related to a class of algorithms which intends to learn from mistakes in a systematic way by managing nogoods.

For independent tasks, the use of QuickXplain is critical to speed up the convergence but the limits seem to be reached for highly constrained and inconsistent problems. Nevertheless, we believe that the difficulty can be overcome through an efficient re-modeling process. The use of an efficient heuristic to guide the CP search is needed on communicating tasks when several hardness aspect compete on the problem. As lot of traditionnal approaches in real time systems are based on heuristics, we hope to benefit from them and more experiments have to be carried out on this point.

Our next step would be to compare our approach with other methods such as traditional constraint and linear programming. We believe it should be also interesting to extend our study to other kinds of network protocols (CAN, TDMA, etc.) and precedence constraints. Moreover, another kind of constraints sometimes occur: disjunction between set of tasks. The disjunction global constraint has not been studied a lot and it could provide accurate modeling and solving tools to tackle the assignment problem with more complex allocation constraints.

Our approach gives a new answer to the problematic of real-time task allocation. It opens new perspectives on integrating techniques coming from a broader horizon than optimization, within CP in a Benders scheme.

References

1. Peter Altenbernd and Hans Hansson. The Slack Method: A New Method for Static Allocation of Hard Real-Time Tasks. *Real-Time Systems*, 15:103–130, 1998.
2. J. F. Benders. Partitionning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4:238–252, 1962.
3. T. Benoist, E. Gaudin, and B. Rottembourg. Constraint programming contribution to benders decomposition: A case study. In *CP'02*, pages 603–617, 2002.

4. G. Borriello and D. Miles. Task Scheduling for Real-Time Multiprocessor Simulations. *11th Workshop on RTOSS*, pages 70–73, 1994.
5. Coen Bron and Joep Kerbosch. Algorithm 457: finding all cliques of an undirected graph. *Commun. ACM*, 16(9):575–577, 1973.
6. E. Ferro, R. Cayssials, and J. Orozco. Tuning the Cost Function in a Genetic/Heuristic Approach to the Hard Real-Time Multitask-Multiprocessor Assignment Problem. *Proceeding of the Third World Multiconference on Systemics Cybernetics and Informatics*, pages 143–147, 1999.
7. M. González Harbour, M.H. Klein, and J.P. Lehoczky. Fixed Priority Scheduling of Periodic Tasks with Varying Execution Priority. *Proceeding of the IEEE Real-Time Systelms Symposium*, pages 116–128, December 1991.
8. J.N. Hooker and G. Ottosson. Logic-based benders decomposition. *Mathematical Programming*, 96:33–60, 2003.
9. J.N. Hooker, G. Ottosson, E. S. Thorsteinsson, and H. Kim. A scheme for unifying optimization and constraint satisfaction methods. *Knowledge Engineering Review, special issue on AI/OR*, 15(1):11–30, 2000.
10. Vipul Jain and I. E. Grossmann. Algorithms for hybrid milp/cp models for a class of optimization problems. *INFORMS Journal on Computing*, 13:258–276, 2001.
11. Ulrich Junker. Quickxplain: Conflict detection for arbitrary constraint propagation algorithms. In *IJCAI'01 Workshop on Modelling and Solving problems with constraints (CONS-1)*, Seattle, WA, USA, August 2001.
12. Narendra Jussien. The versatility of using explanations within constraint programming. RR 03-04-INFO, École des Mines de Nantes, France, 2003.
13. Narendra Jussien and Olivier Lhomme. Local search with constraint propagation and conflict-based heuristics. *Artificial Intelligence*, 139(1):21–45, July 2002.
14. E. L. Lawler. Recent Results in the Theory of Machine Scheduling. *Mathematical Programming: The State of the Art*, pages 202–233, 1983.
15. C. L. Liu and J. W. Layland. Scheduling Algorithms for Multiprogramming in a Hard-Real Time Environment. *Journal ACM*, 20(1):46–61, 1973.
16. Y. Monnier, J.-P. Beauvais, and A.-M. Déplanche. A Genetic Algorithm for Scheduling Tasks in a Real-Time Distributed System. *24th Euromicro Conference*, 2, 1998.
17. K. Ramamritham. Allocation and Scheduling of Complex Periodic Tasks. *10th International Conference on Distributed Computing Systems*, pages 108–115, 1990.
18. J.C. Régin. Generalized arc consistency for global cardinality constraint. *AAAI / IAAI*, pages 209–215, 1996.
19. F. E. Sandnes. A hybrid genetic algorithm applied to automatic parallel controller code generation. *8th IEEE Euromicro Workshop on Real-Time Systems*, 1996.
20. Thomas Schiex and Gérard Verfaillie. Nogood recording for static and dynamic constraint satisfaction problem. *IJAIT*, 3(2):187–207, 1994.
21. Erlendur S. Thorsteinsson. Branch-and-check: A hybrid framework integrating mixed integer programming and constraint logic programming. In *CP'01*, 2001.
22. K. Tindell, A. Burns, and A. Wellings. Allocation Hard Real-Time tasks: An NP-Hard Problem Made Easy. *The Journal of Real-Time Systems*, 4(2):145–165, 1992.
23. K. Tindell and J. Clark. Holistic scheduling Analysis for Distributed Hard Real-Time Systems. *Euromicro Journal*, pages 40–117, 1994.