# *Granularity of models, choice of domains*

## *Examples and application*

Hadrien Cambazard

*G-SCOP - Université de Grenoble*

# Outline

1. Choice of domains, *granularity* of the model

2. Illustration on a *toy problem*
   – Pentaminoes

3. Illustration on a real application: multi-leaf sequencing
   – Direct model
   – Counter model
   – Path model

4. Stepping back, looking for a generic answer
   – Set variables
   – MDD consistency

5. Conclusion

# 1- Choice of domains

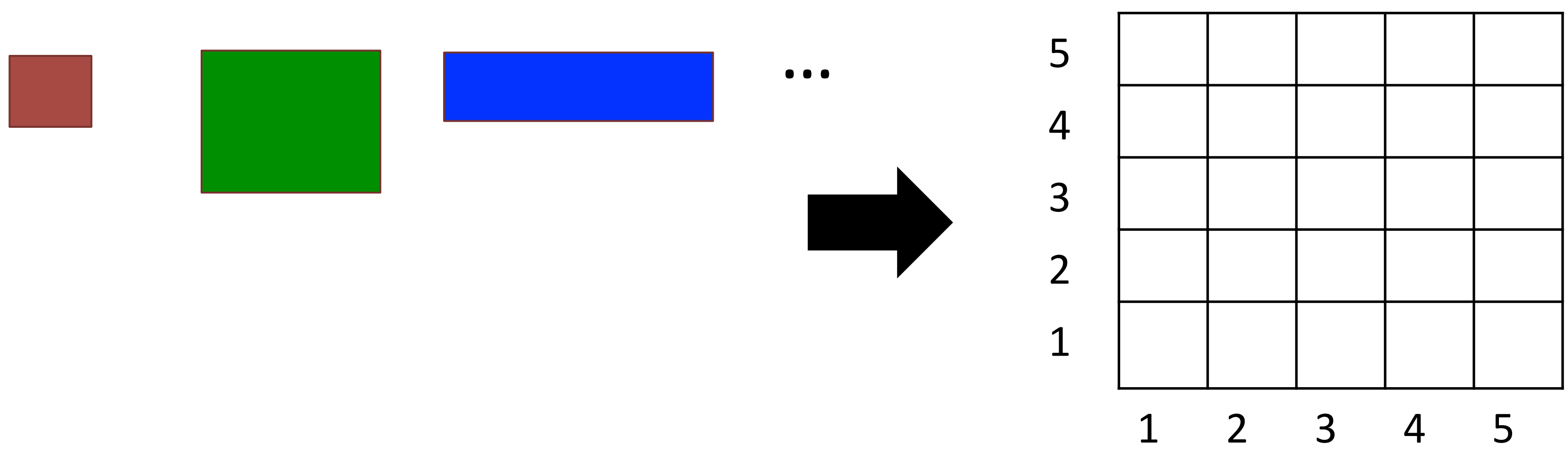
- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning

# 1- Choice of domains

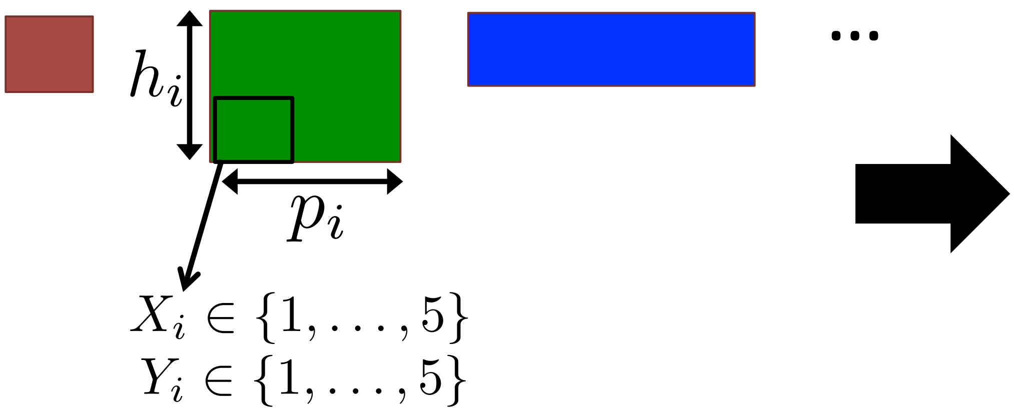
- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning

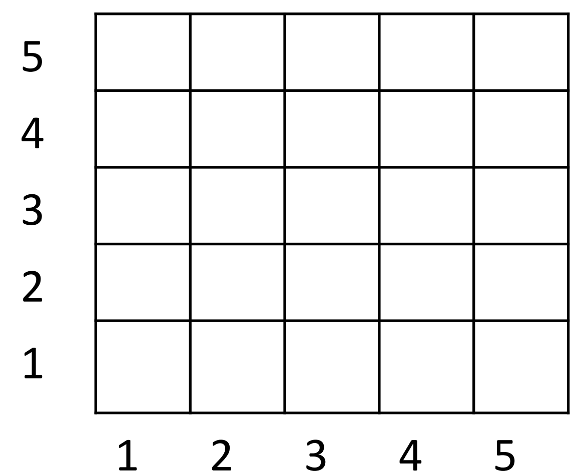



- Let's model a simple packing/placement problem

# 1- Choice of domains

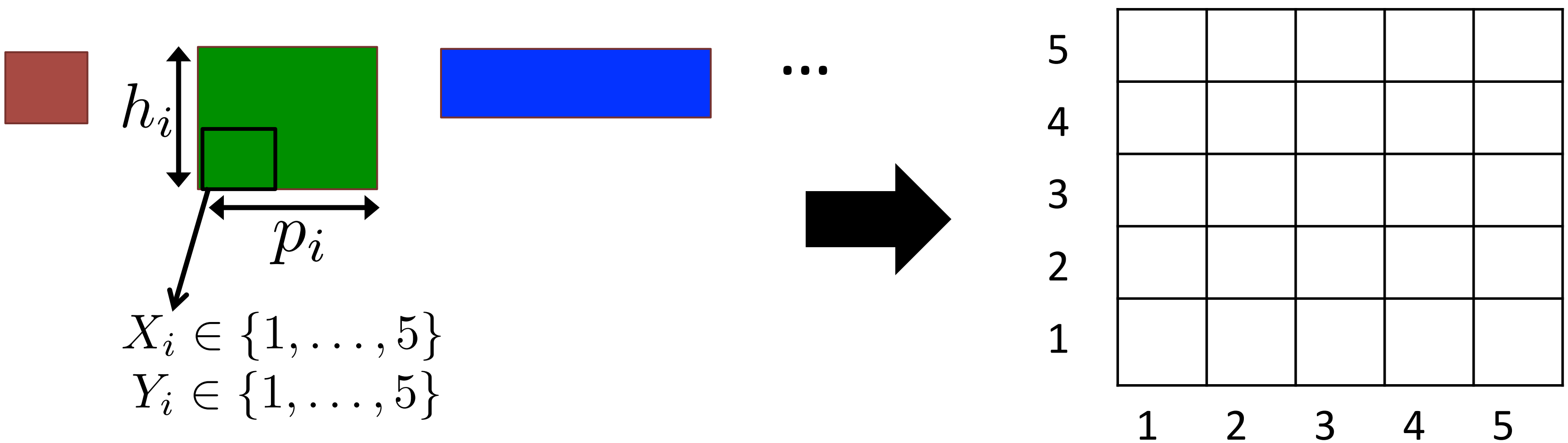
- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning



# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning

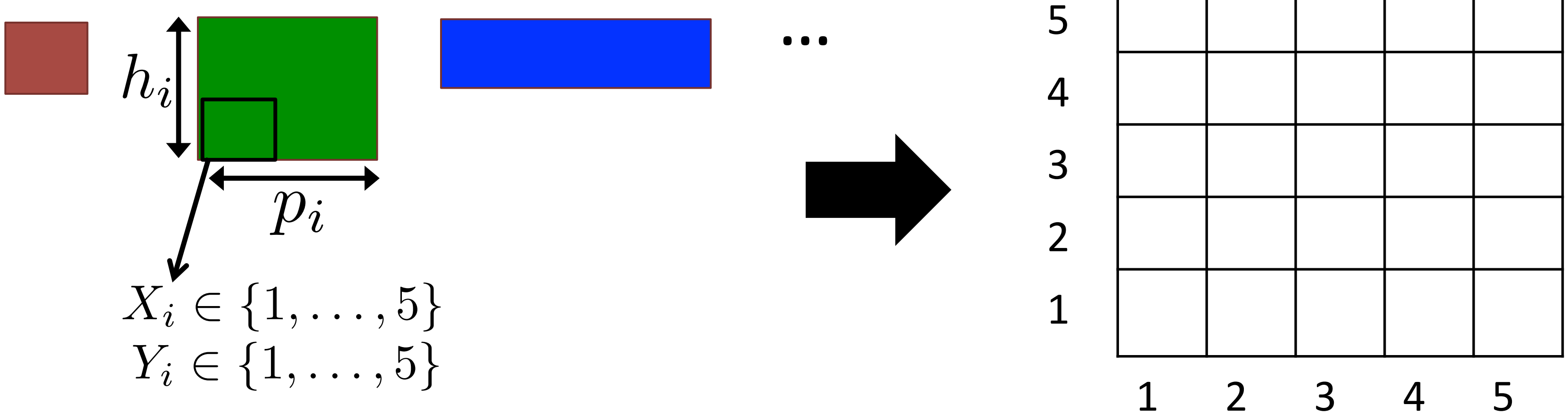



$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j — i above j — i left of j — i right of j

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of C ∩ D on the domain representation
- Choice of domains = choice of possible reasoning



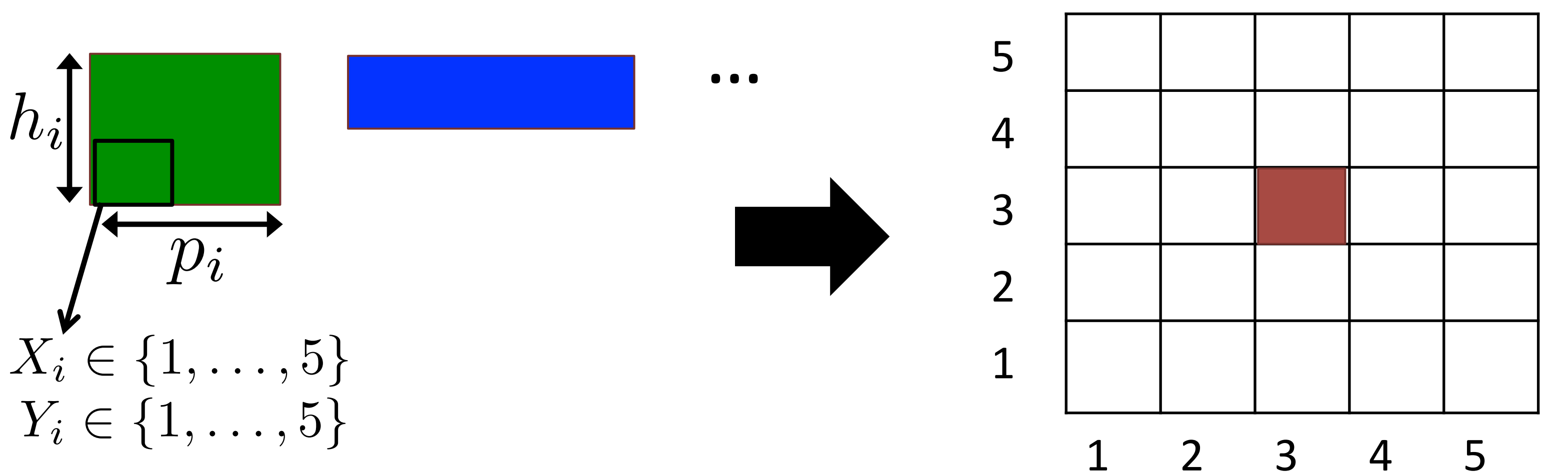

$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j | i above j | i left of j | i right of j

➡ Geost

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of C $\cap$ D on the domain representation
- Choice of domains = choice of possible reasoning



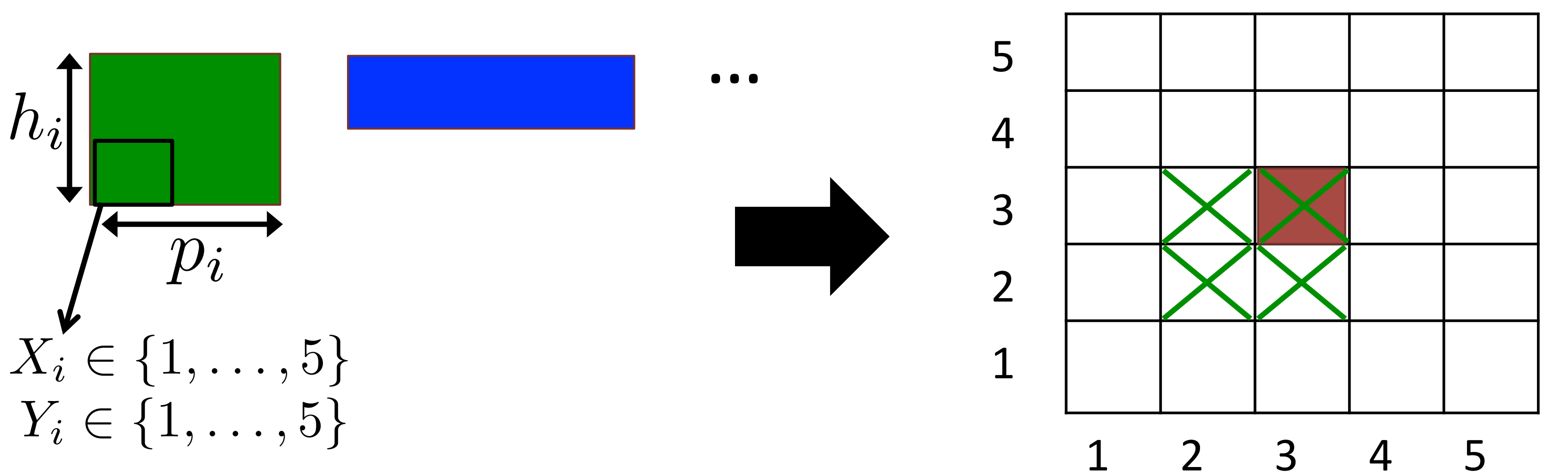

$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j | i above j | i left of j | i right of j

→ GEOST

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning



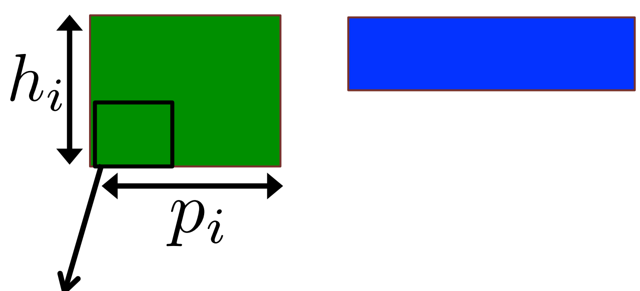

$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j | i above j | i left of j | i right of j

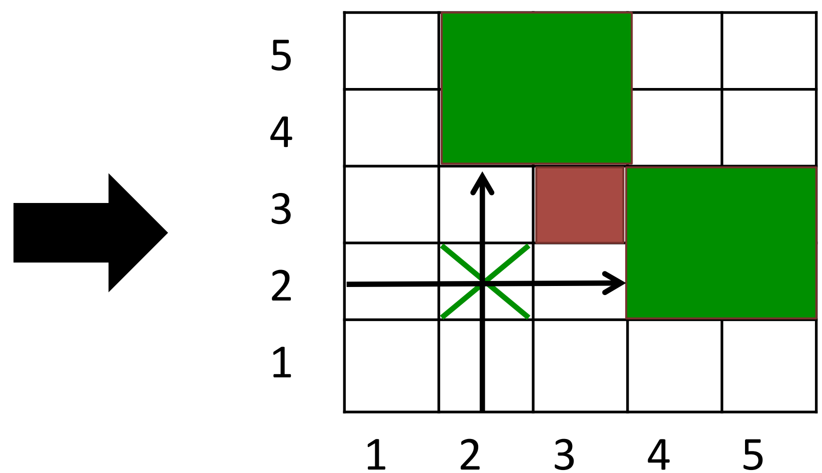
→ GEOST

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning




$X_i \in \{1, \ldots, 5\}$
$Y_i \in \{1, \ldots, 5\}$



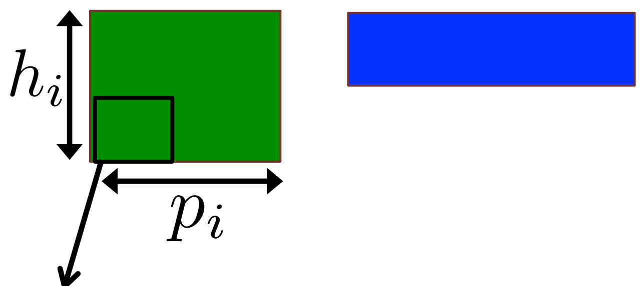

$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j — i above j — i left of j — i right of j

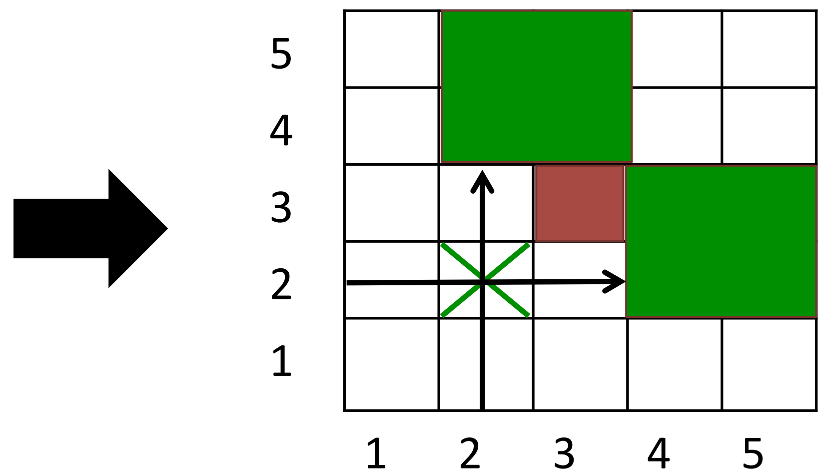
→ GEOST

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning




$X_i \in \{1, \ldots, 5\}$
$Y_i \in \{1, \ldots, 5\}$



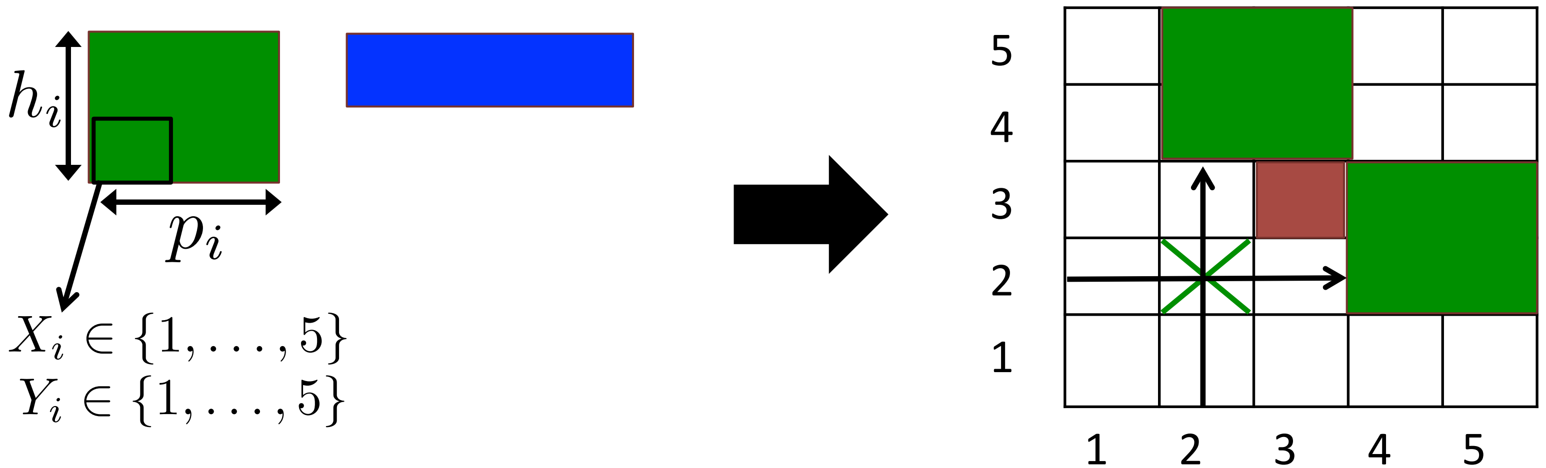

$$(Y_i + h_i \leq Y_j) \vee (Y_j + h_j \leq Y_i) \vee (X_i + p_i \leq X_j) \vee (X_j + p_j \leq X_i)$$

i below j — i above j — i left of j — i right of j

This reasoning can not be expressed in the domains

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning




- This reasoning can not be expressed in the domains
- GEOST probably knows it but can't communicate it to the rest of the model

# 1- Choice of domains

- Given a constraint C and a domain D
  - Filtering = project the relation of $C \cap D$ on the domain representation
- Choice of domains = choice of possible reasoning
- Question variables and domains to find out the proper *granularity* of information that should be represented
  - *A CP model reveals key combinatorial structures*
  - *Identify structures then decide variables-domains ?*

# Outline

1. Choice of domains, *granularity* of the model

2. Illustration on a *toy problem*
   – Pentaminoes

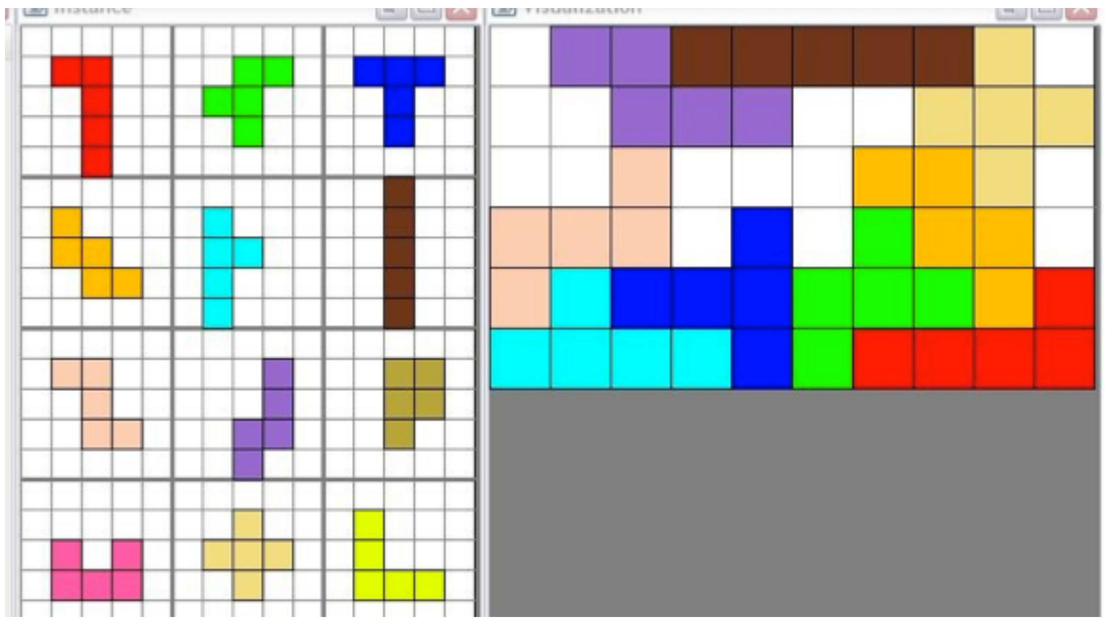
3. Illustration on a real application: multi-leaf sequencing
   – Direct model
   – Counter model
   – Path model

4. Stepping back, looking for a generic answer
   – Set variables
   – MDD consistency

5. Conclusion

# 2- Example of Pentaminoes

- *Modeling Irregular Shape placements problems with regular constraints*, Mikael Z. Lagerkvist, Gilles Pesant 2008.
- Example of Pentaminoes:



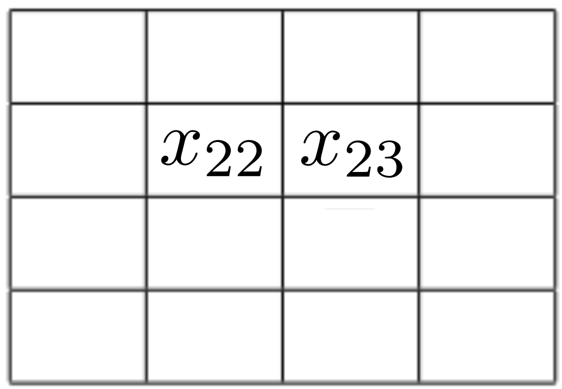
- All shapes you can create with five connected unit squares

# 2- Example of Pentaminoes

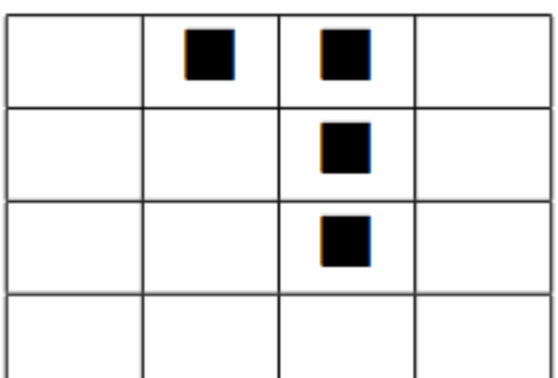
- Since shapes are irregular, “holes” might matter more.
- Let’s consider Boolean domains for a given shape:

# 2- Example of Pentaminoes

- Since shapes are irregular, “holes” might matter more.
- Let’s consider Boolean domains for a given shape:



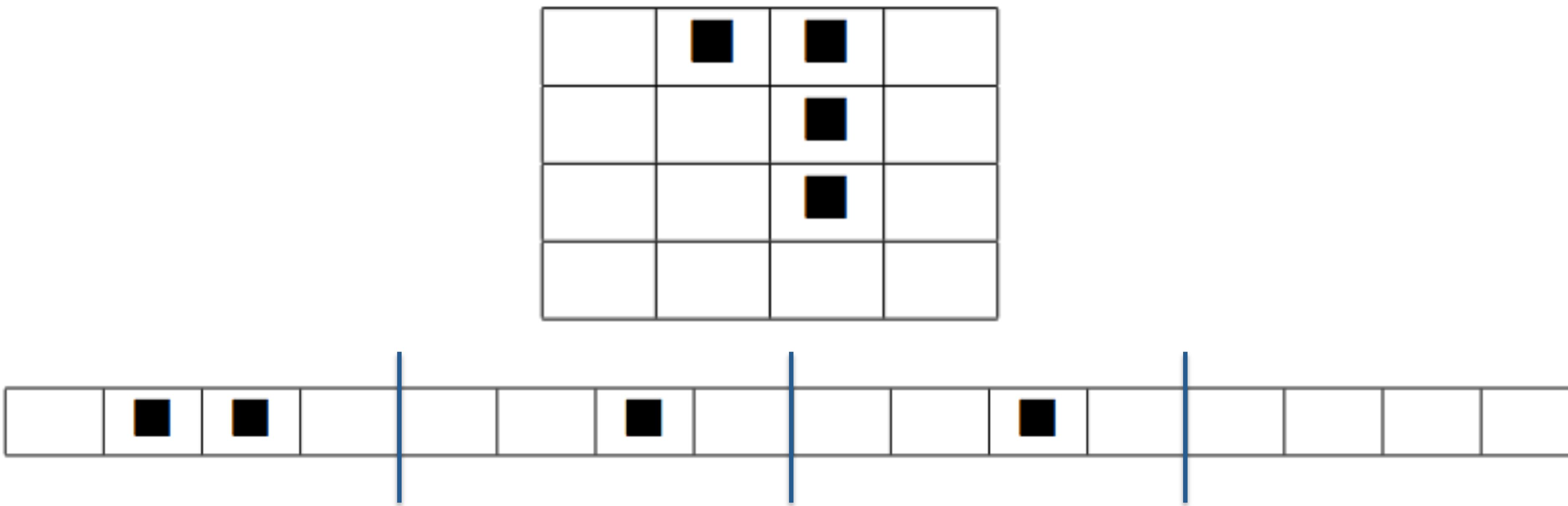

$$x_{ij} \in \{0, 1\}$$



| | | | |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

# 2- Example of Pentaminoes

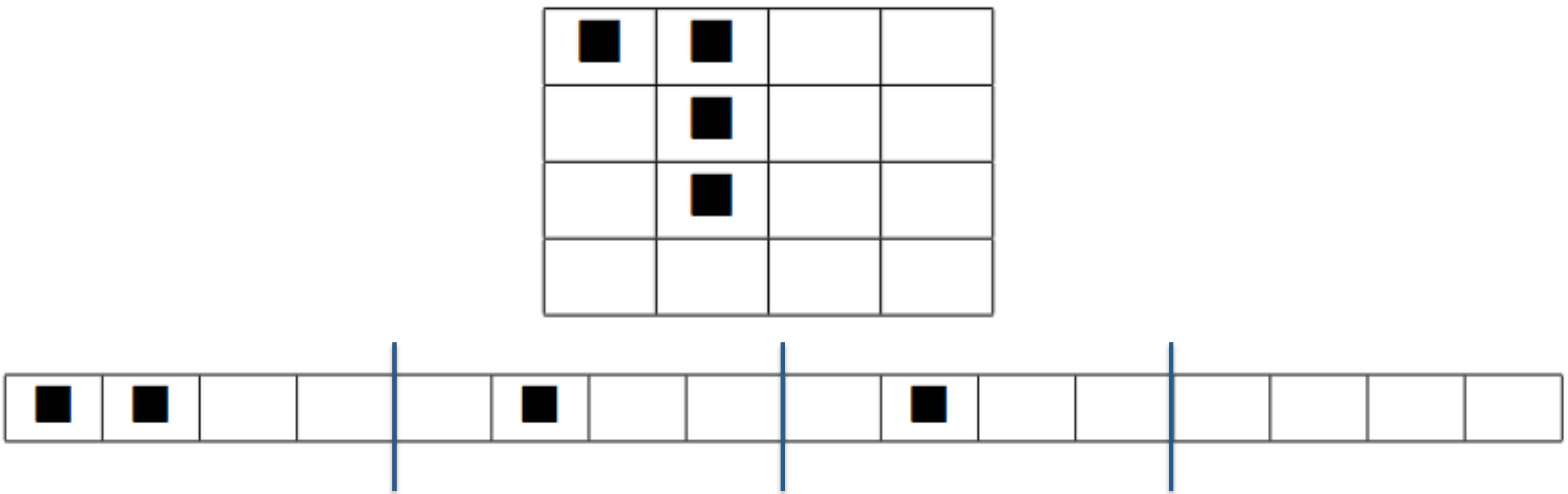
- How to state the placement of the shape ?
- Let's flatten the matrix:



A feasible sequence: 0110001000100000

# 2- Example of Pentaminoes

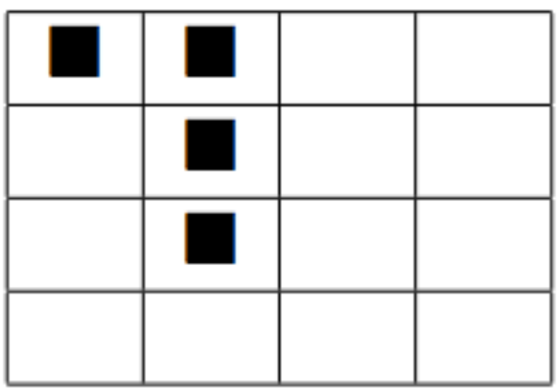
- How to state the placement of the shape ?
- Let's flatten the matrix:



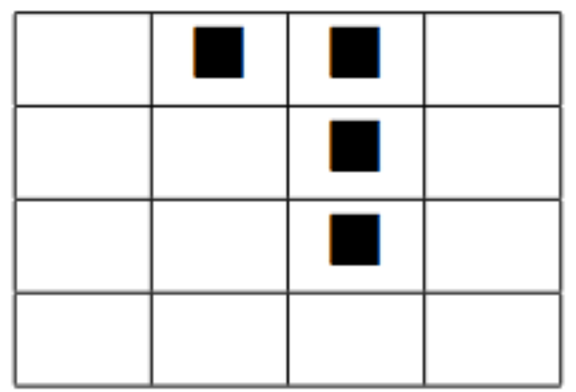
Another feasible sequence : 1100010001000000

# 2- Example of Pentaminoes

- How to state the placement of the shape ?



1100010001000000

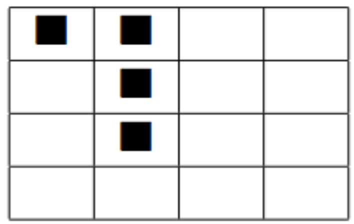


0110001000100000

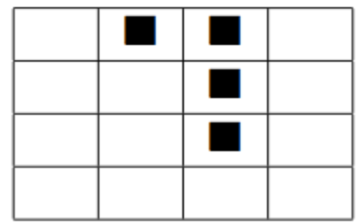
The sequences can be generalized to : $0^{*}11000100010^{*}$

# 2- Example of Pentaminoes

- How to state the placement of the shape ?



1100010001000000



0110001000100000

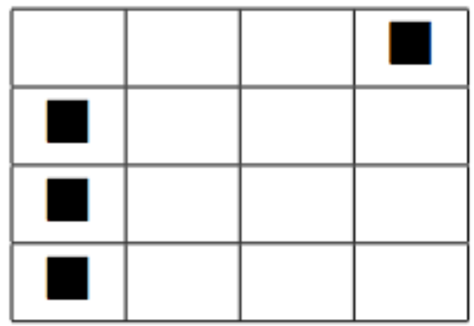
The sequences can be generalized to : $0^*11000100010^*$

Use regular expressions and the Regular constraint:

| | | | |
|---|---|---|---|
| $R$ | $:=$ | $R\,R$ | concatenation |
| | $\mid$ | $R^*$ | repetition |
| | $\mid$ | $R\mid R$ | alternation |
| | $\mid$ | $(R)$ | grouping |
| | $\mid$ | $\epsilon$ | empty string |
| | $\mid$ | $X$ | base values |

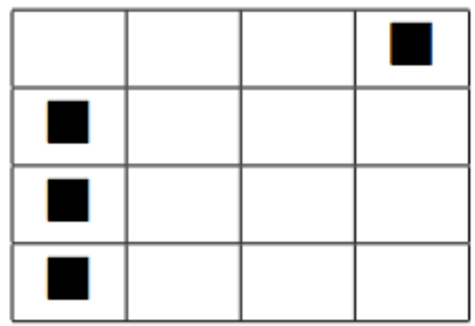
# 2- Example of Pentaminoes

Problem: The expression 0*11000100010* allows the following placement



# 2- Example of Pentaminoes

Problem: The expression 0*11000100010* allows the following placement



Solution: add extra column witch forced zeroes

# 2- Example of Pentaminoes

- *Modeling Irregular Shape placements problems with regular constraints*, Mikael Z. Lagerkvist, Gilles Pesant 2008.

| | regular | | geost | |
|---|---|---|---|---|
| Size | failures | time | failures | time |
| 20 × 3 | 35 680 | 9 320 | 47 381 | 49 740 |
| 15 × 4 | 649 068 | 147 210 | 888 060 | 939 060 |
| 12 × 5 | 2 478 035 | 576 270 | 3 994 455 | 4 112 870 |
| 10 × 6 | 5 998 165 | 1 517 150 | 9 688 985 | 10 726 810 |

Search for all solutions

More precise domains might allow to express more reasoning

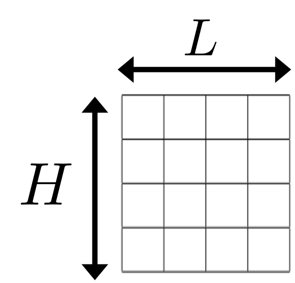
# 2- Example of Pentaminoes

- *Modeling Irregular Shape placements problems with regular constraints*, Mikael Z. Lagerkvist, Gilles Pesant 2008.

- Express placement as regular constraints to capture “holes” in the placement leads to stronger propagation

- Drawback in the size of the model:
  - Standard: $O(n(L+H))$
  - Regular: $O(n(L \times H))$

$n$ pentaminoes




*Question the granularity of representation*

# Outline

1. Choice of domains, *granularity* of the model
2. Illustration on a *toy problem*
   – Pentaminoes
3. Illustration on a real application: multi-leaf sequencing
   – Direct model
   – Counter model
   – Path model
4. Stepping back, looking for a generic answer
   – Set variables
   – MDD consistency
5. Conclusion

# 3- The Multileaf Collimator Sequencing Problem

*Data* : A matrix of integers (**the intensities**)

*Question* : Find a decomposition into a weighted sum of Boolean matrices such that,

- The matrices have the **consecutive ones** property
- The sum of the coefficients (**Beam on time B**) is minimum
- The number of matrices (**Cardinality K**) is minimum

# 3- The Multileaf Collimator Sequencing Problem

*Data* : A matrix of integers (**the intensities**)

*Question* : Find a decomposition into a weighted sum of Boolean matrices such that,

- The matrices have the **consecutive ones** property
- The sum of the coefficients (**Beam on time B**) is minimum
- The number of matrices (**Cardinality K**) is minimum


$$\begin{bmatrix} 0&3&3&0&2&2&0&0\\ 0&0&5&5&6&4&4&1\\ 0&3&3&3&5&5&2&2\\ 0&4&4&6&5&5&2&0\\ 0&3&3&2&3&2&2&0\\ 0&5&5&1&1&1&0&0\\ 0&3&3&0&0&0&0&0\\ 0&3&3&2&2&2&2&0\\ 1&1&1&4&2&2&2&2 \end{bmatrix} = 2 \begin{bmatrix} 0&0&0&0&1&1&0&0\\ 0&0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&0&1&1&1&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&0\\ 0&1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0 \end{bmatrix} + 3 \begin{bmatrix} 0&1&1&0&0&0&0&0\\ 0&0&1&1&1&1&1&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{bmatrix}$$

# 3- The Multileaf Collimator Sequencing Problem

*Data* : A matrix of integers (**the intensities**)

*Question* : Find a decomposition into a weighted sum of Boolean matrices such that,

- The matrices have the **consecutive ones** property
- The sum of the coefficients (**Beam on time B**) is minimum
- The number of matrices (**Cardinality K**) is minimum


B= 6

K= 3

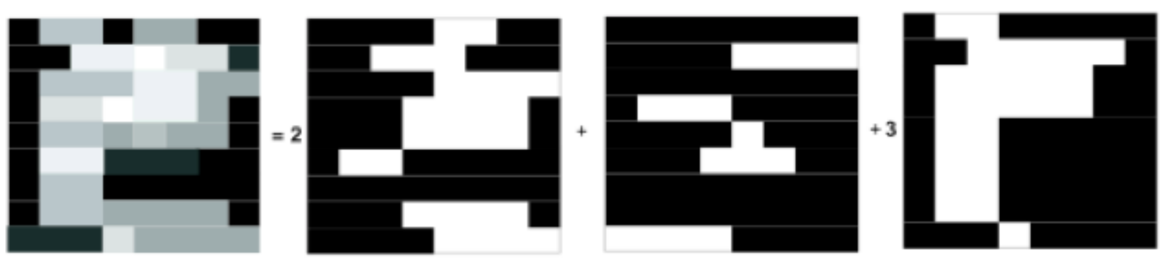
$$\begin{bmatrix} 0&3&3&0&2&2&0&0\\ 0&0&5&5&6&4&4&1\\ 0&3&3&3&5&5&2&2\\ 0&4&4&6&5&5&2&0\\ 0&3&3&2&3&2&2&0\\ 0&5&5&1&1&1&0&0\\ 0&3&3&0&0&0&0&0\\ 0&3&3&2&2&2&2&0\\ 1&1&1&4&2&2&2&2 \end{bmatrix} = 2\begin{bmatrix} 0&0&0&0&1&1&0&0\\ 0&0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&0&1&1&1&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&0\\ 0&1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0 \end{bmatrix} + 3\begin{bmatrix} 0&1&1&0&0&0&0&0\\ 0&0&1&1&1&1&1&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{bmatrix}$$

# 3- The Multileaf Collimator Sequencing Problem

*Data* : A matrix of integers (**the intensities**)

*Question* : Find a decomposition into a weighted sum of Boolean matrices such that,

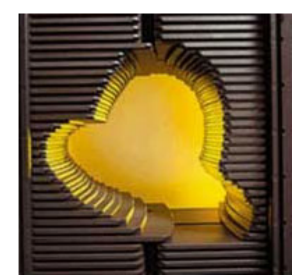
- The matrices have the **consecutive ones** property
- The sum of the coefficients (**Beam on time B**) is minimum
- The number of matrices (**Cardinality K**) is minimum




$$\begin{bmatrix} 0&3&3&0&2&2&0&0\\ 0&0&5&5&6&4&4&1\\ 0&3&3&3&5&5&2&2\\ 0&4&4&6&5&5&2&0\\ 0&3&3&2&3&2&2&0\\ 0&5&5&1&1&1&0&0\\ 0&3&3&0&0&0&0&0\\ 0&3&3&2&2&2&2&0\\ 1&1&1&4&2&2&2&2 \end{bmatrix} = 2 \begin{bmatrix} 0&0&0&0&1&1&0&0\\ 0&0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&0&1&1&1&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&0\\ 0&1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0 \end{bmatrix} + 3 \begin{bmatrix} 0&1&1&0&0&0&0&0\\ 0&0&1&1&1&1&1&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{bmatrix}$$

B= 6

K= 3



$$minimise \;\; w_1 K + w_2 B$$

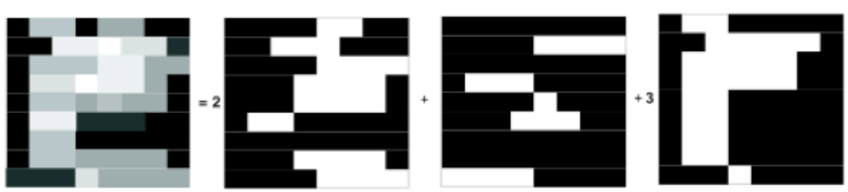
# 3- The Multileaf Collimator Sequencing Problem

*Data* : A matrix of integers (**the intensities**)

*Question* : Find a decomposition into a weighted sum of Boolean matrices such that,

- The matrices have the **consecutive ones** property
- The sum of the coefficients (**Beam on time B**) is minimum
- The number of matrices (**Cardinality K**) is minimum

- In fact B* is easy to compute
- Minimizing K alone is however NP-Hard and so is the weighted sum




$$minimise \;\; w_1 K + w_2 B$$

$$\begin{bmatrix} 0&3&3&0&2&2&0&0\\ 0&0&5&5&6&4&4&1\\ 0&3&3&3&5&5&2&2\\ 0&4&4&6&5&5&2&0\\ 0&3&3&2&3&2&2&0\\ 0&5&5&1&1&1&0&0\\ 0&3&3&0&0&0&0&0\\ 0&3&3&2&2&2&2&0\\ 1&1&1&4&2&2&2&2 \end{bmatrix} = 2\begin{bmatrix} 0&0&0&0&1&1&0&0\\ 0&0&1&1&1&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&0&1&1&1&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&0\\ 0&1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0 \end{bmatrix} + 3\begin{bmatrix} 0&1&1&0&0&0&0&0\\ 0&0&1&1&1&1&1&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&1&1&1&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{bmatrix}$$

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{lll} Variables: & \forall k \leq K & b_k \in \{1, \ldots, M\} \\ & \forall k \leq K, i \leq m, j \leq n, & x_{ij}^k \in \{0, 1\} \end{array}$$

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$b_1 = 2 \qquad b_2 = 2 \qquad b_3 = 1$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x_{23}^2 = 0$$

$$\begin{aligned} Variables: \quad & \forall k \leq K & & b_k \in \{1, \ldots, M\} \\ & \forall k \leq K, i \leq m, j \leq n, & & x_{ij}^k \in \{0, 1\} \end{aligned}$$

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$b_1 = 2$ $\quad b_2 = 2$ $\quad b_3 = 1$

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$x_{23}^{2} = 0$

$$\begin{array}{lll} Variables: & \forall k \leq K & b_k \in \{1, \ldots, M\} \\ & \forall k \leq K, i \leq m, j \leq n, & x_{ij}^{k} \in \{0, 1\} \end{array}$$

$$DM_1: \qquad \textstyle\sum_{k \leq K} b_k = B^*$$

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$b_1 = 2 \qquad b_2 = 2 \qquad b_3 = 1$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x_{23}^2 = 0$$

| | | |
|---|---|---|
| $Variables:$ | $\forall k \leq K$ | $b_k \in \{1, \ldots, M\}$ |
| | $\forall k \leq K, i \leq m, j \leq n,$ | $x_{ij}^k \in \{0, 1\}$ |
| $DM_1:$ | | $\sum_{k \leq K} b_k = B^*$ |
| $DM_4:$ | $\forall i \leq m, j \leq n$ | $\sum_{k \leq K} b_k \times x_{ij}^k = I_{ij}$ |

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$\begin{bmatrix}2&4&3\\3&4&2\end{bmatrix} = 2\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix} + 2\begin{bmatrix}0&1&0\\0&1&0\end{bmatrix} + 1\begin{bmatrix}0&0&1\\1&0&0\end{bmatrix}$$

$b_1 = 2$ $b_2 = 2$ $b_3 = 1$

$x^2_{23} = 0$

| | | |
|---|---|---|
| $Variables:$ | $\forall k \leq K$ | $b_k \in \{1, \ldots, M\}$ |
| | $\forall k \leq K, i \leq m, j \leq n,$ | $x^k_{ij} \in \{0, 1\}$ |
| $DM_1:$ | | $\sum_{k \leq K} b_k = B^*$ |
| $DM_3:$ | $\forall k \leq K, i \leq m,$ | $\textsc{ConsecutiveOnes}(\{x^k_{i1}, \ldots, x^k_{in}\})$ |
| $DM_4:$ | $\forall i \leq m, j \leq n$ | $\sum_{k \leq K} b_k \times x^k_{ij} = I_{ij}$ |

# 3- The Direct model

- Not so easy to model
- Let’s fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = \underset{b_1 = 2}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \underset{b_2 = 2}{2} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \underset{b_3 = 1}{1} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x^2_{23} = 0$$

| | | |
|---|---|---|
| $Variables:$ | $\forall k \leq K$ | $b_k \in \{1, \ldots, M\}$ |
| | $\forall k \leq K, i \leq m, j \leq n,$ | $x^k_{ij} \in \{0, 1\}$ |
| $DM_1:$ | | $\sum_{k \leq K} b_k = B^*$ |
| $DM_3:$ | $\forall k \leq K, i \leq m,$ | $\textsc{Regular}([x^k_{i1}, \ldots, x^k_{in}], A(0^*1^*0^*))$ |
| $DM_4:$ | $\forall i \leq m, j \leq n$ | $\sum_{k \leq K} b_k \times x^k_{ij} = I_{ij}$ |

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = \underset{b_1 = 2}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \underset{b_2 = 2}{2} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \underset{b_3 = 1}{1} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x_{23}^2 = 0$$

$$\begin{array}{lll}
Variables: & \forall k \leq K & b_k \in \{1, \ldots, M\} \\
 & \forall k \leq K, i \leq m, j \leq n, & x_{ij}^k \in \{0, 1\} \\
 & & \\
DM_1: & & \sum_{k \leq K} b_k = B^* \\
DM_2: & & b_1 \geq b_2 \geq \ldots \geq b_K \\
DM_3: & \forall k \leq K, i \leq m, & \textsc{Regular}([x_{i1}^k, \ldots, x_{in}^k], A(0^*1^*0^*)) \\
DM_4: & \forall i \leq m, j \leq n & \sum_{k \leq K} b_k \times x_{ij}^k = I_{ij}
\end{array}$$

# 3- The Direct model

- Not so easy to model
- Let's fix cardinality (K) and Beam-on time (B*) and look for a feasible decomposition

| | | |
|---|---|---|
| $Variables:$ | $\forall k \leq K$ | $b_k \in \{1, \ldots, M\}$ |
| | $\forall k \leq K, i \leq m, j \leq n,$ | $x_{ij}^k \in \{0, 1\}$ |
| $DM_1:$ | | $\sum_{k \leq K} b_k = B^*$ |
| $DM_2:$ | | $b_1 \geq b_2 \geq \ldots \geq b_K$ |
| $DM_3:$ | $\forall k \leq K, i \leq m,$ | $\textsc{Regular}([x_{i1}^k, \ldots, x_{in}^k], A(0^*1^*0^*))$ |
| $DM_4:$ | $\forall i \leq m, j \leq n$ | $\sum_{k \leq K} b_k \times x_{ij}^k = I_{ij}$ |

- Note: Some symmetries remain

$$2\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & \mathbf{1}\end{pmatrix} + 2\begin{pmatrix}0 & 1 & 0\\ 0 & 1 & \mathbf{0}\end{pmatrix} = 2\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & \mathbf{0}\end{pmatrix} + 2\begin{pmatrix}0 & 1 & 0\\ 0 & 1 & \mathbf{1}\end{pmatrix}$$

[Baatar, Boland, Brand, Stuckey 07], [Brand 08]

# 3- The Counter model

- Let's minimize the cardinality (K)

$$\begin{array}{lll} minimise & K & \\ Variables: & \forall b \leq M & N_b \in \{0,\ldots,B^*\} \\ & \forall i \leq m, j \leq n, b \leq M & Q^b_{ij} \in \{0,\ldots,M\} \end{array}$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

# 3- The Counter model

- Let's minimize the cardinality (K)

$$
\begin{array}{lll}
\textit{minimise} & K & \\
\textit{Variables:} & \forall b \leq M & N_b \in \{0,\ldots,B^*\} \\
 & \forall i \leq m, j \leq n, b \leq M & Q_{ij}^b \in \{0,\ldots,M\}
\end{array}
$$

$$N_2 = 2 \qquad N_1 = 1 \quad N_3 = 0 \; N_4 = 0$$

$$
\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}
$$

$$Q_{22}^2 = 2$$

[Baatar, Boland, Brand, Stuckey 07], [Brand 08]

# 3- The Counter model

- Let’s minimize the cardinality (K)

$$
\begin{array}{lll}
minimise & K & \\
Variables: & \forall b \leq M & N_b \in \{0, \ldots, B^*\} \\
 & \forall i \leq m, j \leq n, b \leq M & Q_{ij}^b \in \{0, \ldots, M\}
\end{array}
$$

$N_2 = 2 \qquad N_1 = 1 \quad N_3 = 0 \; N_4 = 0$

$$
\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}
$$

$Q_{22}^2 = 2$

$$
\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}}_{Q^2} + 1 \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{Q^1}
$$

# 3- The Counter model

- Let's minimize the cardinality (K)

$$
\begin{array}{lll}
\textit{minimise} & K & \\
Variables: & \forall b \leq M & N_b \in \{0, \ldots, B^*\} \\
 & \forall i \leq m, j \leq n, b \leq M & Q^b_{ij} \in \{0, \ldots, M\} \\
 & & \\
CM_1: & \forall i \leq m, j \leq n & \sum_{b=1}^{M} b \times Q^b_{ij} = I_{ij} \\
CM_2: & & \sum_{b=1}^{M} b \times N_b = B^* \\
CM_3: & & \sum_{b=1}^{M} N_b = K
\end{array}
$$

$$
\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}}_{Q^2} + 1 \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{Q^1}
$$

- How to express the consecutive one property on Q ?

# 3- The Counter model

- Let's minimize the cardinality (K)

$$
\begin{array}{lll}
minimise & K & \\
Variables: & \forall b \leq M & N_b \in \{0,\ldots,B^*\} \\
 & \forall i \leq m, j \leq n, b \leq M & Q^b_{ij} \in \{0,\ldots,M\} \\
 & & \\
CM_1: & \forall i \leq m, j \leq n & \sum_{b=1}^{M} b \times Q^b_{ij} = I_{ij} \\
CM_2: & & \sum_{b=1}^{M} b \times N_b = B^* \\
CM_3: & & \sum_{b=1}^{M} N_b = K \\
CM_4: & \forall b \leq M, i \leq m, & \textsc{SumOfIncrements}(\{Q^b_{i1},\ldots,Q^b_{in}\}, N_b)
\end{array}
$$

$$
\begin{bmatrix} 2 & 4 & 3 \\ 3 & 4 & 2 \end{bmatrix} = 2 \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}}_{Q^2} + 1 \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{Q^1}
$$

- How to express the consecutive one property on Q ?

$$\sum_{j=0}^{n-1} max(Q^b_{i,j+1} - Q^b_{i,j}, 0) \leq N_b$$

# 3- The shortest path model

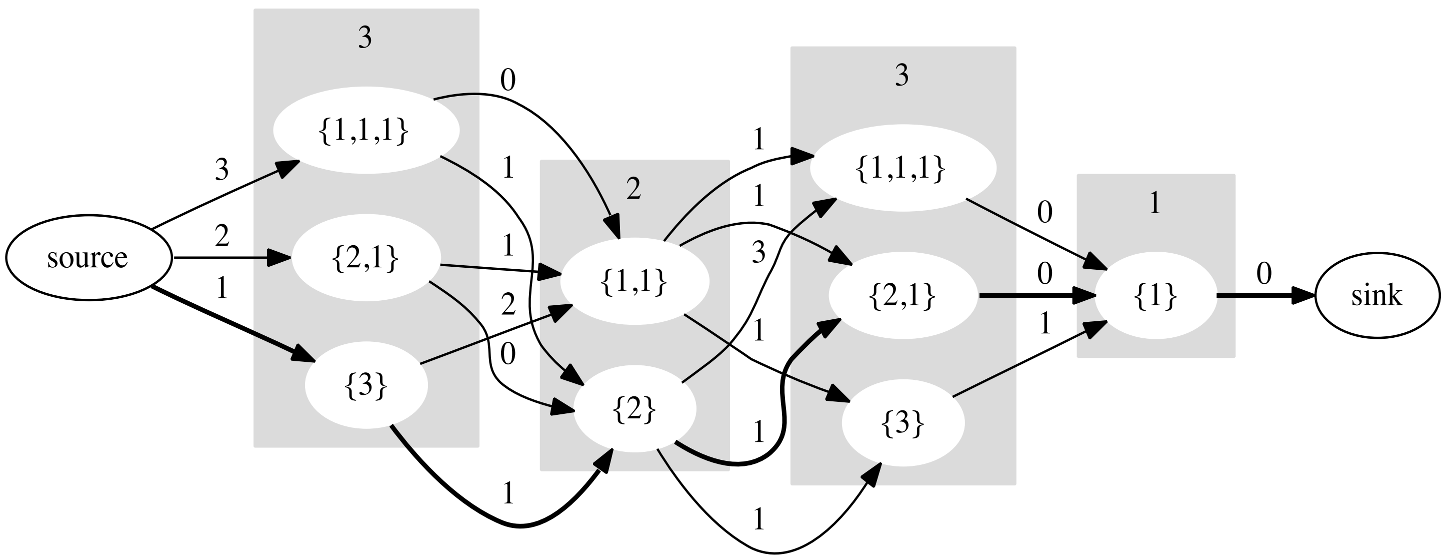
- Problem restricted to one row: [3,2,3,1]

$$[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,1] + 2[0,0,1,0]$$

- The decomposition contains an integer partition of each element:
  - 3 is decomposed with {2,1}
  - 2 with {1,1}
  - 3 with {2,1}
  - 1 with {1}

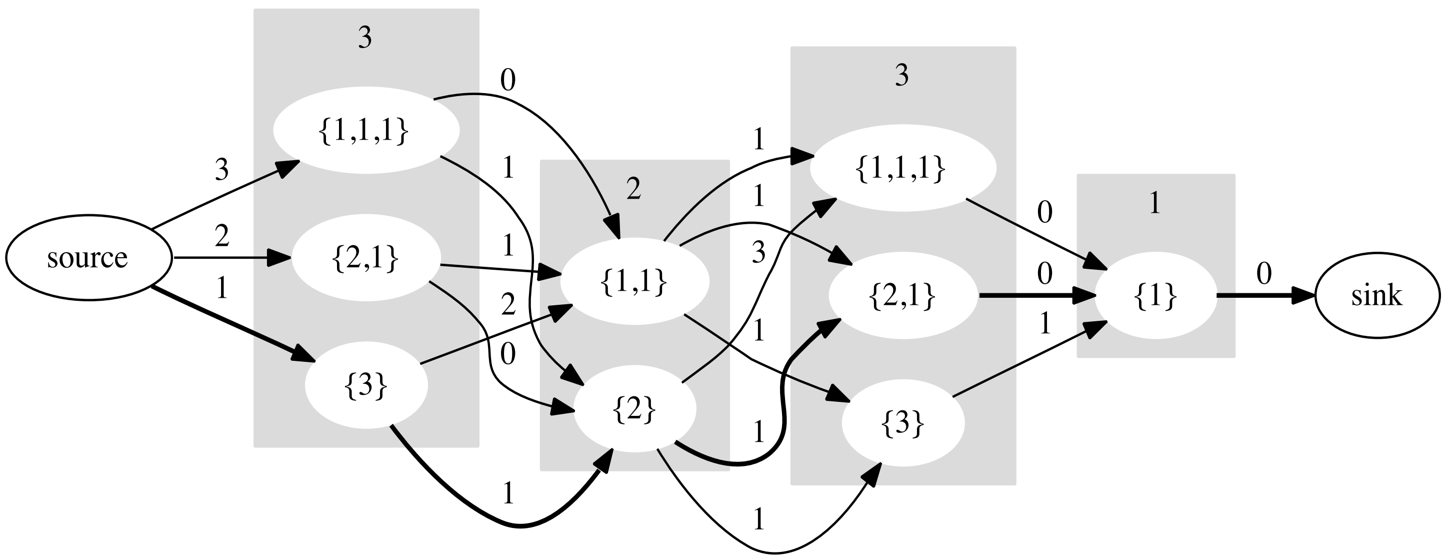
# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions



# 3- The shortest path model

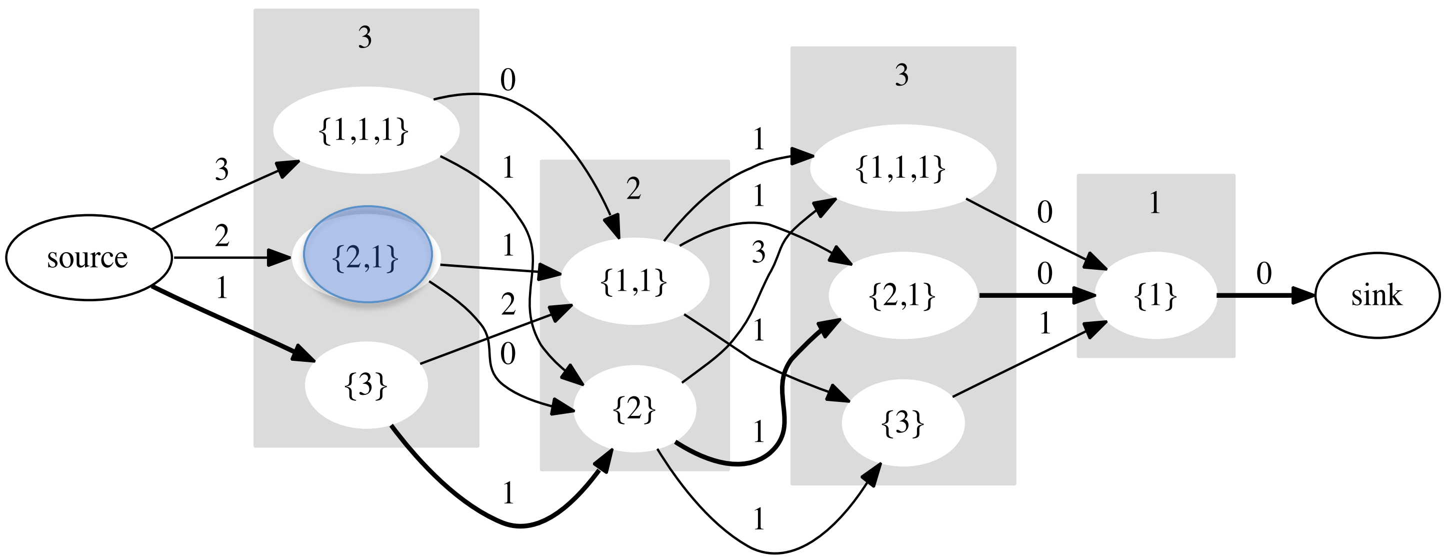
- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




[3,2,3,1] =

# 3- The shortest path model

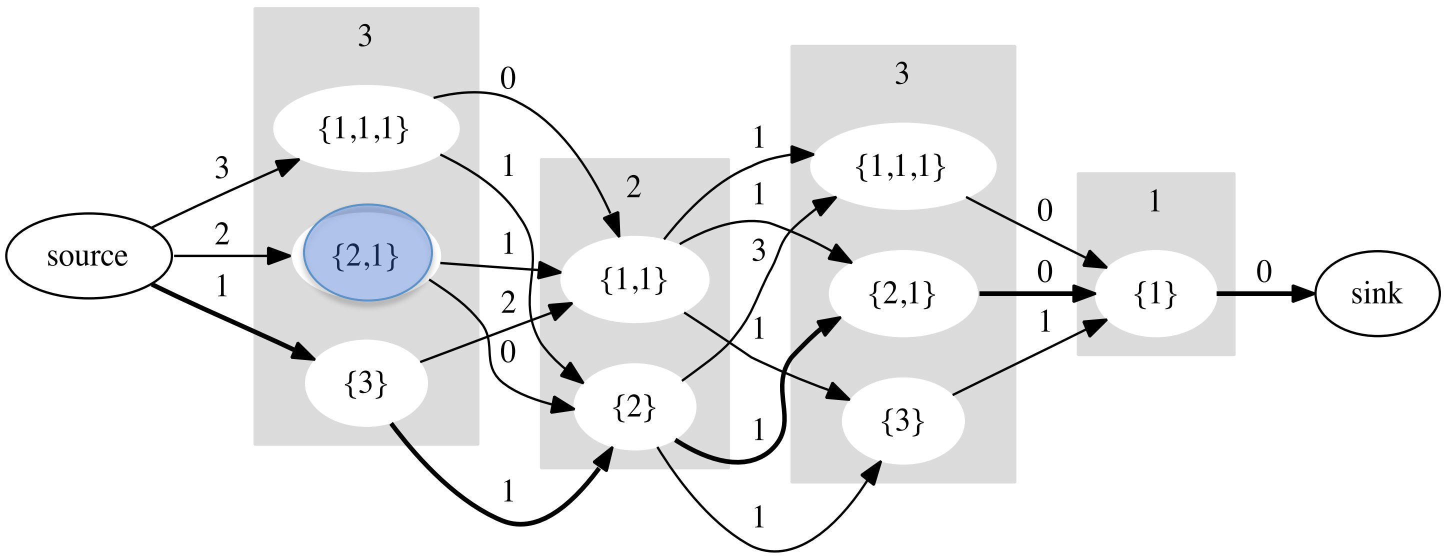
- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




[3,2,3,1] =

# 3- The shortest path model

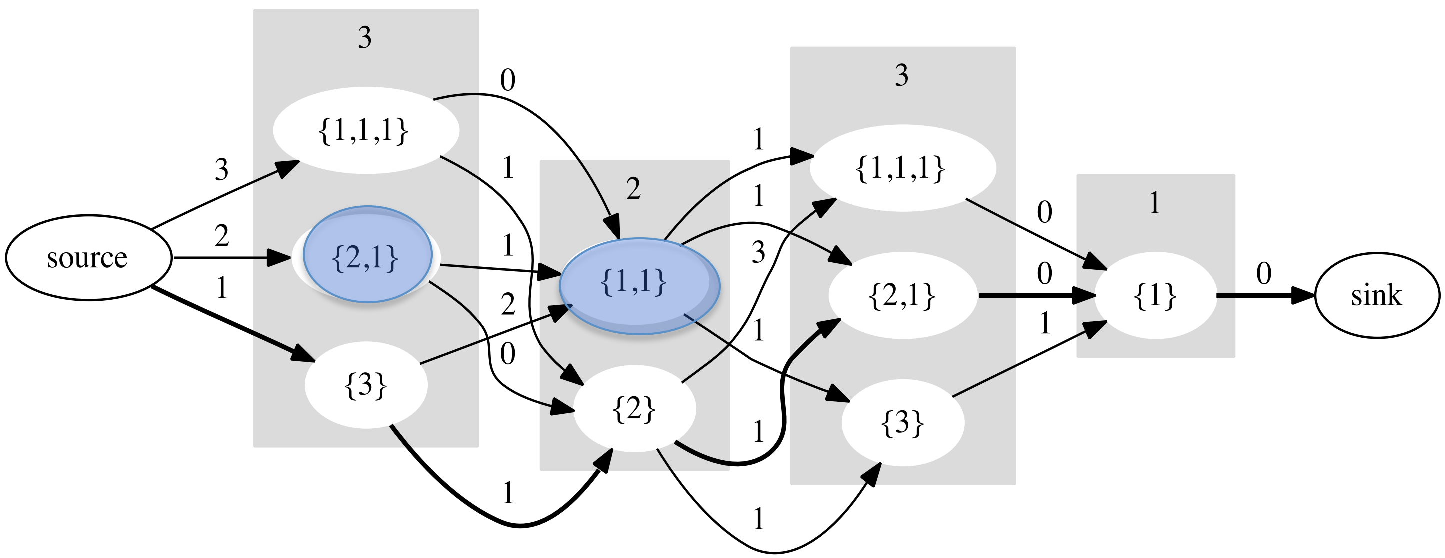
- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

# 3- The shortest path model

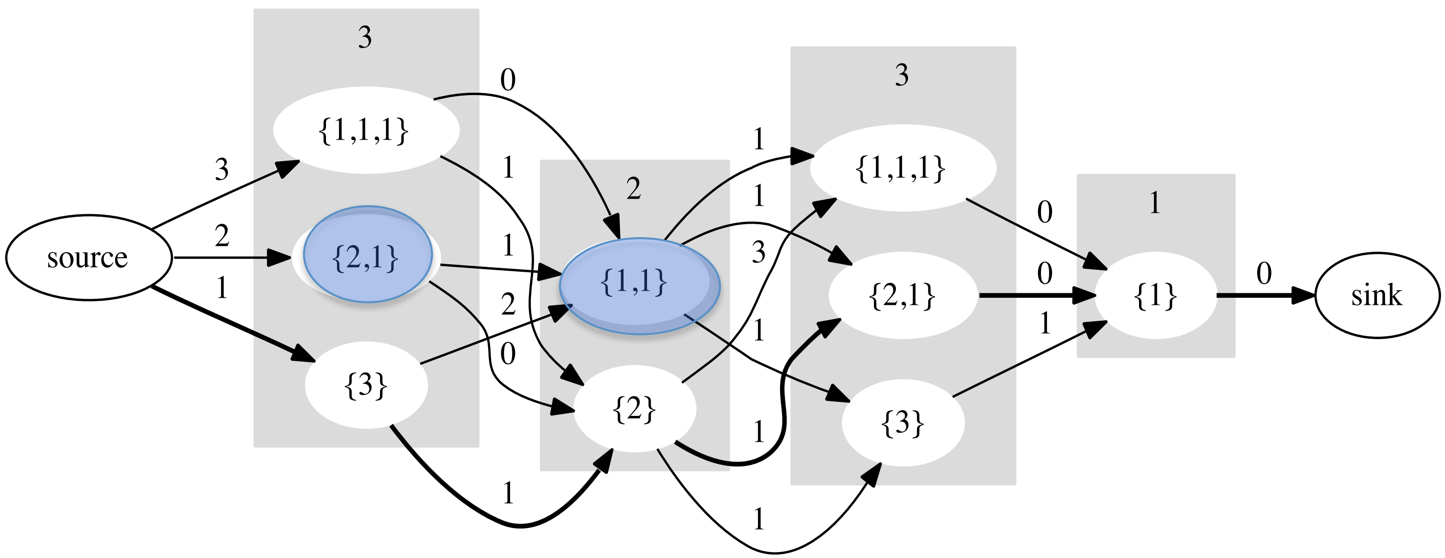
- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions

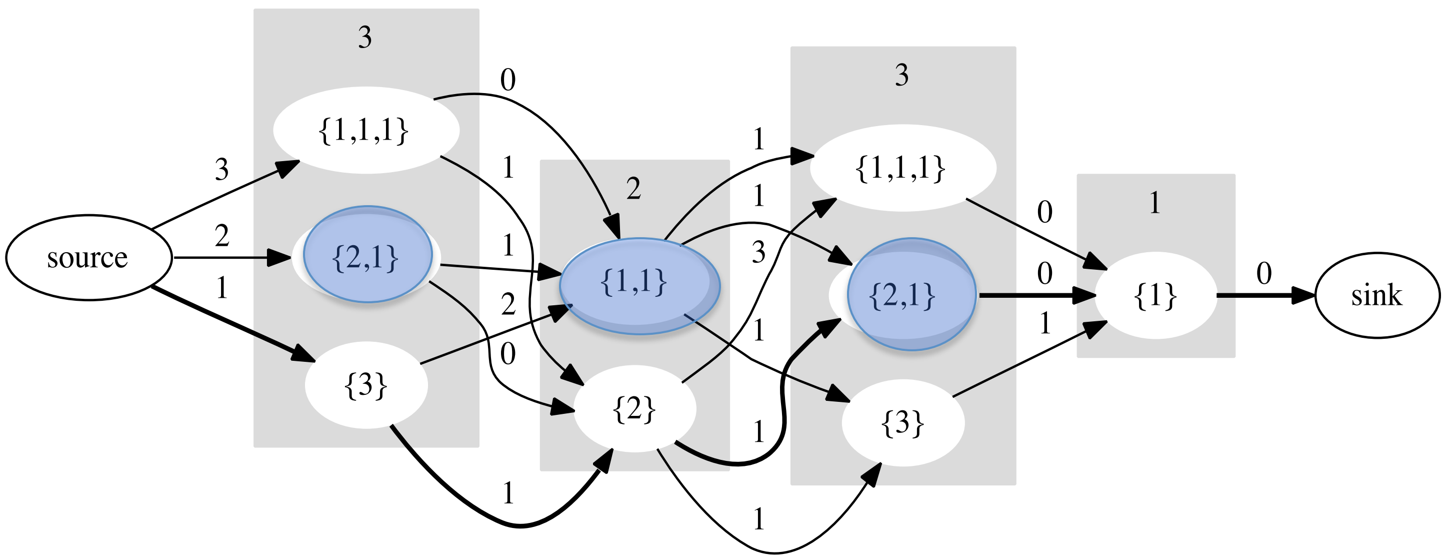



[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions

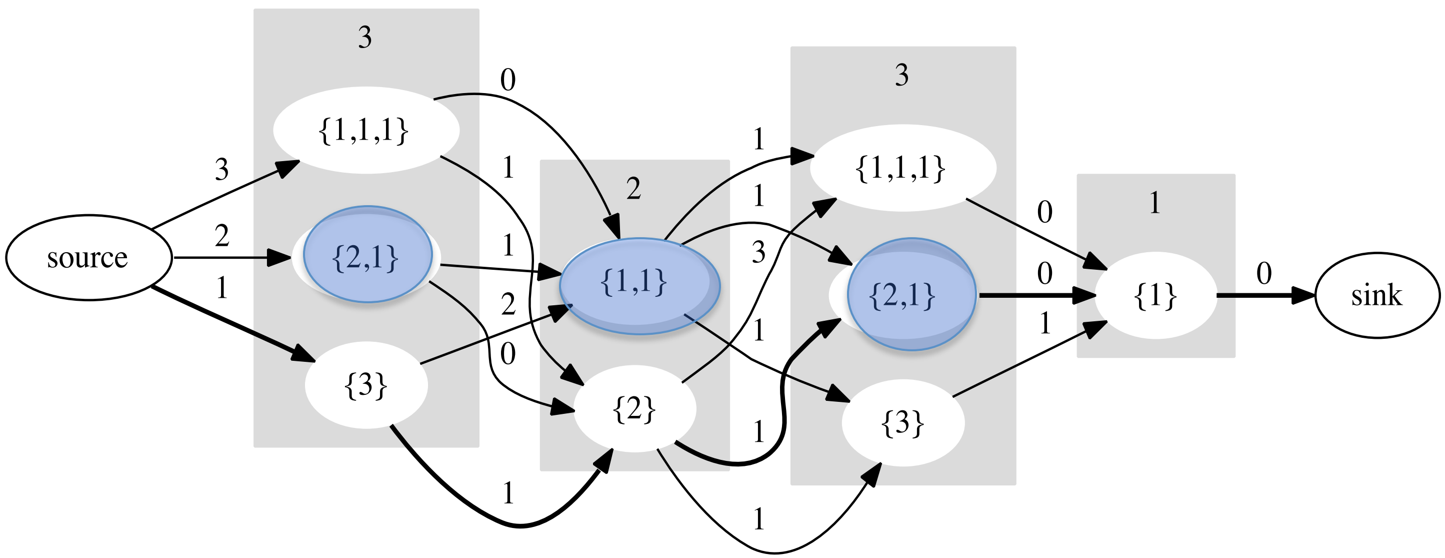



[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions



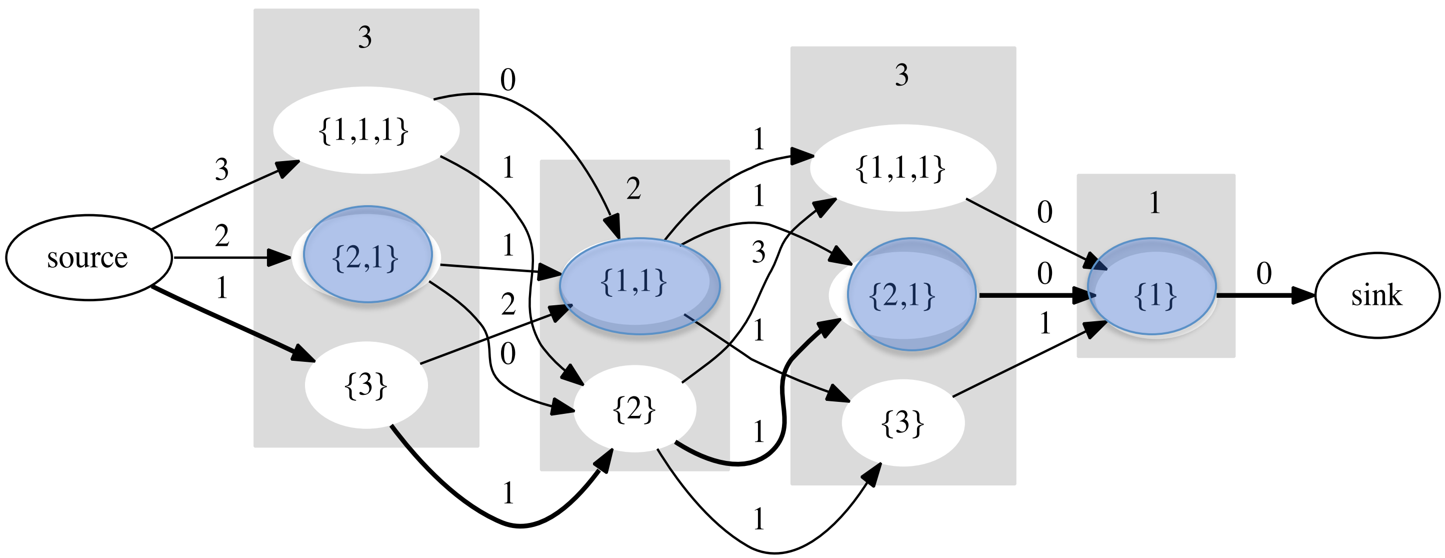

[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,?] + 2[0,0,1,0]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions



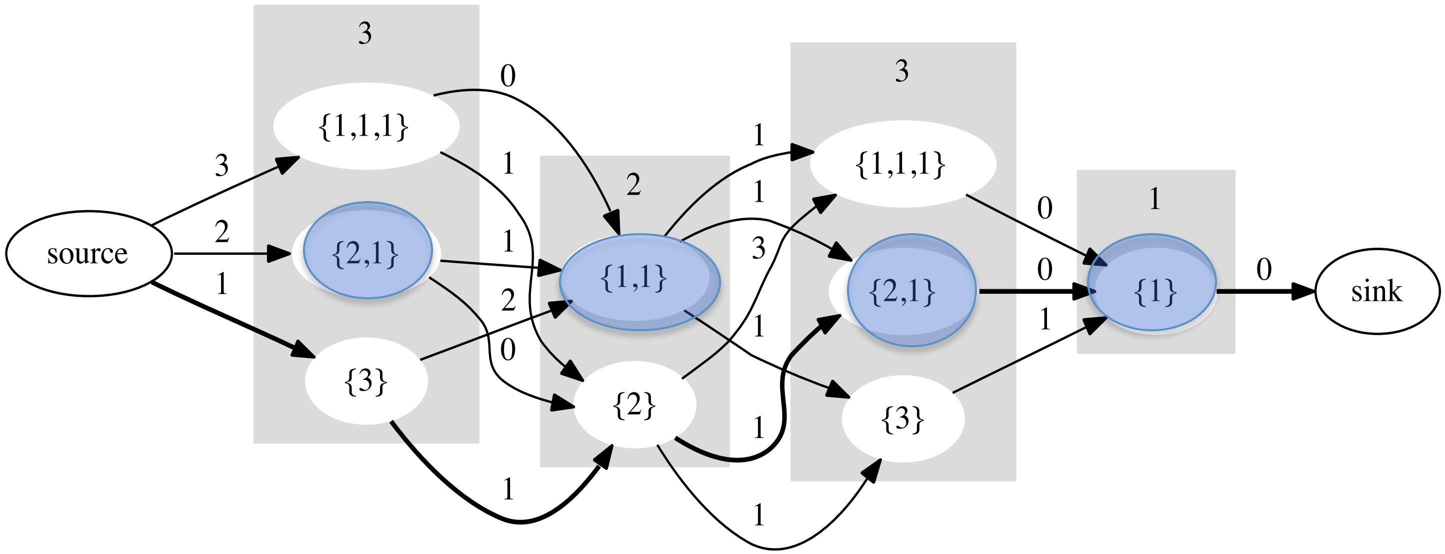

[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,?] + 2[0,0,1,0]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

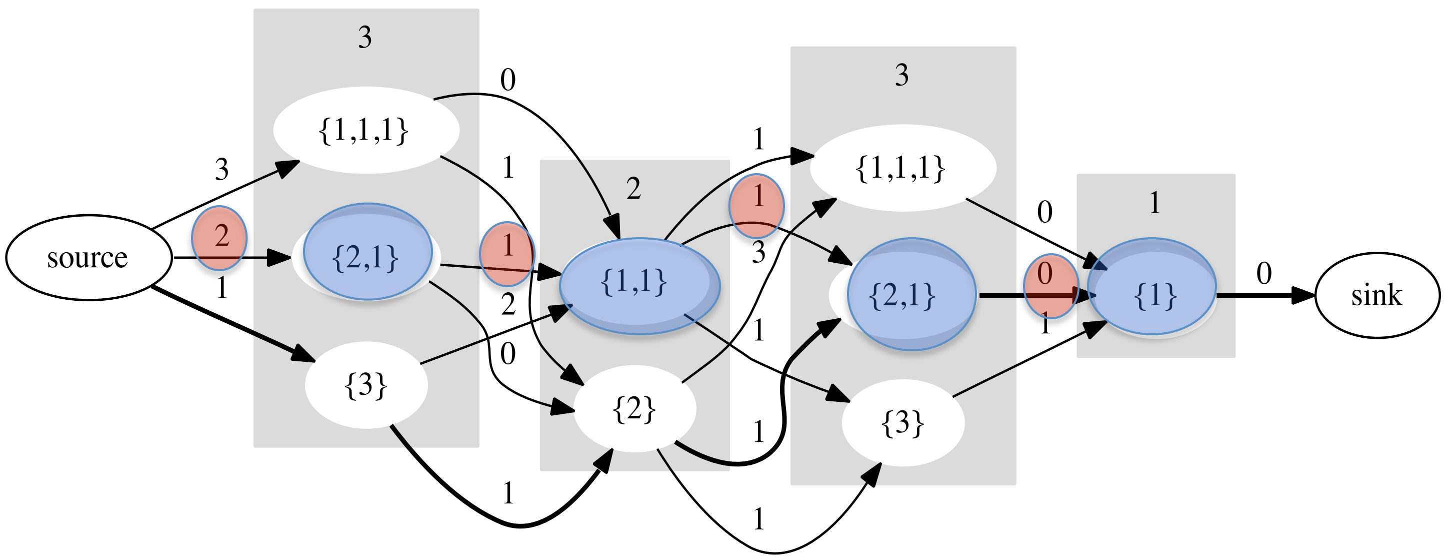
[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,?] + 2[0,0,1,0]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,1] + 2[0,0,1,0]

# 3- The shortest path model

- Minimisation of the cardinality for a single line : [3,2,3,1]
- Explicit representation of the integer partitions




Cardinality K = 2 + 1 + 1 + 0 = 4

[3,2,3,1] = 2[1,?,?,?] + 1[1,?,?,?]

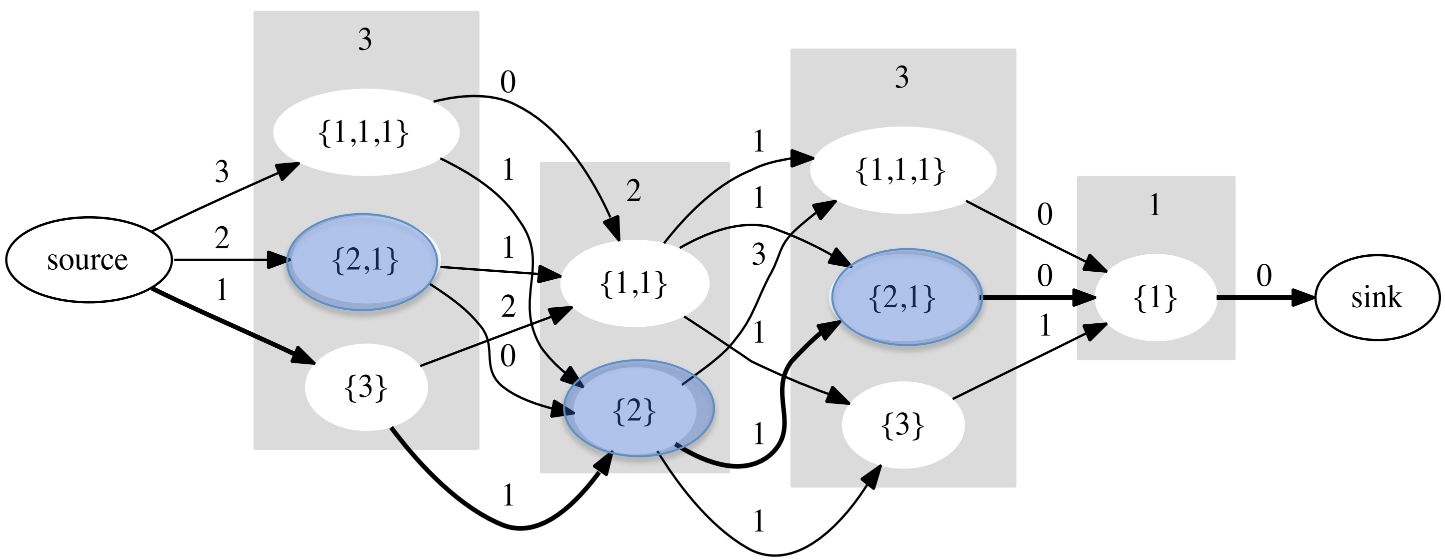
[3,2,3,1] = 2[1,0,0,0] + 1[1,1,?,?] + 1[0,1,?,?]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,?] + 2[0,0,1,0]

[3,2,3,1] = 2[1,0,0,0] + 1[1,1,0,0] + 1[0,1,1,1] + 2[0,0,1,0]

# 3- The shortest path model

- A path encodes a decomposition
- The length of the path gives the cardinality



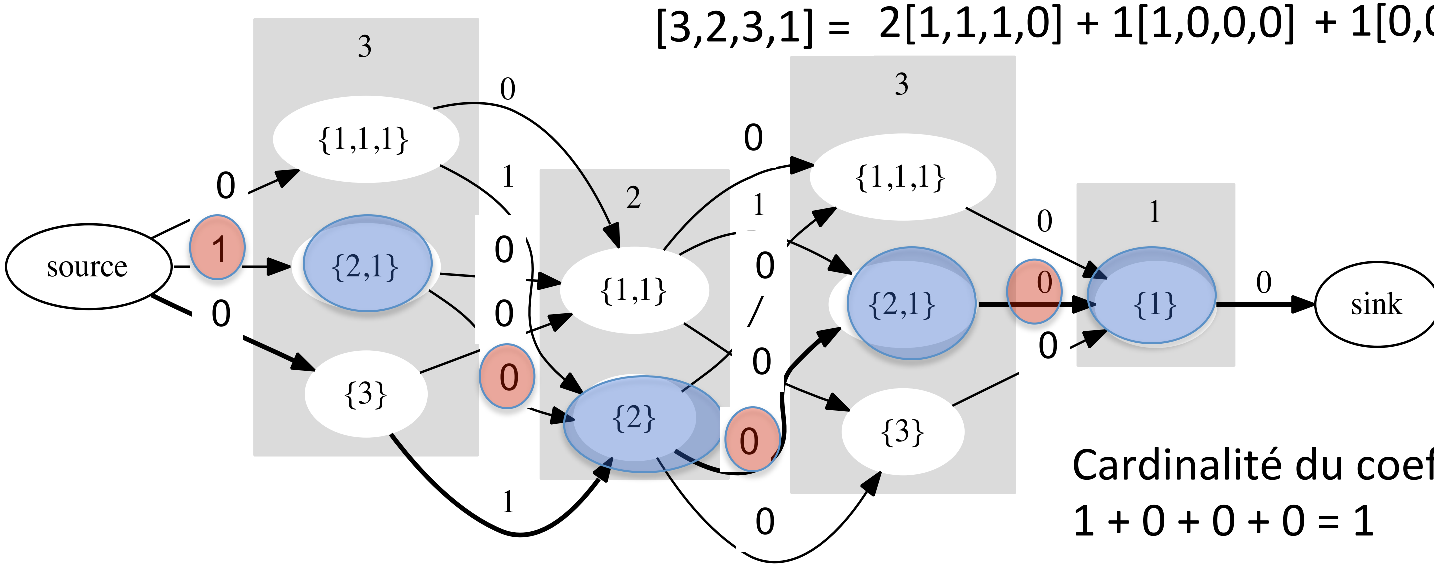

A shortest path gives a minimum cardinality decomposition

[3,2,3,1] = 2[1,1,1,0] + 1[1,0,0,0] + 1[0,0,1,1]     K = 3

# 3- The shortest path model

- Costs can be restricted to a particular coefficient:

[3,2,3,1] = 2[1,1,1,0] + 1[1,0,0,0] + 1[0,0,1,1]




Cardinalité du coefficient 2 :
1 + 0 + 0 + 0 = 1

- Idea: it allows to maintain the minimum number of coefficient « 2 » needed for consecutive ones decomposition

# 3- The shortest path model

- Representations of the integer partition
  - Example with intensity I = 10 :

    $$P \in \{\{10\}, \{9,1\}, \{8,2\}, \{8,1,1\}, \ldots, \{4,4,2\}, \ldots\}$$

  - Occurrence representation:

    *[Baatar, Boland, Brand, Stuckey CPAIOR'07 / Constraint 2011]*

$$Q_{10} = \{0,1\}, Q_9 = \{0,1\}, \ldots, Q_4 = \{0,1,2\}, \ldots, Q_1 = \{0,\ldots,10\}$$

$$P = \{4,4,2\} \quad \Leftrightarrow \quad Q_4 = 2, Q_2 = 1$$

- The filtering of the value {4,4,2} from the domain of P can not be expressed on Q

# 3- The shortest path model

$$
\begin{array}{llll}
minimise & w_1 K + w_2 B \quad \text{with} & & K \in \{0,\dots,nmM\}, B \in \{B^*,...,nmM\} \\
 & \forall b \leq M & & N_b \in \{0,\dots,nmM\} \\
 & \forall i \leq m, j \leq n, & & \boxed{P_{ij} \in \{1,\dots,|P(I_{ij})|\}} \quad \text{Partition variables} \\
CP_1: & & & \sum_{b=1}^{M} b \times N_b = B \\
CP_2: & & & \sum_{b=1}^{M} N_b = K \\
CP_3: & \forall i \leq m, & & \textsc{ShortestPath}(G_1(i), \{P_{i1},\dots,P_{in}\}, K) \\
CP_4: & \forall i \leq m, b \leq M & & \textsc{ShortestPath}(G_2(i,b), \{P_{i1},\dots,P_{in}\}, N_b) \\
CP_5: & \forall i \leq m, & & \textsc{ShortestPath}(G_3(i), \{P_{i1},\dots,P_{in}\}, B) \\
CP_6: & \forall i \leq m, \forall j < m \text{ s.t } I_{ij} = I_{i,j+1} & & P_{ij} = P_{i,j+1}
\end{array}
$$

- Shortest Path with M+2 resource constraints (the graph is identical for all constraints)

# Synthesis

- Size of the models for a given K and **maximum intensity M**
  - Direct model : $O(KM + 2nmK)$
  - Counter model : $O(MB^* + nmM^2)$
  - Path model : $\text{exponential in M}$
- The level of consistency increases in each model
- More fine-grained domains allow for more reasoning
- Similar idea in MIP: column generation
  - An exponential number of variables can lead to a very strong linear relaxation

# Overview of results

- Some results using **CP**:
  - Counter model: 20 x 20 with max intensity 10 *[Baatar, Boland, Brand, Stuckey 07], [Brand 08]*
  - Path model: 40 x 40 with max intensity 10 *[Cambazard, O'Mahony, O'Sullivan 09]*

- ***Dedicated*** *algorithm:*
  - 15 x 15 with max intensity 10 (up to 10h of computation) *[Kalinowski 08]*

- Using ***Benders decomposition***: *[Taskin, Smith, Romeijn, Dempsey ANOR'09]*

  Clinical instances (around 20x20 with max intensity 20) solved optimally with up to 5.8 h of computation

- Results can be improved using **Lagrangian Relaxation** when intensity is small

  *[Cambazard, O'Mahony, O'Sulllivan, 2010]*

- Significant improvement using ***Branch and Price and constraint propagation*** : *[Cambazard, O'Mahony, O'Sulllivan, 2012]*
  - 80 x 80 with max intensity 10
  - 20 x 20 with max intensity 20
  - 12 x 12 with max intensity 25
  - Clinical instances with up to 10 min of computation

⇒ *Question the granularity of representation*

# Outline

1. Choice of domains, *granularity* of the model
2. Illustration on a *toy problem*
   - Pentaminoes
3. Illustration on a real application: multi-leaf sequencing
   - Direct model
   - Counter model
   - Path model
4. Stepping back, looking for a generic answer
   - Set variables
   - MDD consistency
5. Conclusion

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Set Variables [Puget, 1992], [Gervet, 1997]
  - Domain is set of sets $D(X) = \{S \mid S \subseteq \{1,2,3,4,5,6,7,8,9\}\}$
  - A value is a set $X = \{1,4,9\}$

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Set Variables [Puget, 1992], [Gervet, 1997]
  - Domain is set of sets $D(X) = \{S \mid S \subseteq \{1,2,3,4,5,6,7,8,9\}\}$
  - A value is a set $X = \{1,4,9\}$
  - A representation with a lower/upper *bound*: $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

Elements that can still go in the set: $\overline{X} \subseteq \{1,2,4,6,9\}$

Elements that are in the set: $\underline{X} \subseteq \{4,9\}$

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Set Variables [Puget, 1992], [Gervet, 1997]
  - Domain is set of sets $D(X) = \{S \mid S \subseteq \{1,2,3,4,5,6,7,8,9\}\}$
  - A value is a set $X = \{1,4,9\}$
  - A representation with a lower/upper *bound*: $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

Elements that can still go in the set: $\overline{X} \subseteq \{1,2,4,6,9\}$

Elements that are in the set: $\underline{X} \subseteq \{4,9\}$

- Equivalent to boolean variables:

$$b_1, b_2, b_6 \in \{0,1\}$$
$$b_4 = 1,\ b_9 = 1$$
$$b_3 = 0,\ b_5 = 0,\ b_7 = 0,\ b_8 = 0$$

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Set Variables [Puget, 1992], [Gervet, 1997]
  - Domain is set of sets $D(X) = \{S \mid S \subseteq \{1,2,3,4,5,6,7,8,9\}\}$
  - A value is a set $X = \{1,4,9\}$
  - A representation with a lower/upper *bound*: $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

Elements that can still go in the set: $\overline{X} \subseteq \{1,2,4,6,9\}$

Elements that are in the set: $\underline{X} \subseteq \{4,9\}$

- Equivalent to boolean variables:

$$b_1, b_2, b_6 \in \{0,1\}$$
$$b_4 = 1,\ b_9 = 1$$
$$b_3 = 0,\ b_5 = 0,\ b_7 = 0,\ b_8 = 0$$

- Common solver representation adds the cardinality:

$$\overline{X} \subseteq \{1,2,4,6,9\} \quad \underline{X} \subseteq \{4,9\} \quad K = |X|$$

- Some reasoning on K can not be expressed on $\underline{X}$ and $\overline{X}$ alone

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Length-Lex representation [Gervet and Van-Hentenryck, 2006]
  - Order the sets by cardinality and lexicographycally

$$\emptyset << \{1\} << \{2\} << \{3\} << \{1,2\} << \{1,3\} << \{2,3\} << \{1,2,3\}$$

# 4- Stepping back, looking for a generic answer
*Set Variables*

- Length-Lex representation [Gervet and Van-Hentenryck, 2006]
  - Order the sets by cardinality and lexicographycally

$$\emptyset << \{1\} << \{2\} << \{3\} << \{1,2\} << \{1,3\} << \{2,3\} << \{1,2,3\}$$

  - The bounds are refined and encodes cardinality

$D(X) = \{S \mid m << S << M\}$ versus $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Length-Lex representation [Gervet and Van-Hentenryck, 2006]
  - Order the sets by cardinality and lexicographycally

$$\emptyset << \{1\} << \{2\} << \{3\} << \{1,2\} << \{1,3\} << \{2,3\} << \{1,2,3\}$$

  - The bounds are refined and encodes cardinality

$D(X) = \{S \mid m << S << M\}$ versus $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

  - Example with $D(X) = \{\{1,4\},\{2,3\},\{1,2,3\}\}$ so **3** sets

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Length-Lex representation [Gervet and Van-Hentenryck, 2006]
  - Order the sets by cardinality and lexicographycally

$$\emptyset << \{1\} << \{2\} << \{3\} << \{1,2\} << \{1,3\} << \{2,3\} << \{1,2,3\}$$

  - The bounds are refined and encodes cardinality

$D(X) = \{S \mid m << S << M\}$ versus $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

  - Example with $D(X) = \{\{1,4\},\{2,3\},\{1,2,3\}\}$ so **3** sets

Classic: $D(X) = \{S \mid \emptyset \subseteq S \subseteq \{1,2,3,4\}\} \text{ and } K \in \{2,3\}$ represents $\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\ldots,\{2,3,4\}\}$ so **9** sets

# 4- Stepping back, looking for a generic answer

*Set Variables*

- Length-Lex representation [Gervet and Van-Hentenryck, 2006]
  - Order the sets by cardinality and lexicographycally

$$\emptyset << \{1\} << \{2\} << \{3\} << \{1,2\} << \{1,3\} << \{2,3\} << \{1,2,3\}$$

  - The bounds are refined and encodes cardinality

$D(X) = \{S \mid m << S << M\}$ versus $D(X) = \{S \mid \underline{X} \subseteq S \subseteq \overline{X}\}$

  - Example with $D(X) = \{\{1,4\},\{2,3\},\{1,2,3\}\}$ so **3** sets

Classic: $D(X) = \{S \mid \emptyset \subseteq S \subseteq \{1,2,3,4\}\}$ and $K \in \{2,3\}$ represents $\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\ldots,\{2,3,4\}\}$ so **9** sets

Length-Lex: $D(X) = \{S \mid \{1,4\} << S << \{1,2,3\}\}$ represents $\{\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\}\}$ so **5** sets

# 4- Stepping back, looking for a generic answer

*MDD-consistency*

**Limit : “constraints communicate through domains”**

- All structural relationships among variables are projected onto the domains
- Potential solution space implicitly defined by Cartesian product of variable domains
- How to communicate more information between constraints ?
  - ➢Multi-valued Decision Diagram MDD consistency
    [Hooker, Hadzic, Van Hoeve, 2007]
  - ➢Explicit representation of more refined potential solution space
  - ➢Limited width defines **relaxation MDD**

(From Willem-Jan Tutorial)

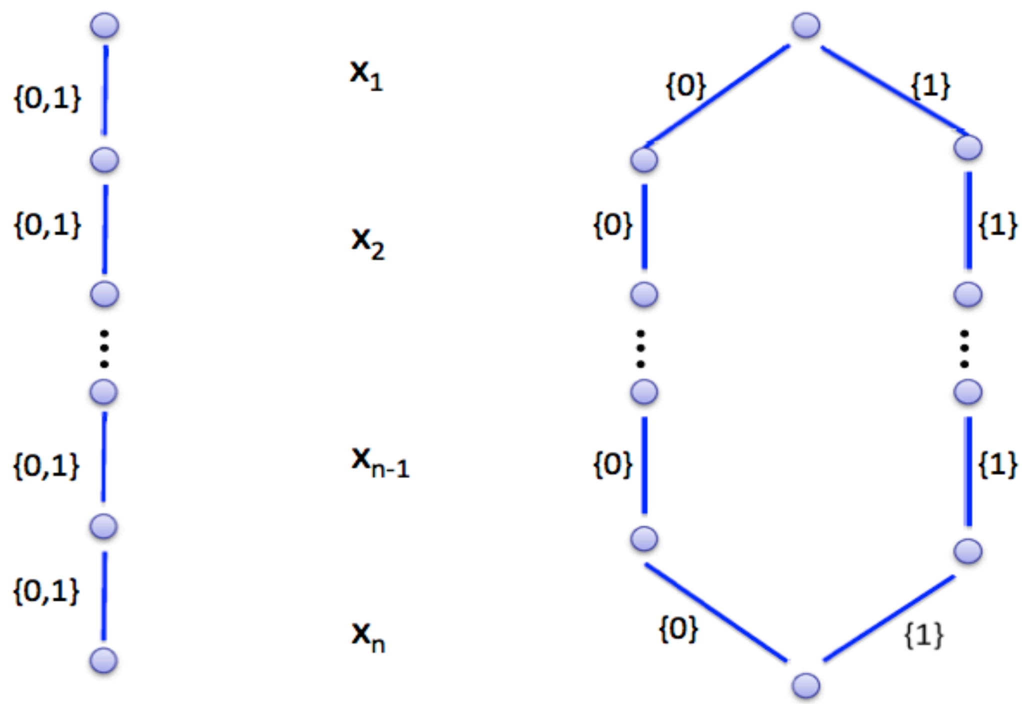
# 4- Stepping back, looking for a generic answer

*MDD-consistency*

**Limit : "constraints communicate through domains"**

*AllEqual*$(x_1, x_2, \ldots, x_n)$, all $x_i$ binary

$x_1 + x_2 + \ldots + x_n \geq n/2$




domain representation, size $2^n$

MDD representation, size 2

(From Willem-Jan Tutorial)

# Outline

1. Choice of domains, *granularity* of the model
2. Illustration on a *toy problem*
   – Pentaminoes
3. Illustration on a real application: multi-leaf sequencing
   – Direct model
   – Counter model
   – Path model
4. Stepping back, looking for a generic answer
   – Set variables
   – MDD consistency
5. Conclusion

# Conclusion

- A CP model reveals combinatorial structures of the problem:
  - TPP (p-median, hitting-set, k-TSP)
  - Matrix decomposition (shortest path with resource constraints)
- Find out the proper *granularity* where key-reasoning can be expressed
  - Find-out what the problem is made of (what structures) and only then, choose the variables/domains
- Modeling the objective function is often neglected
  - Relatively generic mechanisms that do not require an LP solver exist to reason about costs: Lagrangian Relaxation